HW 5 # 6

$$\phi'' + (2-4x)\phi' + \lambda\phi = 0, \qquad \phi(0) = \phi(1) = 0$$

a) Integrating factor is $e^{\int 2-4x\, dx} = e^{2x-2x^2}$

$$\underbrace{e^{2x-2x^2}\phi'' + (2-4x)e^{2x-2x^2}\phi'} + \lambda\phi e^{2x-2x^2} = 0$$

$$\left(e^{2x-2x^2}\phi'\right)' + \lambda\phi e^{2x-2x^2} = 0$$

is in Sturm-Liouville Form $\left(p = e^{2x-2x^2},\ q = 0,\ \sigma = e^{2x-2x^2}\right)$.

b). $\phi(x) = x(1-x)$ is an eigenfunction

$\phi(x) = x - x^2$

$\phi'(x) = 1 - 2x$

$\phi''(x) = -2$

So, if we plug in the equation:

$$\phi'' + (2-4x)\phi' + \lambda\phi =$$

$$= -2 + (2-4x)(1-2x) + \lambda(x-x^2)$$

$$= -2 + 2 - 4x - 4x + 8x^2 + \lambda(x-x^2)$$

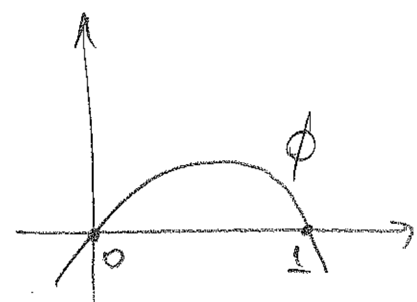
$$= 8(x^2-x) - \lambda(x^2-x)$$

$$= (8-\lambda)(x^2-x) = 0 \text{ if } \underline{\lambda = 8}.$$

So we see that $\phi(x) = x(1-x)$ is an eigenfunction of this Sturm-Liouville problem, with eigenvalue $\lambda = 8$.

c) Since $\phi(x) = x(1-x)$ has no zeroes in the open interval $(0,1)$, this is the eigenfunction associated to the first (smallest) eigenvalue.

Thus $\lambda_1 = 8$.



[Recall that on a regular Sturm-Liouville prob. the eigenfunction ϕ_n associated with the n^{th} eigenvalue λ_n has $(n-1)$ zeroes in (a,b).]