Math 114, Spring 2017 **Renato Ghini Bettiol**

Homework Set 5

DUE: FEB 23, 2017 (IN CLASS)

1. Compute the following limits, or prove that they do not exist:

   (a) $\displaystyle\lim_{(x,y)\to(0,0)} \frac{x^4-y^4}{x^2-y^2}$

   (b) $\displaystyle\lim_{(x,y)\to(0,0)} \frac{x-y}{x^2+y^2}$

   (c) $\displaystyle\lim_{(x,y)\to(0,0)} \frac{x+y}{\sqrt{x^2+y^2}}$

   (d) $\displaystyle\lim_{(x,y)\to(0,0)} \frac{(x^2+y^2)\sin(x^2+y^2)}{x^4+y^4}$

2. Compute the following partial derivatives of the function $f(x,y,z)=x^2y+\sin(z^2-x)$

$$\frac{\partial f}{\partial x},\quad \frac{\partial f}{\partial y},\quad \frac{\partial f}{\partial z},\quad \frac{\partial^2 f}{\partial x^2},\quad \frac{\partial^2 f}{\partial y^2},\quad \frac{\partial^2 f}{\partial z^2},\quad \frac{\partial^2 f}{\partial x\partial y},\quad \frac{\partial^2 f}{\partial x\partial z},\quad \frac{\partial^2 f}{\partial y\partial z}$$

3. Let $z=f(x,y)$ be the function implicitly defined by $z^3+z=x^2+y^2$. Note that when $x=1$ and $y=1$, then $z=1$ as well. Compute $\frac{\partial f}{\partial x}(1,1)$.

4. Consider the parametrization $x(t)=e^{-t}\cos t$, $y(t)=e^{-t}\sin t$, of the logarithmic spiral. Let $f(x,y)=x+y$, and set $w(t)=f(x(t),y(t))$. Compute $w'(t)$ using the Chain Rule for functions of 2 variables, and then verify you obtained the correct answer by substituting and using the explicit formula for $w(t)$.

5. An ice block, in the format of a brick (more precisely, a *rectangular parallelepiped*), is left under the sun to melt. At a given instant, the lengths of its three sides a, b, and c, are $1cm$, $2cm$, and $3cm$. At that same time, you observe that each of the sides a, b, and c, is respectively shrinking at rates of $0.5cm/sec$, $1cm/sec$, and $3cm/sec$. At what rates are the *volume* and the *surface area* of this ice block changing at that instant?