

HW 4 Solutions

1. $r(t) = (2t, t^2, \ln t) \quad 1 \leq t \leq e$

$r'(t) = (2, 2t, \frac{1}{t})$

$S = \int_1^e \sqrt{4 + 4t^2 + \frac{1}{t^2}} dt = \int_1^e (2t + \frac{1}{t}) dt = t^2 + \ln t \Big|_1^e = (e^2 + 1) - (1 + 0) = e^2$

$\kappa = \frac{|r' \times r''|}{|r'|^3}$

$r''(t) = (0, 2, -\frac{1}{t^2})$

$r' \times r'' = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 2t & \frac{1}{t} \\ 0 & 2 & -\frac{1}{t^2} \end{vmatrix} = (-2t^2 - 2t \cdot \frac{2}{t^2}, 4) = (-\frac{4}{t}, \frac{2}{t^2}, 4)$

$|r' \times r''| = \sqrt{\frac{16}{t^2} + \frac{4}{t^4} + 16} = \sqrt{\frac{1}{t^4}(16t^2 + 4 + 16t^4)} = \frac{1}{t^2}(4t^2 + 2) = 4 + \frac{2}{t^2}$

$|r'| = 2t + \frac{1}{t}$

$\kappa = \frac{4 + \frac{2}{t^2}}{(2t + \frac{1}{t})^3}$

5. (2,3) (4,-1)

$y = mx + b$

$m = \frac{3 - (-1)}{2 - 4} = -2$

$y = -2x + 7$

$x = \cos \theta ; y = \sin \theta$

$\sin \theta = -2 \cos \theta + 7$

$r = \frac{7}{\sin \theta + 2 \cos \theta}$

2. $r(t) = (t, e^t, t^2)$

$T(t) = \frac{r'(t)}{|r'(t)|}$

$r'(t) = (1, e^t, 2t)$

$|r'(t)| = \sqrt{1 + e^{2t} + 4t^2}$

$\therefore T(t) = \frac{1}{\sqrt{1 + e^{2t} + 4t^2}} (1, e^t, 2t)$

3. $R = 40 \text{ yds}$

$V_0 = 30 \text{ yds/s}$

$R = \frac{V_0^2 \sin(2\theta)}{g} \Rightarrow 40 = \frac{30^2 \sin(2\theta)}{11.25}$

$\frac{1}{2} = \sin(2\theta)$

$2\theta = \frac{\pi}{6} \Rightarrow \theta = \frac{\pi}{12}$

$40 = V_0 \cos \theta \times t$

$t = \frac{40}{30 \cos(\frac{\pi}{12})} = \frac{40}{30} \frac{4}{\sqrt{2} + \sqrt{6}} = \frac{80}{15(\sqrt{2} + \sqrt{6})}$

4. $r(t) = (3 \sin t, 2 - 2t, 3 \cos t)$

$r'(t) = (3 \cos t, -2, -3 \sin t)$

$r''(t) = (-3 \sin t, 0, -3 \cos t)$

$\Rightarrow |r'(t)| = \sqrt{9 \cos^2 t + 4 + 9 \sin^2 t} = \sqrt{13}$

$a_T = \frac{d}{dt} |r'(t)| = 0 \Rightarrow a_T = 0$

$a_N = \sqrt{|a|^2 - |a_T|^2} = \sqrt{9 - 0} = 3$