

# MATH661 DIFFERENTIAL GEOMETRY II

## SYLLABUS

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Meeting coordinates: Tuesdays and Thursdays, 3pm-4:30pm at DRL 4C4

This course will focus on two central areas of modern Differential Geometry:

- (1) *Lie groups and isometric actions.* We will develop the basic theory of compact Lie groups and their actions on manifolds. Topics to be covered include the fundamental theorems relating Lie groups and their Lie algebras, bi-invariant Riemannian metrics, Killing form, maximal tori, roots, Weyl group, Dynkin diagrams, isometric and proper actions, slice theorem, principal and associated bundles, stratification by orbit types, and homogeneous spaces. The basic reference for this material is [1, Ch. 1-4]. Individual copies of these chapters will be provided for everyone.
- (2) *Comparison geometry.* The central results to be discussed are the comparison theorems of Rauch and Toponogov for sectional curvature, and Bishop-Gromov and Heintze-Karcher for Ricci curvature, together with some applications and related results, including the Grove-Shiohama diameter sphere theorem, Gromoll-Meyer soul theorem, and Gromov's theorem on Betti numbers. The main reference for these topics is [2], besides the original research articles containing the aforementioned results.
- (3) *Extra topics (time permitting).* According to the interests of the audience, if time permits, we might also discuss topics including: cohomogeneity one manifolds, manifolds with positive curvature, general Weitzenböck formulas and the Bochner technique, spectral geometry of the Laplacian, etc.

There will be no assignments/exams for this course; instead, the plan is to point out open research problems connected to the material and discuss recent progress.

### REFERENCES

- [1] M. M. ALEXANDRINO AND R. G. BETTIOL, *Lie Groups and Geometric Aspects of Isometric Actions*. Springer, Cham, 2015. ISBN: 978-3-319-16612-4; 978-3-319-16613-1
- [2] J. CHEEGER AND D. G. EBIN, *Comparison theorems in Riemannian geometry*. Revised reprint of the 1975 original. AMS Chelsea Publishing, Providence, RI, 2008. ISBN: 978-0-8218-4417-5