

Final Exam

May 7, 2012

Name: _____

Instructor:
& Section _____

This exam has 13 pages and contains 24 problems. Each problem is worth 6 points and 6 points will be given for following instructions. You have 2 hours to work on it. No calculators, books, notes, or other aids are allowed. Be sure to write your name on this page and to put your initials at the top of every page in case pages become detached. Good luck!

Honor Pledge: As a member of the Notre Dame community, I will not participate in nor tolerate academic dishonesty.

Signature: _____

Mark your answers to the multiple choice problems here.

Place an \times through your answer to each problem.

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|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| 1. | (a) | (b) | (c) | (d) | (•) | 13. | (a) | (b) | (•) | (d) | (e) |
| 2. | (a) | (b) | (c) | (•) | (e) | 14. | (a) | (•) | (c) | (d) | (e) |
| 3. | (a) | (b) | (•) | (d) | (e) | 15. | (•) | (b) | (c) | (d) | (e) |
| 4. | (a) | (b) | (•) | (d) | (e) | 16. | (a) | (b) | (•) | (d) | (e) |
| 5. | (a) | (b) | (c) | (•) | (e) | 17. | (a) | (b) | (•) | (d) | (e) |
| 6. | (a) | (b) | (c) | (•) | (e) | 18. | (a) | (•) | (c) | (d) | (e) |
| 7. | (a) | (•) | (c) | (d) | (e) | 19. | (a) | (•) | (c) | (d) | (e) |
| 8. | (a) | (b) | (•) | (d) | (e) | 20. | (a) | (b) | (•) | (d) | (e) |
| 9. | (•) | (b) | (c) | (d) | (e) | 21. | (•) | (b) | (c) | (d) | (e) |
| 10. | (a) | (b) | (c) | (•) | (e) | 22. | (a) | (b) | (c) | (d) | (•) |
| 11. | (a) | (b) | (c) | (•) | (e) | 23. | (a) | (•) | (c) | (d) | (e) |
| 12. | (•) | (b) | (c) | (d) | (e) | 24. | (a) | (b) | (c) | (d) | (•) |

Please do not write below here.

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The next four problems refer to the two matrices

$$A = \begin{bmatrix} 1 & 13 & 2 & -2 & 4 & 3 \\ -1 & -13 & 5 & 9 & 10 & -2 \\ -2 & -26 & 3 & 11 & 6 & -5 \\ 7 & 91 & 2 & -26 & 4 & 11 \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} 1 & 13 & 0 & -4 & 0 & 0 \\ 0 & 0 & 1 & 1 & 2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

where B is the reduced row echelon form of A .

1.(6pts) Which set of vectors below is a basis for the column space of A ?

$$\begin{aligned} \text{(a)} & \left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix} \right\} & \text{(b)} & \left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} -4 \\ 1 \\ 0 \\ 0 \end{bmatrix} \right\} & \text{(c)} & \left\{ \begin{bmatrix} 1 \\ -1 \\ -2 \\ 7 \end{bmatrix}, \begin{bmatrix} 2 \\ 5 \\ 3 \\ 2 \end{bmatrix}, \begin{bmatrix} 4 \\ 10 \\ 6 \\ 4 \end{bmatrix}, \begin{bmatrix} 3 \\ -2 \\ -5 \\ 11 \end{bmatrix} \right\} \\ \text{(d)} & \left\{ \begin{bmatrix} 1 \\ -1 \\ -2 \\ 7 \end{bmatrix}, \begin{bmatrix} 2 \\ 5 \\ 3 \\ 2 \end{bmatrix} \right\} & \text{(e)} & \left\{ \begin{bmatrix} 1 \\ -1 \\ -2 \\ 7 \end{bmatrix}, \begin{bmatrix} 2 \\ 5 \\ 3 \\ 2 \end{bmatrix}, \begin{bmatrix} 3 \\ -2 \\ -5 \\ 11 \end{bmatrix} \right\} \end{aligned}$$

2.(6pts) Which set of vectors below is a basis for the row space of A ?

$$\begin{aligned} \text{(a)} & \left\{ \begin{bmatrix} 1 \\ 13 \\ 2 \\ -2 \\ 4 \\ 3 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 7 \\ 7 \\ 14 \\ 1 \end{bmatrix} \right\} & \text{(b)} & \left\{ \begin{bmatrix} 1 \\ 13 \\ 2 \\ -2 \\ 4 \\ 3 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 7 \\ 7 \\ 14 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 13 \\ 9 \\ 5 \\ 18 \\ 4 \end{bmatrix} \right\} \\ \text{(c)} & \left\{ \begin{bmatrix} 1 \\ 13 \\ 2 \\ -2 \\ 4 \\ 3 \end{bmatrix}, \begin{bmatrix} -1 \\ -13 \\ 5 \\ 9 \\ 10 \\ -2 \end{bmatrix}, \begin{bmatrix} -2 \\ -26 \\ 3 \\ 11 \\ 6 \\ -5 \end{bmatrix} \right\} & \text{(d)} & \left\{ \begin{bmatrix} 1 \\ 13 \\ 0 \\ -4 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \\ 1 \\ 2 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} \right\} \\ \text{(e)} & \left\{ \begin{bmatrix} 1 \\ 13 \\ 2 \\ -2 \\ 4 \\ 3 \end{bmatrix}, \begin{bmatrix} -1 \\ -13 \\ 5 \\ 9 \\ 10 \\ -2 \end{bmatrix}, \begin{bmatrix} -2 \\ -26 \\ 3 \\ 11 \\ 6 \\ -5 \end{bmatrix}, \begin{bmatrix} 7 \\ 91 \\ 2 \\ -26 \\ 4 \\ 11 \end{bmatrix} \right\} \end{aligned}$$

For your convenience, here are the two matrices from the previous page:

$$A = \begin{bmatrix} 1 & 13 & 2 & -2 & 4 & 3 \\ -1 & -13 & 5 & 9 & 10 & -2 \\ -2 & -26 & 3 & 11 & 6 & -5 \\ 7 & 91 & 2 & -26 & 4 & 11 \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} 1 & 13 & 0 & -4 & 0 & 0 \\ 0 & 0 & 1 & 1 & 2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

where B is the reduced row echelon form of A .

3.(6pts) Which set of vectors below is a basis for the null space of A ?

$$(a) \left\{ \begin{bmatrix} 13 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 4 \\ 0 \\ 1 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 2 \\ 0 \\ 1 \\ 0 \end{bmatrix} \right\} \quad (b) \left\{ \begin{bmatrix} -13 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 4 \\ 0 \\ -3 \\ 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ -2 \\ 0 \\ 1 \\ 0 \end{bmatrix} \right\}$$

$$(c) \left\{ \begin{bmatrix} -13 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 4 \\ 0 \\ -1 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ -2 \\ 0 \\ 1 \\ 0 \end{bmatrix} \right\} \quad (d) \left\{ \begin{bmatrix} -13 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 4 \\ 0 \\ -1 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} -9 \\ 1 \\ -1 \\ 0 \\ 0 \\ 0 \end{bmatrix} \right\}$$

(e) The null space has dimension 0.

4.(6pts) Which set of vectors below is a basis for $(\text{row } A)^\perp$, the orthogonal complement to the row space of the matrix A above?

$$(a) \left\{ \begin{bmatrix} 13 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 4 \\ 0 \\ 1 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 2 \\ 0 \\ 1 \\ 0 \end{bmatrix} \right\} \quad (b) \left\{ \begin{bmatrix} -13 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 4 \\ 0 \\ -3 \\ 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ -2 \\ 0 \\ 1 \\ 0 \end{bmatrix} \right\}$$

$$(c) \left\{ \begin{bmatrix} -13 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 4 \\ 0 \\ -1 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ -2 \\ 0 \\ 1 \\ 0 \end{bmatrix} \right\} \quad (d) \left\{ \begin{bmatrix} -13 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 4 \\ 0 \\ -1 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} -9 \\ 1 \\ -1 \\ 0 \\ 0 \\ 0 \end{bmatrix} \right\}$$

(e) The space $(\text{row } A)^\perp$ has dimension 0.

5.(6pts) Which number below is the determinant of $\begin{bmatrix} 1 & 2 & 3 \\ 3 & 2 & 1 \\ 1 & -1 & 1 \end{bmatrix}$?

(a) 5

(b) 16

(c) -5

(d) -16

(e) 0

6.(6pts) If $\begin{bmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{bmatrix}$ is the inverse to $\begin{bmatrix} 1 & 2 & 3 \\ 5 & 6 & 4 \\ -1 & -1 & 1 \end{bmatrix}$ find b_{31} .

(a) $b_{31} = -0.6$

(b) $b_{31} = 0.6$

(c) $b_{31} = -1$

(d) $b_{31} = -0.2$

(e) $b_{31} = 1$

7.(6pts) Suppose \mathbf{a} , \mathbf{b} , \mathbf{x} and \mathbf{y} are n -vectors and suppose the various dot products are as follows: $\mathbf{a} \cdot \mathbf{a} = -1$, $\mathbf{a} \cdot \mathbf{b} = 1$, $\mathbf{a} \cdot \mathbf{x} = 3$, $\mathbf{a} \cdot \mathbf{y} = -3$, $\mathbf{b} \cdot \mathbf{b} = 7$, $\mathbf{b} \cdot \mathbf{x} = 4$, $\mathbf{b} \cdot \mathbf{y} = -4$, $\mathbf{x} \cdot \mathbf{x} = 7$, $\mathbf{x} \cdot \mathbf{y} = 2$ and $\mathbf{y} \cdot \mathbf{y} = -2$. The number $(2\mathbf{a} + \mathbf{x}) \cdot (-\mathbf{y} + \mathbf{b})$ is which answer below?

- (a) -5 (b) 10 (c) 3 (d) 0
 (e) Can not be computed from the given data.

8.(6pts) Let $W = \text{Span} \left\{ \begin{bmatrix} 1 \\ 2 \\ 3 \\ -1 \end{bmatrix}, \begin{bmatrix} 2 \\ 2 \\ -1 \\ 3 \end{bmatrix} \right\}$. Which vector below is the projection of $\begin{bmatrix} 21 \\ -3 \\ 9 \\ -3 \end{bmatrix}$ into W ?

- (a) $\begin{bmatrix} 11 \\ 12 \\ -2 \\ 14 \end{bmatrix}$ (b) $\begin{bmatrix} 24 \\ 1 \\ 11 \\ -1 \end{bmatrix}$ (c) $\begin{bmatrix} 5 \\ 8 \\ 8 \\ 0 \end{bmatrix}$ (d) $\begin{bmatrix} 3 \\ 4 \\ 2 \\ 2 \end{bmatrix}$ (e) $\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$

9.(6pts) If $W = \text{Span} \left\{ \begin{bmatrix} 1 \\ 2 \\ 3 \\ 1 \end{bmatrix}, \begin{bmatrix} 8 \\ 10 \\ 0 \\ 2 \end{bmatrix} \right\}$, which set of vectors below is an orthogonal basis for W ?

(a) $\left\{ \begin{bmatrix} 1 \\ 2 \\ 3 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ -1 \\ 0 \end{bmatrix} \right\}$

(b) $\left\{ \begin{bmatrix} 1 \\ 2 \\ 3 \\ 1 \end{bmatrix}, \begin{bmatrix} 5 \\ 2 \\ -3 \\ 0 \end{bmatrix} \right\}$

(c) $\left\{ \begin{bmatrix} 1 \\ 2 \\ 3 \\ 1 \end{bmatrix}, \begin{bmatrix} 5 \\ 1 \\ 0 \\ -7 \end{bmatrix} \right\}$

(d) $\left\{ \begin{bmatrix} 1 \\ 2 \\ 3 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 3 \\ 0 \\ -7 \end{bmatrix} \right\}$

(e) $\left\{ \begin{bmatrix} 1 \\ 2 \\ 3 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ -1 \\ -2 \end{bmatrix} \right\}$

10.(6pts) Find the least squares solution to the equation $\begin{bmatrix} 2 & -1 & 0 \\ 0 & 2 & 0 \\ 2 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \mathbf{x} = \begin{bmatrix} -2 \\ 3 \\ -2 \\ 10 \end{bmatrix}$.

(a) $\begin{bmatrix} -1 \\ 3 \\ 4 \end{bmatrix}$

(b) $\begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}$

(c) $\begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$

(d) $\begin{bmatrix} -1 \\ 1 \\ 10 \end{bmatrix}$

(e) $\begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$

11.(6pts) Let U be any $n \times n$ matrix with orthonormal columns. Which of the following are sometimes false?

- (1) The rows of U are orthonormal.
- (2) $U^T U$ is the identity.
- (3) For any n -vectors \mathbf{x} and \mathbf{y} , $U\mathbf{x} \cdot \mathbf{y} = \mathbf{x} \cdot \mathbf{y}$
- (4) The determinant of U is ± 1 .
- (5) UU^T has a non-trivial kernel.

- (a) (1) and (4) (b) (1) and (5) (c) (2) and (3)
(d) (3) and (5) (e) All five statements are always true.

12.(6pts) Let y be the solution to $(4xy + 1)dx + (2x^2 + \cos y)dy = 0$ which goes through the point $(0, \pi/2)$. Which implicit equation below is satisfied by this solution?

- (a) $2x^2y + x + \sin y = 1$ (b) $2x^2y + x - \sin y = 0$ (c) $2x^2y + y + \sin y = \pi/2 + 1$
(d) $2x^2y + x - \sin y = -1$ (e) $2x^2y + y - \sin y = \pi/2 - 1$

- 13.(6pts) Let \mathbb{P}_2 be the vector space of polynomials of degree 2 or less. Let $L: \mathbb{P}_2 \rightarrow \mathbb{P}_2$ be the linear transformation defined by $L[y] = t^2y'' + ty' + y$. Determine the matrix of L with respect to the basis $\{1, t, t^2\}$ for \mathbb{P}_2 .

(a)
$$\begin{bmatrix} 0 & 0 & 1 \\ 0 & 2 & 0 \\ 5 & 0 & 0 \end{bmatrix}$$

(b)
$$\begin{bmatrix} 1 & 2 & 5 \\ 2 & 3 & -1 \\ 5 & -1 & 0 \end{bmatrix}$$

(c)
$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 5 \end{bmatrix}$$

(d)
$$\begin{bmatrix} 1 & 0 & 1 \\ 0 & 2 & 5 \\ 0 & 0 & 0 \end{bmatrix}$$

(e)
$$\begin{bmatrix} 1 & 3 & 1 \\ 3 & 2 & 5 \\ 1 & 5 & 0 \end{bmatrix}$$

- 14.(6pts) Suppose Y is the solution to the equation

$$y' + \tan(t)y = \frac{t}{t^2 + 1} \quad \text{and} \quad y(\pi) = 3$$

Which interval below is the largest interval over which Y is guaranteed to exist?

- (a) $0 < t < \pi$ (b) $\pi/2 < t < 3\pi/2$ (c) $0 < t < 2\pi$ (d) $-\pi/2 < t < \pi/2$
(e) $-3\pi/2 < t < 3\pi/2$

15.(6pts) If y is the solution to

$$y'' - 2y' - 8y = 9e^t \quad \text{and} \quad y(0) = y'(0) = 0$$

which number below is $y(1)$?

(a) $\frac{e^{-2} - 2e + e^4}{2}$

(b) $\frac{e^3 - 6e^{-1} + 3}{2}$

(c) $\frac{e^4 + 6e + 5}{6e^2}$

(d) $\frac{e^3 + 6e^{-1} + 3}{2}$

(e) $\frac{e^6 + 6e + 5}{6e^2}$

16.(6pts) If y is the solution to

$$y' + 4xy = x \quad \text{with} \quad y(1) = 2$$

find $y(0)$.

(a) $\frac{1}{4} - \frac{7e^2}{4}$

(b) $\frac{1}{4} - \frac{4e^2}{7}$

(c) $\frac{1}{4} + \frac{7e^2}{4}$

(d) $\frac{1}{7} - \frac{4e^2}{7}$

(e) $\frac{7e^2}{4} - \frac{1}{4}$

17.(6pts) Consider the equation

$$y' = (y^3 - 4y)(9 - y^2)$$

with initial value $y(-1) = 1$. Assuming $\lim_{t \rightarrow \infty} y(t)$ exists, find it without solving the equation.

(a) -2

(b) 3

(c) 0

(d) -3

(e) 2

18.(6pts) An object of mass m is released from rest from a boat into the water and allowed to sink. Gravity is pulling the object down and distance is to be measured upwards. By Archimedes's Principle, there is a buoyant force pushing up equal to $bm g$ where b is a constant. Resistance imparts a force on the object proportional to its velocity but in the opposite direction with proportionality constant $k > 0$. Determine a differential equation for the velocity v of the object.

(a) $v' + \frac{k}{m}v = bg$

(b) $v' + \frac{k}{m}v = (b - 1)g$

(c) $v' - \frac{k}{m}v = (b - 1)g$

(d) $v' - \frac{k}{m}v = bg$

(e) $v' - kv = (b - 1)g$

19.(6pts) Which function below is the Wronskian of the two functions $y_1 = t^2$ and $y_2 = t^{-1}$?

- (a) $\frac{t^2 + t^{-1}}{2t - t^{-1}}$ (b) -3 (c) 5 (d) $2t$ (e) $t^2 \ln |t| - \frac{t^2}{3}$

20.(6pts) Given that $y = t$ is one solution to the homogeneous equation

$$y'' - \left(1 + \frac{2}{t}\right) y' + \left(\frac{1}{t} + \frac{2}{t^2}\right) y = 0$$

use reduction of order to find a second independent solution. Which function below is such a function?

- (a) t^3 (b) e^{-t} (c) te^t (d) $t^2 - 3t$ (e) $t^2 e^{-2t}$

21.(6pts) Find the form of a particular solution to

$$y'' + 49y = t^2 \sin(7t)$$

using the method of Undetermined Coefficients.

(a) $y = (A_0t + A_1t^2 + A_2t^3) \sin(7t) + (B_0t + B_1t^2 + B_2t^3) \cos(7t)$

(b) $y = (A_0t + A_1t^2 + A_2t^3) \sin(7t) + (A_0t + A_1t^2 + A_2t^3) \cos(7t)$

(c) $y = (A_0 + A_1t + A_2t^2) \sin(7t) + (B_0 + B_1t + B_2t^2) \cos(7t)$

(d) $y = (A_0t + A_1t^2 + A_2t^3) \sin(7t) + (A_2t + A_1t^2 + A_0t^3) \cos(7t)$

(e) $y = (A_0t^2 + A_1t^3 + A_2t^4) \sin(7t) + (B_0t^2 + B_1t^3 + B_2t^4) \cos(7t)$

22.(6pts) Which of these equations is 2nd order and non-linear?

(1) $y'' + (\sin x)y' + (\tan x)y = e^x$

(5) $y''' + e^{ty} = 0$

(2) $y'' + (\sin x)y' + \tan(xy) = e^x$

(6) $y''' + e^t y = 0$

(3) $y' + \tan(xy) = e^x$

(7) $y'' + e^t y = \sin(t)$

(4) $y' + (\tan x)y = e^x$

(8) $y'' + e^{ty} = 0$

(a) (2), (6) and (8)

(b) (1), (3) and (4)

(c) (2) and (7)

(d) (5) and (8)

(e) (2) and (8)

23.(6pts) Suppose t and $\sin t$ are two solutions to the homogeneous system

$$y'' + p(t)y' + q(t)y = 0$$

Use Variation of Parameters to find a solution to the equation

$$y'' + p(t)y' + q(t)y = t \cos(t) - \sin(t)$$

Which function below is a solution?

(a) $\frac{t^3 \sin t + 3t \cos t}{3}$

(b) $\frac{t^2 \sin t + 2t \cos t}{2}$

(c) $\frac{2t \sin t + t^2 \cos t}{2}$

(d) $\frac{2t \sin t - 3t^2 \cos t}{2}$

(e) $\frac{t^2 \sin t + 3t \cos t}{3}$

24.(6pts) The vector $\begin{bmatrix} 7 \\ 1 \\ 1 \\ 0 \\ 0 \end{bmatrix}$ is an eigenvector for the matrix $\begin{bmatrix} 1 & 2 & 5 & 7 & 10 \\ 0 & -1 & 3 & 4 & 12 \\ 0 & 0 & 2 & 8 & 8 \\ 0 & 0 & 0 & -2 & 8 \\ 0 & 0 & 0 & 0 & 6 \end{bmatrix}$. What is the corresponding eigenvalue?

(a) 0

(b) -3

(c) 3

(d) -2

(e) 2