Economics 43535: Assignment 3 Key

Professor Jensen Spring 2014

Assume a consumer's utility function is

$U(x,y,q) = x^{1/3}y^{2/3}q$

where x is the quantity of good x consumed, y is the quantity of good y consumed, and q is environmental quality.

Also that the prices of goods x and y are p_x and p_y and that the consumer's income is I.

1. Derive this consumer's indirect utility function $V(p_x,p_y,I,q)$.

Using the same approach as in class, the demand functions for goods x and y are defined by the following two equations:

$$\mathbf{MU_x/p_x = MU_y/p_y \text{ and } p_xx + p_yy = I,}$$

where $MU_x = (1/3)x^{-2/3}y^{1/3}q$ and $MU_y = (2/3)x^{1/3}y^{-2/3}q$.

The first equation reduces to $2p_xx = p_yy$. Substituting this into the budget constraint and solving for x gives the demand function

$$\mathbf{d_x(p_x,p_y,I,q) = I/3p_x .}$$

Substituting this back into $p_yy = p_xx$ and solving for y gives the other demand function

$$\mathbf{d_y(p_x,p_y,I,q) = 2I/3p_y .}$$

Therefore $V(p_x,p_y,I,q) = (d_x(p_x,p_y,I,q))^{1/3}(d_y(p_x,p_y,I,q))^{2/3}q = (I/3p_x)^{1/3}(2I/3p_y)^{2/3}q$

$$= (1/3p_x)^{1/3}(2/3p_y)^{1/3}Iq$$

2. Suppose income and environmental quality are initially I_0 and q_0. Also suppose that a new policy is introduced that would change environmental quality to q_1 (and have no effect on prices).

a. How much income would need to be give to (or taken away) from this consumer to make them indifferent to the new policy?

Let CV (compensating variation) be the amount of income given to or taken away. Then as noted in class it is defined by

$$\mathbf{V(p_x,p_y,I_0-CV,q_1) = V(p_x,p_y,I_0,q_0), \text{ or}}$$

$$(1/3p_x)^{1/3}(2/3p_y)^{2/3}(I_0-CV)q_1 = (1/3p_x)^{1/3}(2/3p_y)^{2/3}I_0q_0.$$

Solving this gives

$$CV = I_0(q_1-q_0)/q_1 .$$

b. When do you need to give the consumer more income to keep her indifferent?

Notice her income is greater after compensation if CV<0 (so $I_0-CV>I_0$). That is, $CV = I_0(q_1-q_0)/q_1 < 0$ when $q_1 < q_0$, or you need to give her more income to make her indifferent to the policy if it reduces environmental quality.

3. Suppose instead that the consumer's utility function is

$$U(x,y,q) = x^{1/3}y^{2/3}q^2.$$

a. Rework questions 1 and 2 above for this utility function.

Now $MU_x = (1/3)x^{-2/3}y^{2/3}q^2$ and $MU_y = (2/3)x^{1/3}y^{-1/3}q^2$,

but $MU_x/p_x = MU_y/p_y$ and $p_xx + p_yy = I$ still imply that

$d_x(p_x,p_y,I,q) = I/3p_x$ and $d_y(p_x,p_y,I,q) = 2I/3p_y$, so

$$V(p_x,p_y,I,q) = (d_x(p_x,p_y,I,q))^{1/3}(d_y(p_x,p_y,I,q))^{2/3}q^2 = (1/3p_x)^{1/3}(2/3p_y)^{2/3}Iq^2.$$

Therefore CV now defined by $V(p_x,p_y,I_0-CV,q_1) = V(p_x,p_y,I_0,q_0)$, or

$$(1/3p_x)^{1/3}(2/3p_y)^{2/3}(I_0-CV)(q_1)^2 = (1/3p_x)^{1/3}(2/3p_y)^{2/3}I_0(q_0)^2,$$

Solving this gives

$$CV = I_0[(q_1)^2-(q_0)^2]/(q_1)^2 .$$

b. If your answer to 2a is different for this utility function, explain why.

The answer is different because the marginal utility of environmental quality has changed from $MU_q = x^{1/3}y^{2/3}$ in the first to $MU_q = 2x^{1/3}y^{2/3}q$ in the second.