Department of Mathematics
University of Notre Dame
Math 20580 - Spring 2013

Version #1

Name: ____________________

Section: ____________________

Exam 2

March 19, 2013

This exam is in 2 parts on 10 pages and contains 15 problems worth a total of 100 points.You have 1 hour and 15 minutes to work on it. No calculators, books, notes, or other aids are allowed except for one piece of paper you brought with you. Be sure to write your name on this page and to put your initials at the top of every page in case pages become detached. *On the partial credit problems you must show your work and all important steps to receive credit.* Good luck!

Honor Pledge: As a member of the Notre Dame community, I will not participate in nor tolerate academic dishonesty.

Signature:__________________________________

Mark your answers to the multiple choice problems here.
Place an × through your answer to each problem.

1.	(a)	(b)	(•)	(d)	(e)
2.	(a)	(b)	(c)	(d)	(•)
3.	(•)	(b)	(c)	(d)	(e)
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9.	(a)	(•)	(c)	(d)	(e)
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11.	(a)	(b)	(•)	(d)	(e)
12.	(•)	(b)	(c)	(d)	(e)

Please do not write below this line.

MC. __________

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1.(5pts) Let A, B, C be 2×2 matrices with $\det(A) = -2$, $\det(B) = 5$, and $\det(C) = \dfrac{1}{3}$.
What is $\det\Big((A^{-1}B)^T C^{-1}\Big)$?

Note: Remember that $(A^{-1}B)^T$ means the **transpose** of $(A^{-1}B)$.

(a) -30 (b) $\dfrac{15}{2}$ (c) $-\dfrac{15}{2}$ (d) $\dfrac{6}{5}$ (e) $-\dfrac{6}{5}$

2.(5pts) Mary wants to solve the matrix equation $A\mathbf{x} = \mathbf{b}$ in her father's thirty-year-old notebook. However, due to the ink stain on the notebook, she can only read the partial information

$$A = \begin{bmatrix} \blacksquare & 0 & -3 \\ \blacksquare & 5 & 1 \\ 1 & 2 & 3 \end{bmatrix}, \ \det(A) = -4, \ \mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}, \text{ and } \mathbf{b} = \begin{bmatrix} 1 \\ 0 \\ -3 \end{bmatrix}.$$

($\blacksquare$ means the ink stain.) Although she cannot solve the whole system, she observed that she can still find x_1 using Cramer's rule. What is x_1?

(a) 0 (b) 4 (c) -8 (d) -4 (e) 8

3.(5pts) If the row space of a 7×4 matrix A is of dimension 2, then what is the dimension of the null space of A^T (the **transpose** of A)?

(a) 5 (b) 3 (c) 1 (d) 4 (e) 2

4.(5pts) Let $\mathbb{P}_n$ be the vector space of all polynomials of degree at most n. Which of the following statement is **NOT true**?

(a) $\mathbb{P}_5$ is a subspace of $\mathbb{P}_9$.

(b) If $T : \mathbb{P}_5 \to \mathbb{R}^2$ is a linear transformation, then $\ker(T)$ is a subspace of $\mathbb{R}^2$.

(c) The function $D : \mathbb{P}_{10} \to \mathbb{P}_{10}$ defined by $D(f(t)) = f'(t)$ (the derivative of f) is a linear transformation.

(d) The set of $f(t)$ in $\mathbb{P}_3$ such that $f(-1) = f(2)$ is a subspace of $\mathbb{P}_3$.

(e) The function $T : \mathbb{P}_2 \to \mathbb{R}^2$ defined by $T(f(t)) = \begin{bmatrix} f(0) \\ f(-2) \end{bmatrix}$ is a linear transformation.

5.(5pts) The set $\mathcal{B} = \{1, 1-t, 1-t-t^2\}$ forms a basis for the vector space $\mathbb{P}_2$. Find the coordinate vector $[p]_{\mathcal{B}}$ of $p(t) = 4t^2 + 5t - 2$.

(a) $\begin{bmatrix} 3 \\ 1 \\ 4 \end{bmatrix}$ (b) $\begin{bmatrix} -4 \\ 1 \\ 3 \end{bmatrix}$ (c) $\begin{bmatrix} 4 \\ 5 \\ -2 \end{bmatrix}$ (d) $\begin{bmatrix} 3 \\ -1 \\ -4 \end{bmatrix}$ (e) $\begin{bmatrix} -2 \\ 5 \\ 4 \end{bmatrix}$

6.(5pts) Which of the following statements is **false** for a **non-invertible**, $n \times n$ matrix A?

(a) $\det(A) = 0$.

(b) The row space of A is $\mathbb{R}^n$.

(c) A^T is not invertible.

(d) $\mathrm{Nul}(A) \neq \{\mathbf{0}\}$.

(e) $\lambda = 0$ is an eigenvalue of A.

7.(5pts) Suppose that $\mathcal{B} = \{\mathbf{b}_1, \mathbf{b}_2\}$ and $\mathcal{C} = \{\mathbf{c}_1, \mathbf{c}_2\}$ are two bases for a vector space V, and the change-of-coordinate matrix **from** $\mathcal{B}$ **to** $\mathcal{C}$ is given as $\underset{\mathcal{C}\leftarrow\mathcal{B}}{P} = \begin{bmatrix} 2 & -3 \\ -1 & 2 \end{bmatrix}$.

For $\mathbf{v} = 2\mathbf{b}_1 + \mathbf{b}_2$, what is $[\mathbf{v}]_{\mathcal{C}}$, the $\mathcal{C}$-coordinate for $\mathbf{v}$?

(a) $\begin{bmatrix} 0 \\ 1 \end{bmatrix}$ (b) $\begin{bmatrix} -3 \\ 2 \end{bmatrix}$ (c) $\begin{bmatrix} 2 \\ 1 \end{bmatrix}$ (d) $\begin{bmatrix} 1 \\ 0 \end{bmatrix}$ (e) $\begin{bmatrix} 2 \\ -1 \end{bmatrix}$

8.(5pts) The linear transformation $T : \mathbb{R}^2 \to \mathbb{R}^2$ is defined as $T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} x - y \\ 2x + 3y \end{bmatrix}$.

For the basis $\mathcal{B} = \left\{ \begin{bmatrix} 1 \\ -1 \end{bmatrix}, \begin{bmatrix} 2 \\ -1 \end{bmatrix} \right\}$ of $\mathbb{R}^2$, what is $[T]_{\mathcal{B}}$ (the $\mathcal{B}$-matrix of T)?

(a) $\begin{bmatrix} 1 & 2 \\ -1 & 3 \end{bmatrix}$ (b) $\begin{bmatrix} 1 & 2 \\ -1 & -1 \end{bmatrix}$ (c) $\begin{bmatrix} 0 & 1 \\ -5 & 4 \end{bmatrix}$ (d) $\begin{bmatrix} 0 & -4 \\ 1 & -5 \end{bmatrix}$ (e) $\begin{bmatrix} 0 & -5 \\ 1 & 4 \end{bmatrix}$

9.(5pts) The vector $\begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$ is an eigenvector of the matrix $\begin{bmatrix} 0 & 5 & 2 \\ -1 & 6 & 2 \\ 3 & -3 & 7 \end{bmatrix}$.

What is the corresponding eigenvalue?

(a) 0 (b) 5 (c) 7 (d) 3 (e) 1

10.(5pts) Find the eigenvalues of the matrix $\begin{bmatrix} 0 & -2 \\ 5 & 2 \end{bmatrix}$.

(a) $1+3i$ and $1-3i$. (b) 5 and -2. (c) 0 and 2.

(d) $1+6i$ and $1-6i$. (e) $1+\sqrt{11}$ and $1-\sqrt{11}$.

 Initials: ____________

11.(5pts) Which of the following vector is orthogonal to the vector $\begin{bmatrix} -1 \\ 6 \\ -2 \end{bmatrix}$?

(a) $\begin{bmatrix} \sqrt{6} \\ 1 \\ -\sqrt{3} \end{bmatrix}$ (b) $\begin{bmatrix} 0 \\ 1 \\ -3 \end{bmatrix}$ (c) $\begin{bmatrix} 2\sqrt{5} \\ 0 \\ -\sqrt{5} \end{bmatrix}$ (d) $\begin{bmatrix} 6 \\ -1 \\ -3 \end{bmatrix}$ (e) $\begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix}$

12.(5pts) If $A = \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}^{-1} \begin{bmatrix} -1 & 0 \\ 0 & \frac{1}{2} \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$, what is the $(2,1)$-entry of A^5 ?

(a) $-\dfrac{33}{32}$ (b) $\dfrac{1}{32}$ (c) $-\dfrac{31}{32}$ (d) $\dfrac{33}{32}$ (e) $\dfrac{31}{32}$

13.(10pts) The matrix $A = \begin{bmatrix} 2 & -1 & 4 & 4 & 3 \\ 3 & 0 & 3 & 2 & -7 \\ 5 & -1 & 7 & 6 & -4 \\ 1 & 1 & -1 & 5 & -3 \end{bmatrix}$ is row equivalent to $\begin{bmatrix} 1 & 0 & 1 & 0 & -3 \\ 0 & 1 & -2 & 0 & -5 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$.

(You do **not** need to check for this.)

(a) Find a basis for the row space of A.

(b) Find a basis for the column space of A.

(c) Find a basis for the null space of A.

14.(10pts) For the matrix

$$A = \begin{bmatrix} -1 & 6 & -2 \\ 0 & 2 & 5 \\ 0 & 0 & 3 \end{bmatrix},$$

find an invertible matrix P and a diagonal matrix D such that $A = PDP^{-1}$.

Note: You do **not** need to calculate P^{-1}

15.(10pts) Let $\mathcal{B} = \left\{ \begin{bmatrix} 7 \\ -4 \end{bmatrix}, \begin{bmatrix} 6 \\ 1 \end{bmatrix} \right\}$ and $\mathcal{C} = \left\{ \begin{bmatrix} 5 \\ -2 \end{bmatrix}, \begin{bmatrix} 3 \\ -1 \end{bmatrix} \right\}$ be two basis of $\mathbb{R}^2$.

Find the change-of-coordinate matrix from $\mathcal{B}$ to $\mathcal{C}$, i.e., $\underset{\mathcal{C} \leftarrow \mathcal{B}}{P}$.

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2.(5pts) If the row space of a 7×4 matrix A is of dimension 2, then what is the dimension of the null space of A^T (the **transpose** of A)?

(a) 3 (b) 1 (c) 2 (d) 4 (e) 5

3.(5pts) Let $\mathbb{P}_n$ be the vector space of all polynomials of degree at most n. Which of the following statement is **NOT true**?

(a) If $T : \mathbb{P}_5 \to \mathbb{R}^2$ is a linear transformation, then $\ker(T)$ is a subspace of $\mathbb{R}^2$.

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(e) $\mathbb{P}_5$ is a subspace of $\mathbb{P}_9$.

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(b) $\det(A) = 0$.

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(a) $-\dfrac{31}{32}$ (b) $\dfrac{33}{32}$ (c) $\dfrac{31}{32}$ (d) $-\dfrac{33}{32}$ (e) $\dfrac{1}{32}$

7.(5pts) Let A, B, C be 2×2 matrices with $\det(A) = -2$, $\det(B) = 5$, and $\det(C) = \dfrac{1}{3}$.
What is $\det\Big((A^{-1}B)^T C^{-1}\Big)$?

Note: Remember that $(A^{-1}B)^T$ means the **transpose** of $(A^{-1}B)$.

(a) $-\dfrac{6}{5}$ (b) $\dfrac{15}{2}$ (c) -30 (d) $\dfrac{6}{5}$ (e) $-\dfrac{15}{2}$

8.(5pts) Mary wants to solve the matrix equation $A\mathbf{x} = \mathbf{b}$ in her father's thirty-year-old notebook. However, due to the ink stain on the notebook, she can only read the partial information

$$A = \begin{bmatrix} \blacksquare & 0 & -3 \\ \blacksquare & 5 & 1 \\ 1 & 2 & 3 \end{bmatrix}, \ \det(A) = -4, \ \mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}, \text{ and } \mathbf{b} = \begin{bmatrix} 1 \\ 0 \\ -3 \end{bmatrix}.$$

($\blacksquare$ means the ink stain.) Although she cannot solve the whole system, she observed that she can still find x_1 using Cramer's rule. What is x_1?

(a) -8 (b) 8 (c) 0 (d) -4 (e) 4

9.(5pts) The set $\mathcal{B} = \{1, 1-t, 1-t-t^2\}$ forms a basis for the vector space $\mathbb{P}_2$. Find the coordinate vector $[p]_{\mathcal{B}}$ of $p(t) = 4t^2 + 5t - 2$.

(a) $\begin{bmatrix} 3 \\ 1 \\ 4 \end{bmatrix}$ (b) $\begin{bmatrix} 3 \\ -1 \\ -4 \end{bmatrix}$ (c) $\begin{bmatrix} -2 \\ 5 \\ 4 \end{bmatrix}$ (d) $\begin{bmatrix} -4 \\ 1 \\ 3 \end{bmatrix}$ (e) $\begin{bmatrix} 4 \\ 5 \\ -2 \end{bmatrix}$

10.(5pts) The linear transformation $T : \mathbb{R}^2 \to \mathbb{R}^2$ is defined as $T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} x - y \\ 2x + 3y \end{bmatrix}$.

For the basis $\mathcal{B} = \left\{\begin{bmatrix} 1 \\ -1 \end{bmatrix}, \begin{bmatrix} 2 \\ -1 \end{bmatrix}\right\}$ of $\mathbb{R}^2$, what is $[T]_{\mathcal{B}}$ (the $\mathcal{B}$-matrix of T)?

(a) $\begin{bmatrix} 1 & 2 \\ -1 & -1 \end{bmatrix}$ (b) $\begin{bmatrix} 1 & 2 \\ -1 & 3 \end{bmatrix}$ (c) $\begin{bmatrix} 0 & -5 \\ 1 & 4 \end{bmatrix}$ (d) $\begin{bmatrix} 0 & -4 \\ 1 & -5 \end{bmatrix}$ (e) $\begin{bmatrix} 0 & 1 \\ -5 & 4 \end{bmatrix}$

11.(5pts) Find the eigenvalues of the matrix $\begin{bmatrix} 0 & -2 \\ 5 & 2 \end{bmatrix}$.

(a) $1+6i$ and $1-6i$. (b) $1+3i$ and $1-3i$. (c) $1+\sqrt{11}$ and $1-\sqrt{11}$.

(d) 0 and 2. (e) 5 and -2.

12.(5pts) Which of the following vector is orthogonal to the vector $\begin{bmatrix} -1 \\ 6 \\ -2 \end{bmatrix}$?

(a) $\begin{bmatrix} 2\sqrt{5} \\ 0 \\ -\sqrt{5} \end{bmatrix}$ (b) $\begin{bmatrix} \sqrt{6} \\ 1 \\ -\sqrt{3} \end{bmatrix}$ (c) $\begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix}$ (d) $\begin{bmatrix} 6 \\ -1 \\ -3 \end{bmatrix}$ (e) $\begin{bmatrix} 0 \\ 1 \\ -3 \end{bmatrix}$

13.(10pts) The matrix $A = \begin{bmatrix} 2 & -1 & 4 & 4 & 3 \\ 3 & 0 & 3 & 2 & -7 \\ 5 & -1 & 7 & 6 & -4 \\ 1 & 1 & -1 & 5 & -3 \end{bmatrix}$ is row equivalent to $\begin{bmatrix} 1 & 0 & 1 & 0 & -3 \\ 0 & 1 & -2 & 0 & -5 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$.

(You do **not** need to check for this.)

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14.(10pts) For the matrix

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find an invertible matrix P and a diagonal matrix D such that $A = PDP^{-1}$.

Note: You do **not** need to calculate P^{-1}

 Initials: ____________

15.(10pts) Let $\mathcal{B} = \left\{ \begin{bmatrix} 7 \\ -4 \end{bmatrix}, \begin{bmatrix} 6 \\ 1 \end{bmatrix} \right\}$ and $\mathcal{C} = \left\{ \begin{bmatrix} 5 \\ -2 \end{bmatrix}, \begin{bmatrix} 3 \\ -1 \end{bmatrix} \right\}$ be two basis of $\mathbb{R}^2$.

Find the change-of-coordinate matrix from $\mathcal{B}$ to $\mathcal{C}$, i.e., $\underset{\mathcal{C} \leftarrow \mathcal{B}}{P}$.

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Version #3

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14. __________

15. __________

1.(5pts) The set $\mathcal{B} = \{1, 1-t, 1-t-t^2\}$ forms a basis for the vector space $\mathbb{P}_2$. Find the coordinate vector $[p]_{\mathcal{B}}$ of $p(t) = 4t^2 + 5t - 2$.

(a) $\begin{bmatrix} 3 \\ 1 \\ 4 \end{bmatrix}$ (b) $\begin{bmatrix} 4 \\ 5 \\ -2 \end{bmatrix}$ (c) $\begin{bmatrix} 3 \\ -1 \\ -4 \end{bmatrix}$ (d) $\begin{bmatrix} -2 \\ 5 \\ 4 \end{bmatrix}$ (e) $\begin{bmatrix} -4 \\ 1 \\ 3 \end{bmatrix}$

2.(5pts) Let A, B, C be 2×2 matrices with $\det(A) = -2$, $\det(B) = 5$, and $\det(C) = \frac{1}{3}$. What is $\det\Big((A^{-1}B)^T C^{-1}\Big)$?

Note: Remember that $(A^{-1}B)^T$ means the **transpose** of $(A^{-1}B)$.

(a) -30 (b) $-\frac{15}{2}$ (c) $\frac{15}{2}$ (d) $\frac{6}{5}$ (e) $-\frac{6}{5}$

3.(5pts) Which of the following vector is orthogonal to the vector $\begin{bmatrix} -1 \\ 6 \\ -2 \end{bmatrix}$?

(a) $\begin{bmatrix} 2\sqrt{5} \\ 0 \\ -\sqrt{5} \end{bmatrix}$ (b) $\begin{bmatrix} \sqrt{6} \\ 1 \\ -\sqrt{3} \end{bmatrix}$ (c) $\begin{bmatrix} 0 \\ 1 \\ -3 \end{bmatrix}$ (d) $\begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix}$ (e) $\begin{bmatrix} 6 \\ -1 \\ -3 \end{bmatrix}$

4.(5pts) If the row space of a 7×4 matrix A is of dimension 2, then what is the dimension of the null space of A^T (the **transpose** of A)?

(a) 1 (b) 5 (c) 4 (d) 2 (e) 3

5.(5pts) The linear transformation $T : \mathbb{R}^2 \to \mathbb{R}^2$ is defined as $T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} x - y \\ 2x + 3y \end{bmatrix}$.

For the basis $\mathcal{B} = \left\{ \begin{bmatrix} 1 \\ -1 \end{bmatrix}, \begin{bmatrix} 2 \\ -1 \end{bmatrix} \right\}$ of $\mathbb{R}^2$, what is $[T]_{\mathcal{B}}$ (the $\mathcal{B}$-matrix of T)?

(a) $\begin{bmatrix} 1 & 2 \\ -1 & -1 \end{bmatrix}$ (b) $\begin{bmatrix} 0 & -4 \\ 1 & -5 \end{bmatrix}$ (c) $\begin{bmatrix} 1 & 2 \\ -1 & 3 \end{bmatrix}$ (d) $\begin{bmatrix} 0 & -5 \\ 1 & 4 \end{bmatrix}$ (e) $\begin{bmatrix} 0 & 1 \\ -5 & 4 \end{bmatrix}$

6.(5pts) Find the eigenvalues of the matrix $\begin{bmatrix} 0 & -2 \\ 5 & 2 \end{bmatrix}$.

(a) 0 and 2. (b) $1 + 6i$ and $1 - 6i$. (c) 5 and -2.

(d) $1 + \sqrt{11}$ and $1 - \sqrt{11}$. (e) $1 + 3i$ and $1 - 3i$.

Initials: ____________

7.(5pts) The vector $\begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$ is an eigenvector of the matrix $\begin{bmatrix} 0 & 5 & 2 \\ -1 & 6 & 2 \\ 3 & -3 & 7 \end{bmatrix}$.

What is the corresponding eigenvalue?

(a) 0 (b) 5 (c) 1 (d) 7 (e) 3

8.(5pts) Mary wants to solve the matrix equation $A\mathbf{x} = \mathbf{b}$ in her father's thirty-year-old notebook. However, due to the ink stain on the notebook, she can only read the partial information

$$A = \begin{bmatrix} \blacksquare & 0 & -3 \\ \blacksquare & 5 & 1 \\ 1 & 2 & 3 \end{bmatrix}, \ \det(A) = -4, \ \mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}, \text{ and } \mathbf{b} = \begin{bmatrix} 1 \\ 0 \\ -3 \end{bmatrix}.$$

($\blacksquare$ means the ink stain.) Although she cannot solve the whole system, she observed that she can still find x_1 using Cramer's rule. What is x_1?

(a) 0 (b) -4 (c) 8 (d) -8 (e) 4

9.(5pts) Let $\mathbb{P}_n$ be the vector space of all polynomials of degree at most n. Which of the following statement is **NOT true**?

(a) The function $T : \mathbb{P}_2 \to \mathbb{R}^2$ defined by $T(f(t)) = \begin{bmatrix} f(0) \\ f(-2) \end{bmatrix}$ is a linear transformation.

(b) $\mathbb{P}_5$ is a subspace of $\mathbb{P}_9$.

(c) If $T : \mathbb{P}_5 \to \mathbb{R}^2$ is a linear transformation, then $\ker(T)$ is a subspace of $\mathbb{R}^2$.

(d) The set of $f(t)$ in $\mathbb{P}_3$ such that $f(-1) = f(2)$ is a subspace of $\mathbb{P}_3$.

(e) The function $D : \mathbb{P}_{10} \to \mathbb{P}_{10}$ defined by $D(f(t)) = f'(t)$ (the derivative of f) is a linear transformation.

10.(5pts) If $A = \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}^{-1} \begin{bmatrix} -1 & 0 \\ 0 & \frac{1}{2} \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$, what is the $(2,1)$-entry of A^5?

(a) $\dfrac{1}{32}$ (b) $\dfrac{31}{32}$ (c) $-\dfrac{31}{32}$ (d) $\dfrac{33}{32}$ (e) $-\dfrac{33}{32}$

11.(5pts) Suppose that $\mathcal{B} = \{\mathbf{b}_1, \mathbf{b}_2\}$ and $\mathcal{C} = \{\mathbf{c}_1, \mathbf{c}_2\}$ are two bases for a vector space V, and the change-of-coordinate matrix **from** $\mathcal{B}$ **to** $\mathcal{C}$ is given as $\underset{\mathcal{C}\leftarrow\mathcal{B}}{P} = \begin{bmatrix} 2 & -3 \\ -1 & 2 \end{bmatrix}$.

For $\mathbf{v} = 2\mathbf{b}_1 + \mathbf{b}_2$, what is $[\mathbf{v}]_{\mathcal{C}}$, the $\mathcal{C}$-coordinate for $\mathbf{v}$?

(a) $\begin{bmatrix} 1 \\ 0 \end{bmatrix}$ (b) $\begin{bmatrix} 2 \\ -1 \end{bmatrix}$ (c) $\begin{bmatrix} 2 \\ 1 \end{bmatrix}$ (d) $\begin{bmatrix} -3 \\ 2 \end{bmatrix}$ (e) $\begin{bmatrix} 0 \\ 1 \end{bmatrix}$

12.(5pts) Which of the following statements is **false** for a **non-invertible**, $n \times n$ matrix A?

(a) A^T is not invertible. (b) $\text{Nul}(A) \neq \{\mathbf{0}\}$. (c) $\lambda = 0$ is an eigenvalue of A.

(d) The row space of A is $\mathbb{R}^n$.(e) $\det(A) = 0$.

13.(10pts) The matrix $A = \begin{bmatrix} 2 & -1 & 4 & 4 & 3 \\ 3 & 0 & 3 & 2 & -7 \\ 5 & -1 & 7 & 6 & -4 \\ 1 & 1 & -1 & 5 & -3 \end{bmatrix}$ is row equivalent to $\begin{bmatrix} 1 & 0 & 1 & 0 & -3 \\ 0 & 1 & -2 & 0 & -5 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$.

(You do **not** need to check for this.)

(a) Find a basis for the row space of A.

(b) Find a basis for the column space of A.

(c) Find a basis for the null space of A.

14.(10pts) For the matrix

$$A = \begin{bmatrix} -1 & 6 & -2 \\ 0 & 2 & 5 \\ 0 & 0 & 3 \end{bmatrix},$$

find an invertible matrix P and a diagonal matrix D such that $A = PDP^{-1}$.

Note: You do **not** need to calculate P^{-1}

15.(10pts) Let $\mathcal{B} = \left\{ \begin{bmatrix} 7 \\ -4 \end{bmatrix}, \begin{bmatrix} 6 \\ 1 \end{bmatrix} \right\}$ and $\mathcal{C} = \left\{ \begin{bmatrix} 5 \\ -2 \end{bmatrix}, \begin{bmatrix} 3 \\ -1 \end{bmatrix} \right\}$ be two basis of $\mathbb{R}^2$.

Find the change-of-coordinate matrix from $\mathcal{B}$ to $\mathcal{C}$, i.e., $\underset{\mathcal{C} \leftarrow \mathcal{B}}{P}$.