Homework Set 5

DUE: MAR 2, 2016 (IN CLASS)

1. Bretscher Section 5.1: Exercises 32, 33

2. Bretscher Section 5.2: Exercises 32, 33, 36

3. Bretscher Section 5.3: Exercises 45, 46, 60, 61

4. Bretscher Section 5.5: Exercises 10, 17, 18, 30

5. Let $V = \{A \in M_{2\times 2}(\mathbb{R}) : \langle A, \mathrm{Id}\rangle_{HS} = 0\}$ be the subspace of 2×2 real matrices that are orthogonal to the identity matrix with respect to the Hilbert-Schmidt inner product $\langle A, B\rangle_{HS} = \mathrm{tr}(AB^t)$. Find an orthonormal basis of $(V, \langle\cdot,\cdot\rangle_{HS})$.

6. Given $A \in M_{n\times n}(\mathbb{R})$, recall that the *operator norm* of A is given by
$$\|A\| = \max_{\|v\|=1} \|Av\|,$$
and its Hilbert-Schmidt norm is $\|A\|_{HS} = \sqrt{\langle A, A\rangle_{HS}}$. Prove that $\|A\| \leq \|A\|_{HS}$ for all diagonal matrices A. When does equality hold?

7. CHALLENGE (2 BONUS PTS) Prove that $\|A\| \leq \|A\|_{HS}$ for *any* matrix $A \in M_{n\times n}(\mathbb{R})$, and also characterize when equality holds.