GRAD STUDENT GEOMETRY AND TOPOLOGY SEMINAR
SPRING 2017 PROGRAM

Meeting coordinates: Mondays, 3.15pm-4:15pm at DRL 3C2

The topics we elected to cover this semester, not necessarily in this order, are:

(1) 4**-manifolds with positive curvature and continuous symmetry**, following Hsiang and Kleiner [HK89]. The main result is that a closed simply-connected Riemannian 4-manifold M with sec > 0 and continuous isometry group must be homeomorphic to S^4 or $\mathbb{C}P^2$, which is proved by showing that $\chi(M) \leq 3$. A possible follow-up is the refinement of Grove and Wilking [GW14], that improves the conclusion from *homeomorphism* to *equivariant diffeomorphism* through different techniques. In both results, a version for sec ≥ 0 is also available, where the list of manifolds is expanded to include $S^2 \times S^2$ and $\mathbb{C}P^2 \# \pm \mathbb{C}P^2$.

(2) **Curvature homogeneous spaces**. These are Riemannian manifolds such that at all points the curvature operator is "the same", even though the manifold need not be isometric to a homogeneous space. We will hear about results on such manifolds, and relations with nonnegativity of curvature and low co-nullity for the curvature tensor.

(3) **Constant scalar curvature Kähler metrics on fibered complex surfaces**, following Fine [Fin04]. This paper uses singular perturbation methods to construct constant scalar curvature Kähler metrics on certain surface bundles over surfaces.

(4) **Factoring totally geodesic maps**, following Vilms [Vil70]. A smooth map between manifolds that takes geodesics to geodesics factors as a composition of a totally geodesic Riemannian submersion and a totally geodesic immersion.

(5) **Closed geodesics on noncompact manifolds**, following [Ban80] and [AM]. These papers address the existence of infinitely many closed geodesics on noncompact manifolds: the case of surfaces is a classic result of [Ban80], while the higher dimensional case is studied in [AM] adapting ideas of Gromoll and Meyer [GM69] to the noncompact realm.

(6) **Trisections of** 4**-manifolds**, following mainly Gay and Kirby [GK16] and Abrams, Gay, and Kirby [AGK], as well as [MSZ16, MZb, MZa]. This recently developed theory is a 4-dimensional analog of Heegaard splittings for 3-manifolds, which establishes a bijective correspondence between closed connected oriented 4-manifolds modulo diffeomorphisms and group trisections modulo isomorphisms and stabilizations. Since it provides complete invariants for the diffeomorphism type of 4-manifolds, it allows to rewrite statements such as the smooth 4-dimensional Poincaré conjecture in purely group-theoretic terms. Some of our goals are to:

- Develop a working knowledge of trisections, through a detailed study of the foundational papers [GK16, AGK] and working out examples (such as trisections corresponding to S^4, $\mathbb{C}P^2$, $S^2 \times S^2$, and $\mathbb{C}P^2 \# \pm \mathbb{C}P^2$);
- Understand how to relate the trisection of a 4-manifold and its intersection form via purely algebraic procedures. In particular, try to exhibit (nonequivalent) trisections that correspond to different smooth structures on the same topological 4-manifold;
- Connections with Geometry: Search for new topological obstructions to curvature conditions (e.g., $\sec > 0$) that are manifested in terms of trisections. Conversely, produce criteria to ensure that 4-manifolds with certain trisections admit metrics with certain curvature conditions (e.g., by "gluing" the trisected pieces).

(7) **Kähler Ricci flow** and **Cohomogeneity one Ricci flow**. Review the basic properties of these geometric flows and the literature on long term behavior and singularity models. In particular, understand the status of classification efforts in low dimensions.

References

[AGK] A. Abrams, D. Gay, and R. Kirby. *Group trisections and smooth 4-manifolds.* arXiv:1605.06731.

[AM] L. Asselle and M. Mazzucchelli. *On the existence of infinitely many closed geodesics on non-compact manifolds.* arXiv:1602.03679.

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[Fin04] J. Fine. *Constant scalar curvature Kähler metrics on fibred complex surfaces.* J. Differential Geom., 68 (2004), 397–432.

[GK16] D. Gay and R. Kirby. *Trisecting 4-manifolds.* Geom. Topol., 20 (2016), 3097–3132.

[GM69] D. Gromoll and W. Meyer. *Periodic geodesics on compact Riemannian manifolds.* J. Differential Geometry, 3 (1969), 493–510.

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[MSZ16] J. Meier, T. Schirmer, and A. Zupan. *Classification of trisections and the generalized property R conjecture.* Proc. Amer. Math. Soc., 144 (2016), 4983–4997.

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[MZb] J. Meier and A. Zupan. *Genus two trisections are standard.* arXiv:1410.8133.

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