HW 10 Solutions.

1. $\gamma(t) = (\cos(t), \sin(t)) \rightarrow d\gamma = (-\sin t, \cos t)\,dt \quad , t \in [-\frac{\pi}{2}, \frac{\pi}{2}]$

a. $\vec{F} = (x,y) \Rightarrow \vec{F}(t) = (\cos t, \sin t)$

$$\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} (\cos t, \sin t)\cdot(-\sin t, \cos t)\,dt = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} 0\,dt = 0$$

b. $\vec{F} = (x+y, 2xy-3) = (\sin t + \cos t, 2\sin t\cos t - 3)$

$$\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} (\sin t + \cos t, 2\cos t\sin t - 3)\cdot(-\sin t, \cos t)\,dt = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} (-\sin^2 t - \sin t\cos t + 2\cos^2 t\sin t - 3\cos t)\,dt =$$

$$= -\frac{\pi}{2} - 6$$

c. $\vec{F}(x,y) = (6xy - y^3, 3y^2 + 3x^2 - 3xy^2)$

$\vec{F}$ is conservative ; $\nabla f = \vec{F}$, $\int \vec{F}\cdot dr = f(\text{end}) - f(\text{st})$

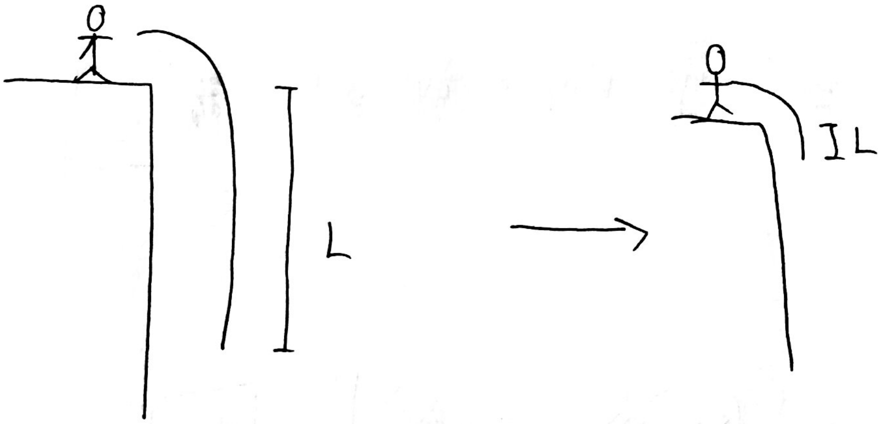
$f = 3x^2y - xy^3 + y^3$

$\gamma(-\frac{\pi}{2}) = (0,-1),\ \gamma(\frac{\pi}{2}) = (0,1)$

$f(0,1) - f(0,-1) = 2$

2. $W = \int \vec{F}\cdot dr$

$\vec{F} = m\vec{a} = \rho L\vec{a} \Rightarrow \vec{F} = \vec{F}(L)$; $\vec{F}$ is a function of L. $\Rightarrow \bar{F} = 2*L*10 = 20L$



shorter the rope, smaller the mass

$$W = \int F*dr = \int 20L\,dr \Rightarrow dr = dL = \int_{L_i}^{L_f} 20L\,dL = \int_{50}^{30} 20L\,dL =$$

$$= 16000\text{ J}$$

3. $\vec{F} = \nabla \varphi$; $\varphi(x,y,z) = x^2 + y^2 + e^z$

* Field is conservative ; $\int \vec{F} \cdot d\vec{r} = \varphi(\text{end}) - \varphi(\text{start})$

a. $I_1 = f(1,1,1) - f(0,0,0) = (2+e) - (1) = 1+e$

b. $I_2 = f(2,0,1) - f(1,1,1) = (4+e) - (2+e) = 2$

c. $I_3 = f(0,0,0) - f(2,0,1) = (1) - (4+e) = -3-e$

$S = I_1 + I_2 + I_3 = 0 \Rightarrow \oint \vec{F} \cdot dr$ for closed curve of conservative field must be 0

4. $\vec{F} = (x,y,z)$

$f = \frac{1}{2}x^2 + \frac{1}{2}y^2 + \frac{1}{2}z^2$

b. $\vec{F} = (e^{xz}, x+y+z, 1)$

Not conservative

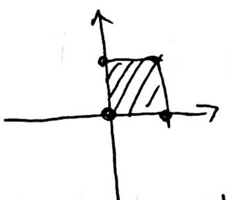
c. $\vec{F} = (2xy, x^2 + \cos(y), 0)$

$f = x^2 y + \sin(y)$

d. $\vec{F} = (z^2, x+y, 4\sin(xz))$

Not conservative

5. $\int_\gamma x^2 y\,dx + xy^3\,dy$



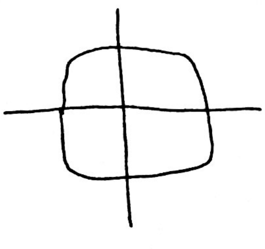
$\parallel$

$$\int_0^1 \int_0^1 (y^3 - x^2)\,dx\,dy = \int_0^1 y^3 x - \frac{1}{3}x^3 \Big|_0^1 = \int_0^1 (y^3 - 1/3)\,dy = \frac{1}{4}y^4 - \frac{1}{3}y \Big|_0^1 = \boxed{\frac{-1}{12}}$$

6. $\int (x+2y)\,dx + (x-2y)\,dy$

$$\iint_A (1 - 2)\,dA = -1 \int_0^1 \int_{x^2}^{x} dy\,dx = -\int_0^1 (x - x^2)\,dx = -\left(\frac{1}{2}x^2 - \frac{1}{3}x^3\right)\Big|_0^1 = \boxed{\frac{-1}{6}}$$

c. $\int_\gamma x^2 dx + y^2 dy$



$x^6 + y^6 = 1$
closed curve

$\iint 0\,dA = \boxed{0}$