Consider the following multi-stage game. Two firms produce a homogeneous product with demand curve

$$
\mathrm{P}\left(\mathrm{q}_{1}, \mathrm{q}_{2}\right)=30-\mathrm{q}_{1}-\mathrm{q}_{2},
$$

where $q_{i}$ is firm i's quantity $(i=1,2)$ and $P$ is price. Initially, each firm's total cost is $C_{i}\left(q_{i}\right)=15 q_{i}$,
but if firm i adopts and the new process succeeds, then its cost is

$$
\mathrm{C}_{\mathrm{si}}\left(\mathrm{q}_{\mathrm{i}}\right)=9 \mathrm{q}_{\mathrm{i}} .
$$

The probability that the new process succeeds is $p \in[0,1]$.

An inventor owns the patent rights to the new process, which it can license to either or both firms, in the first stage. That is, in stage one, the inventor decides how many licenses to sell (one or two), and whether to sell them with an auction or a fixed fee (posted price). The firms them play the licensing game. If either firm buys a license and adopts, it pays the license fee plus a cost of adoption of $k=1$. Then whether the innovation is a success of failure is revealed to all firms (i.e., even if only one firm adopts, the other firm also learns if the innovation is a success or not). Both firms then play a standard Cournot output game and collect profits.

The patent expires at the end of stage one (i.e., length of the time period $=$ the life of the patent). If either firm adopted in stage one, then both use the new process in stage two if succeeds and both use the old technology if not. A firm that did not adopt in stage one must pay the adoption cost $\mathrm{k}=1$ if it adopts in stage two. If neither firm adopted in stage one, then they play a single-stage game of adoption under uncertainty (each adopts or not, success or failure is revealed if either adopted, then they play a Cournot output game). Whenever there are multiple equilibria in this one-stage game, use the symmetric mixed strategy equilibrium to determine the equilibrium payoffs needed to solve for the subgame perfect equilibria of the entire game.

Show that for some probability of success, inventor profit (sales revenue) is maximized by selling one license by fixed fee for all sufficiently large discount factors.

