Name: _____

Department of Mathematics
University of Notre Dame
Math 20580 - Fall 2012 Version #1

Instructor:
Misiolek | Taylor _____

# Exam 3

**November 13, 2012**

This exam is in 2 parts on 9 pages and contains 12 problems worth a total of 100 points.You have 1 hour and 15 minutes to work on it. No calculators, books, notes, or other aids are allowed. Be sure to write your name on this page and to put your initials at the top of every page in case pages become detached. *On the partial credit problems you must show your work and all important steps to receive credit.* Good luck!

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**Mark your answers to the multiple choice problems here.**
Place an × through your answer to each problem.

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|---|---|---|---|---|---|
| 1. | (a) | (b) | (c) | (•) | (e) |
| 2. | (a) | (b) | (c) | (d) | (•) |
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| 6. | (a) | (b) | (c) | (d) | (•) |
| 7. | (a) | (b) | (c) | (•) | (e) |
| 8. | (•) | (b) | (c) | (d) | (e) |

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**1.**(6pts) Let $A = \begin{bmatrix} \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{3}} \\ \frac{1}{\sqrt{6}} & \frac{-1}{\sqrt{2}} & \frac{1}{\sqrt{3}} \\ \frac{2}{\sqrt{6}} & 0 & -\frac{1}{\sqrt{3}} \end{bmatrix}$, let $\mathbf{u} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$ and let $\mathbf{v} = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}$. Compute $(A\mathbf{u}) \bullet (A\mathbf{v})$.

(a) $\frac{3-\sqrt{2}}{\sqrt{6}}$ (b) 3 (c) 0 (d) 5 (e) $\frac{\sqrt{14}-\sqrt{3}}{\sqrt{6}}$

**2.**(6pts) If $W = \text{Span}\left\{ \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ -5 \end{bmatrix} \right\}$ compute $\text{proj}_W \left( \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \right)$.

(a) $\begin{bmatrix} -1/5 \\ 3/5 \\ 0 \end{bmatrix}$ (b) $\begin{bmatrix} 1/5 \\ -1/2 \\ 0 \end{bmatrix}$ (c) $\begin{bmatrix} 1/6 \\ 3/5 \\ 0 \end{bmatrix}$ (d) $\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$ (e) $\begin{bmatrix} 1/5 \\ 2/5 \\ 0 \end{bmatrix}$

**3.**(6pts) Find the least squares solution of the equation $A = \begin{bmatrix} 1 & 2 \\ 2 & 1 \\ -1 & 4 \end{bmatrix}$ $\mathbf{x} = \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}$.

(a) $\begin{bmatrix} \frac{1}{19} \\ \frac{11}{19} \end{bmatrix}$ (b) $\begin{bmatrix} \frac{1}{19} \\ \frac{11}{21} \end{bmatrix}$ (c) $\begin{bmatrix} \frac{1}{6} \\ \frac{11}{21} \end{bmatrix}$ (d) $\begin{bmatrix} \frac{1}{11} \\ \frac{11}{19} \end{bmatrix}$ (e) $\begin{bmatrix} 2 \\ 1 \end{bmatrix}$

**4.**(6pts) If $W = \text{Span}\left\{ \begin{bmatrix} 1 \\ 3 \\ 5 \end{bmatrix} \right\}$ find a basis for $W^{\perp}$.

(a) $\left\{ \begin{bmatrix} -3 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix} \right\}$ (b) $\left\{ \begin{bmatrix} -3 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -5 \\ 0 \\ 1 \end{bmatrix} \right\}$ (c) $\left\{ \begin{bmatrix} -8 \\ 1 \\ 1 \end{bmatrix} \right\}$

(d) $\left\{ \begin{bmatrix} -5 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} -1 \\ -2 \\ 1 \end{bmatrix} \right\}$ (e) $\left\{ \begin{bmatrix} 0 \\ 5 \\ -3 \end{bmatrix} \right\}$

**5.**(6pts) Classify the differential equation $\left(\frac{dy}{dx}\right)^2 + x^3 = \sin x$ as one of the following.

(a) Linear (b) Third order (c) First order

(d) Second order (e) Autonomous

**6.**(6pts) The solution to the equation $y' = xy + e^{x^2/2}\sin x$ with $y(0) = 2$ is given by

(a) $y = e^{x^2/2}\tan x + 2e^{x^2/2}$ (b) $y = e^{x^2/2}\cos x + e^{x^2/2}$ (c) $y = 2 + \int_0^x e^{t^2/2}\sin t dt$

(d) $y = -e^{x^2/2}\tan x + 2e^{x^2/2}$ (e) $y = -e^{x^2/2}\cos x + 3e^{x^2/2}$

**7.**(6pts) The general solution to the equation $\dfrac{dy}{dx} = \dfrac{x^2y^2 - 4y^2}{x-2}$ is given by

(a) $y = \dfrac{1}{C}(x^2 + 4x + C)$ (b) $y = x^2 + 4x + C$ (c) $y = \dfrac{1}{C}(x^2 + 4x)$

(d) $y = \dfrac{-2}{x^2 + 4x + C}$ (e) $y = e^{x^2} + 4x + C$

**8.**(6pts) Compute the distance between the two vectors $\begin{bmatrix}1\\2\\3\\5\end{bmatrix}$ and $\begin{bmatrix}2\\2\\3\\3\end{bmatrix}$.

(a) $\sqrt{5}$ (b) $\sqrt{11} - \sqrt{10}$ (c) $\sqrt{39} - \sqrt{26}$ (d) 2 (e) $\sqrt{13}$

**9.**(13pts) Let $W = \text{Span}\left\{ \begin{bmatrix} 1 \\ 2 \\ -1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 4 \\ -1 \\ 2 \end{bmatrix}, \begin{bmatrix} 5 \\ 6 \\ 7 \\ 8 \end{bmatrix} \right\}$. Find an orthonormal basis for $W$.

**10.**(13pts) Find the general solution to the equation $y' = \frac{y}{x} + x^2$.

**11.**(13pts) By Newton's Law of cooling, the rate of change in the temperature of a body in a room is directly proportional to the difference between the current temperature of the body and the temperature of the room which is assumed to be a constant, $R_0$. Write the differential equation expressing this law. Let $T$ be the temperature of the body at time $t$ and let $k$ be the constant of proportionality. Solve the equation. Why, in your experience, is $k \leqslant 0$?

**12.**(13pts) Find the unique solution to the equation $y' = y^4$ with $y(0) = \frac{1}{2}$. What is the largest interval containing 0 over which this solution is defined?

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