Name: ____________________

Department of Mathematics
University of Notre Dame
Math 20580 – Fall 2012 Version #1

Instructor:
Misiolek |Taylor ______________

Exam 1

September 18, 2012

This exam is in 2 parts on 9 pages and contains 12 problems worth a total of 100 points.You have 1 hour and 15 minutes to work on it. No calculators, books, notes, or other aids are allowed. Be sure to write your name on this page and to put your initials at the top of every page in case pages become detached. *On the partial credit problems you must show your work and all important steps to receive credit.* Good luck!

Honor Pledge: As a member of the Notre Dame community, I will not participate in nor tolerate academic dishonesty.

Signature:________________________________

Mark your answers to the multiple choice problems here.
Place an × through your answer to each problem.

1.	(a)	(b)	(•)	(d)	(e)
2.	(a)	(b)	(c)	(•)	(e)
3.	(a)	(b)	(c)	(d)	(•)
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5.	(a)	(•)	(c)	(d)	(e)
6.	(a)	(b)	(c)	(•)	(e)
7.	(•)	(b)	(c)	(d)	(e)
8.	(•)	(b)	(c)	(d)	(e)

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1.(6pts) If $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$, which matrix below is AA^T for all choices of real numbers a, b, c and d?

(a) $\begin{bmatrix} a^2 & bc \\ bc & d^2 \end{bmatrix}$ (b) $\begin{bmatrix} ad-bc & 0 \\ 0 & ad-bc \end{bmatrix}$ (c) $\begin{bmatrix} a^2+b^2 & ac+bd \\ ac+bd & c^2+d^2 \end{bmatrix}$

(d) $\begin{bmatrix} 2a & b+c \\ b+c & 2d \end{bmatrix}$ (e) The matrices A and A^T can not be multiplied.

2.(6pts) Which column in the matrix below is the first column from the left which is a linear combination of the preceding ones?

$$\begin{bmatrix} 1 & 2 & -1 & -3 & 2 \\ 2 & 5 & -1 & -6 & 6 \\ 3 & 7 & -2 & -8 & 8 \\ 4 & 9 & -3 & -11 & 10 \\ 1 & 2 & -1 & -5 & 3 \end{bmatrix}$$

(a) $\begin{bmatrix} 2 \\ 5 \\ 7 \\ 9 \\ 2 \end{bmatrix}$ (b) $\begin{bmatrix} 2 \\ 6 \\ 8 \\ 10 \\ 3 \end{bmatrix}$ (c) $\begin{bmatrix} -3 \\ -6 \\ -8 \\ -11 \\ -5 \end{bmatrix}$ (d) $\begin{bmatrix} -1 \\ -1 \\ -2 \\ -3 \\ -1 \end{bmatrix}$ (e) $\begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \\ 1 \end{bmatrix}$

3.(6pts) Consider the subspace of all vectors $\begin{bmatrix} a \\ b \\ c \end{bmatrix}$ in the span of $\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$ and $\begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix}$. Which relation below must always hold?

(a) $abc = 0$ (b) $a + c - 2b = 2$ (c) $a + c + 2b = 2$

(d) $a + c + 2b = 0$ (e) $a + c - 2b = 0$

4.(6pts) Which number below is the $2, 1$ entry in the product

$$\begin{bmatrix} 1 & 2 & 3 \\ -1 & 2 & 1 \\ 0 & 4 & -2 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 3 & 4 \\ -2 & 1 \end{bmatrix} \qquad ?$$

(a) 16 (b) 13 (c) 3 (d) -4 (e) 0

5.(6pts) Which sets of vectors below are linearly independent?

$$S_1 = \left\{ \begin{bmatrix}1\\2\\3\\4\end{bmatrix}, \begin{bmatrix}2\\4\\6\\8\end{bmatrix} \right\},\ S_2 = \left\{ \begin{bmatrix}1\\2\\3\end{bmatrix}, \begin{bmatrix}0\\1\\8\end{bmatrix}, \begin{bmatrix}2\\1\\-2\end{bmatrix}, \begin{bmatrix}5\\13\\17\end{bmatrix} \right\},\ S_3 = \left\{ \begin{bmatrix}11\\23\\17\\18\end{bmatrix}, \begin{bmatrix}17\\13\\23\\19\end{bmatrix}, \begin{bmatrix}0\\0\\0\\0\end{bmatrix} \right\}$$

(a) Only S_2.

(b) They are all linearly dependent

(c) Only S_1 and S_2.

(d) Only S_1

(e) Only S_3.

6.(6pts) Let $T\colon \mathbf{R}^3 \to \mathbf{R}^2$ satisfy $T\left(\begin{bmatrix}1\\0\\0\end{bmatrix}\right) = \begin{bmatrix}4\\7\end{bmatrix}$, $T\left(\begin{bmatrix}0\\1\\0\end{bmatrix}\right) = \begin{bmatrix}7\\4\end{bmatrix}$ and $T\left(\begin{bmatrix}1\\2\\1\end{bmatrix}\right) = \begin{bmatrix}0\\0\end{bmatrix}$.

Which matrix below is the standard matrix for T?

(a) $\begin{bmatrix}7 & 4 & 0\\4 & 7 & 0\end{bmatrix}$

(b) $\begin{bmatrix}4 & 7 & 0\\7 & 4 & 0\end{bmatrix}$

(c) $\begin{bmatrix}7 & 4 & -11\\4 & 7 & -11\end{bmatrix}$

(d) $\begin{bmatrix}4 & 7 & -18\\7 & 4 & -15\end{bmatrix}$

(e) Can not be determined from the given information.

7.(6pts) The rank of the matrix $A = \begin{bmatrix} 1 & 2 & -3 \\ 2 & 5 & -6 \\ 3 & 7 & -9 \\ 4 & 9 & -12 \\ 1 & 2 & -3 \end{bmatrix}$ is clearly 2. What is the dimension of Nul(A)?

(a) 3 (b) 5 (c) 2 (d) 4 (e) 1

8.(6pts) Exactly one of the subsets listed in the answers is NOT a subspace. Which one is it? Here A is an $m \times n$ matrix, $\mathbf{x} \in \mathbf{R}^n$ and $\mathbf{b} \in \mathbf{R}^m$.

(a) The subset of all $\mathbf{x}$ for which $A\mathbf{x} = \mathbf{b}$ for a fixed $\mathbf{b}$, $\mathbf{b} \neq \mathbf{0}$.

(b) The subset of all $\mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}$ such that $x_1 = 0$.

(c) The subset of all $\mathbf{b}$ for which $A\mathbf{x} = \mathbf{b}$ has a solution.

(d) The subset of all $\mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}$ such that $x_1^2 + \cdots + x_n^2 = 0$.

(e) The subset of all $\mathbf{x}$ for which $\mathbf{x}^T A^T = \mathbf{0}$

9.(13pts) If V and W are subspaces of $\mathbf{R}^n$, define $V \cap W$ to be the subset of all vectors $\mathbf{v}$ in $\mathbf{R}^n$ such that $\mathbf{v}$ is in V and $\mathbf{v}$ is in W. Show that $V \cap W$ is a subspace.

For some credit, indicate what three things you need to show even if you are unable to show them.

10.(13pts) Compute the inverse to the matrix $\begin{bmatrix} 1 & 2 & 4 \\ 2 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}$.

11.(13pts) Find a basis for the null space and the column space of the matrix

$$A = \begin{bmatrix} 1 & 0 & 5 & 6 & 4 \\ 2 & 3 & -2 & 2 & -2 \\ 3 & 1 & 11 & 11 & 5 \end{bmatrix}$$

12.(13pts) Let $\mathcal{B} = \left\{ \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \end{bmatrix}, \begin{bmatrix} 6 \\ 5 \\ 4 \\ 3 \\ 2 \\ 1 \end{bmatrix} \right\}$ be a basis for a subspace V of $\mathbf{R}^6$ Find a 6×2 matrix A so that given any vector $\mathbf{w} \in V$ with $\mathcal{B}$-coordinate vector $\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}_{\mathcal{B}}$, $A \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}_{\mathcal{B}} = \mathbf{w}$.

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Version #2

Instructor:
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(a) 0 (b) 3 (c) 16 (d) 13 (e) -4

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