The potential hazard of one aircraft encountering the trailing vortex wake of another aircraft during either take-off or landing is well known. The wake vortex hazard in the terminal area has been virtually eliminated using the operational procedures and aircraft separation criteria mandated by the air traffic authorities. This paper presents arguments that wake encounters at cruise altitude are a potential safety issue that needs to be examined. Wake vortex encounters at cruise altitudes are likely to increase due to three factors. These factors include; the reduction of the minimum vertical separation distances between aircraft, the increased air traffic allowed by the reduced vertical separations, and the large difference in the size of aircraft operating at the cruise altitudes. The arguments are supported by simple analysis and a review of simulation and flight test results obtained from earlier studies of wake encounters at low altitudes. Simple analysis techniques show that the rolling moment induced on a following aircraft is comparable in magnitude to that which could occur during take-off or landing. In addition reported wake encounters at cruise altitude are discussed.

**Nomenclature**

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>AR</td>
<td>wing aspect ratio</td>
</tr>
<tr>
<td>b</td>
<td>wing span</td>
</tr>
<tr>
<td>c</td>
<td>wing chord</td>
</tr>
<tr>
<td>b_v</td>
<td>trailing vortex span</td>
</tr>
<tr>
<td>C_{\alpha}</td>
<td>section lift curve slope</td>
</tr>
<tr>
<td>C_f</td>
<td>roll moment coefficient</td>
</tr>
<tr>
<td>k_t</td>
<td>proportionality constant</td>
</tr>
<tr>
<td>L_v</td>
<td>vortex induced rolling moment</td>
</tr>
<tr>
<td>Q</td>
<td>dynamic pressure</td>
</tr>
<tr>
<td>\dot{p}</td>
<td>roll acceleration</td>
</tr>
<tr>
<td>t^*</td>
<td>time to linking</td>
</tr>
<tr>
<td>V_\infty</td>
<td>flight velocity</td>
</tr>
<tr>
<td>V_\theta</td>
<td>tangential velocity of the vortex</td>
</tr>
<tr>
<td>V_t</td>
<td>maximum tangential velocity</td>
</tr>
<tr>
<td>W</td>
<td>aircraft weight</td>
</tr>
<tr>
<td>X</td>
<td>downstream distance</td>
</tr>
<tr>
<td>\psi</td>
<td>heading angle</td>
</tr>
<tr>
<td>\theta</td>
<td>pitch angle</td>
</tr>
<tr>
<td>\nu</td>
<td>kinematic viscosity</td>
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</tbody>
</table>

**Subscripts**

<table>
<thead>
<tr>
<th>Subscript</th>
<th>Meaning</th>
</tr>
</thead>
<tbody>
<tr>
<td>F</td>
<td>following</td>
</tr>
<tr>
<td>G</td>
<td>generating</td>
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</table>
I. Introduction

The potential hazard of trailing vortex encounters has been the subject of numerous conferences and symposia during the past 50 years. During this period the research was focused primarily on aircraft encounters during either take-off or landing. The severity of wake vortex encounters in the terminal area was clearly established through flight test programs, ground-based flight simulators, and computer simulation. Based upon studies conducted by NASA and the FAA, a series of operational rules and separation criteria to avoid hazardous wake encounters were developed. Aircraft separation criteria are based upon the size of the wake generating aircraft and size of the following aircraft. The separation times were selected based upon flight experiments and analysis so that the risk of an unsafe wake encounter could be eliminated. The separation times are summarized in Table 1.

<table>
<thead>
<tr>
<th>Leader Aircraft (max. TO weight)</th>
<th>Following Aircraft</th>
<th>Separation (nautical miles)</th>
<th>Time delay (sec) (appr. speed 70m/sec)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Heavy</td>
<td>Heavy</td>
<td>4</td>
<td>106</td>
</tr>
<tr>
<td>Heavy</td>
<td>Medium</td>
<td>5</td>
<td>132</td>
</tr>
<tr>
<td>Heavy</td>
<td>Light</td>
<td>6</td>
<td>159</td>
</tr>
</tbody>
</table>

Table 1: ICAO Separation Distances to avoid wake vortex encounters.

The Wake Vortex Problem

All aircraft create a trailing vortex wake that can persist for many miles behind the generating aircraft depending upon atmospheric conditions. Much of what is known about aircraft trailing vortices is summarized in two excellent review articles by Rossow and Gerz, Holzapfel and Darracq. These articles provide a comprehensive overview of vortex decay, movement of the vortices in the atmosphere, and their influence on following aircraft.

The trailing vortex wake is often referred to as wake turbulence and can pose a potential safety hazard to an aircraft passing through the trailing vortex wake of another aircraft. Aircraft/Wake Vortex Encounters are most likely to occur during the take-off or landing phase of flight due to the close proximity of landing or departing aircraft. The wake vortex upset problem has been the subject of much research and numerous conferences. References 4-10 are some of conferences held in North America. A comprehensive annotated bibliography of aircraft wake vortices was published by James Hallock. An updated online version of the Wake Vortex Bibliography can be found at the Volpe Center homepage. In addition there have also been numerous conferences in Europe on aircraft wake vortex research during the past decade.

Figure 1 is a sketch depicting three potential aircraft encounters with a trailing vortex wake. The type of upset that can occur depends upon how the airplane penetrates the wake. McGowan (Reference 12-13) examined the effect on an aircraft crossing perpendicular to the trailing vortex wake of another aircraft. His analysis showed that crossing the wake in this manner could result in significant structural loads during the wake encounter particularly for general aviation aircraft. Another possible encounter is for the airplane to fly between the trailing vortices (i.e. in the downwash region), in this case the penetrating airplane will experience a reduction in its rate of climb (for a climbing aircraft) or an increased rate of descent for a descending aircraft. The final encounter depicted in Figure 1 is when the airplane penetrates along the vortex axis. The velocity field of the vortex wake induces a roll moment on the penetrating aircraft. This type of encounter is believed to be the most dangerous because the vortex may cause a large roll excursion that would be unacceptable at a low altitude. Flight test results have shown that relatively large aircraft can be susceptible to significant roll excursions (References 14-17). Computer simulations of aircraft/vortex wake encounters have shown that the magnitude of the response is very dependent on the entry angle into the vortex system (References 18-24) and that the pilot or autopilot control system input can aggravate the upset excursion.

Why should Cruise Altitude be a concern?

Although no accidents have been attributed to wake vortex encounters at cruise altitude there have been a number of wake upsets that have caused large roll and heading excursions. These encounters resulted in split coffee and frightened passengers. While these incidents have been rare the likelihood of wake encounters will increase due to a number of factors. These factors include reduced vertical minimum separation (RVMS), the increase traffic at cruise altitudes due to the adoption of RVMS, and aircraft disparity in terms of size and weight at cruise altitudes.

There have been very few studies that have focused on aircraft/wake vortex encounters at cruise altitude. Rossow and James examined wake vortex decay and cross track encounters at cruise altitudes. They showed that
the wake intensity (circulation) at cruise altitudes was comparable to that during the take-off and landing. The study focused on wake decay issues and cross track penetrations. They concluded that it was not possible to predict when a safe cross track encounter was possible due to the variability of vortex decay time caused by atmospheric effects. The paper also recommended that additional analysis of cross track penetration was needed to identify and define potential safety issues at cruise altitudes.

![Figure 1. Vortex Penetration Scenarios.](image)

II. Cruise and Take-off Conditions

A comparison of the vortex induced rolling moment at low speeds such as take-off, landing and cruise altitude is developed here and is similar to that presented by Rossow and James. The circulation of the trailing vortex wake can be shown to be

\[ \Gamma = \frac{W}{\rho V_\infty b_v} \]  

where, \( W \) is the weight of the generating airplane, \( \rho \) is the air density, \( V_\infty \) is the flight speed and \( b_v \) is the span of the trailing vortices. Now if we form the ratio of the circulation estimate at take-off to the circulation at cruise conditions we obtain the following equation.

\[ \frac{\Gamma_{TO}}{\Gamma_{cruise}} = \left[ \frac{W_{TO}}{W_{cruise}} \right] \left[ \frac{\rho_{cruise}}{\rho_{SL}} \right] \left[ \frac{V_{cruise}}{V_{TO}} \right] \]  

Assuming typical take-off and cruise conditions one can easily show that the circulation at cruise altitude is nearly the same as at sea level.

\[ \frac{\Gamma_{TO}}{\Gamma_{cruise}} \approx 1.0 \]  

If we assume that the vortex field at altitude behaves in a manner similar to low altitudes, the rolling moment induced on an aircraft wing penetrating along the vortex axis can be estimated.
\[ L_v = 2 \int_{b/2}^{b/2} \sqrt{\frac{\rho}{\rho_0}} V_\theta(y) C_{f_v} \left( \frac{V_\theta(y)}{V_\infty} \right) Q c dy \]  

(4)

where \( V_\theta(y)/V_\infty \) is the local angle of attack induced by the vortex field across the penetrating wing. Rearranging this equation yields,

\[ L_v = \rho V_\infty \int_{b/2}^{b/2} \sqrt{\frac{\rho}{\rho_0}} V_\theta(y) c dy \]  

(5)

The integral is only a function of the vortex velocity field which in turn is a direct function of circulation of the generating aircraft, and the aerodynamic and geometric characteristics of the aircraft penetrating the wake. Having shown that the vortex strength at cruise is roughly the same as during take-off or landing and assuming the vortex velocity field is similar, then the integral would yield the same value for either cruise or sea level flight. This assumption implies that the vortex wake is the same age at cruise altitude and sea level. The ratio of the roll moment at cruise to that at sea level can be expressed as follows:

\[ \frac{L_{v,\text{cruise}}}{L_{v,\text{SL}}} = \left[ \frac{\rho_{\text{cruise}}}{\rho_{\text{SL}}} \right] \left[ \frac{V_{\text{cruise}}}{V_{\text{TO}}} \right] \geq 1 \]  

(6)

Equation (6) tell us that if the vortices are of the same age and strength then the vortex induced rolling moment would be essentially the same at cruise altitude as during take-off and landing.

\[ \frac{L_{v,\text{cruise}}}{L_{v,\text{SL}}} = \geq 1 \]  

(7)

This simple analysis leads us to conclude the following two points regarding aircraft wake encounters.

**Wake strength and Vortex Induced Loads**

**Point # 1.** Vortex wake strength at cruise altitude is comparable to that at take-off and landing conditions

**Point # 2.** The induced roll moment on a following aircraft at cruise altitude is the same order of magnitude as would occur during take-off and landing for a wake generated by an aircraft of similar weight and vortex age.

---

### III. Wake Movement and Reported Cruise Upsets

Woodfield\textsuperscript{26-29} examined wake encounters at cruise altitude and discusses the implications of Reduced Vertical Separation Minimums (RVSM) in European airspace. RVSM initiative reduces the minimum vertical separation requirement between aircraft for cruise altitudes of 29,000 ft. to 41,000 ft. from 2,000 feet to 1,000 feet. At cruise altitude the turbulence level can be quite low and the vortices decay slowly. The downward movement of the vortices is of interest in examining cruise altitude encounters. In Reference 26, Woodfield developed a model for wake descent similar to models proposed by Greene\textsuperscript{30} and Sarpkaya\textsuperscript{31}. The vortex oval containing the two counter-rotating trailing vortices descends due to the mutual downward velocity of the two trailing vortex filaments. However, the fluid within the oval has increasing buoyancy as it descends due to the lower density air entrained at the altitude where the vortices were created. From his analysis he concludes that the wakes generated at any of the RVSM altitudes would not descend far enough to be a safety concern for aircraft flying at the next lower (1,000 ft. below) RVSM altitude. This is consistent with the observations made in the flight tests of wake encounters by
NASA and Boeing in the early 1970s. They found that the trailing vortex wakes descended approximately 700 to 900 feet before leveling off to a constant altitude. The flight tests were conducted under low turbulence conditions.

The motion of the vortices is also influenced by local crosswinds and the vertical movement of the air mass in which the vortices are imbedded. Both updrafts and downdrafts can significantly affect the vertical movement of the vortices. Under certain atmospheric conditions the leeward flow over a mountain range can cause a standing wave or mountain wave. Depending on where the vortices are located relative to the standing wave, the vortices could either experience rapid upward or downward movement. In the case of a strong downdraft the wake could descend well beyond the 700 to 900 ft. measured under calm atmospheric conditions.

In another report Woodfield examined reported roll excursions from wake vortex encounters. Prior to the introduction of RVSM in European airspace he reports that there were a small but steady number of significant wake encounters. Figure 2 shows the maximum roll excursion data presented in Reference 27. All but two of the encounters occurred while the airplane was climbing or descending through the RVSM cruise altitude range. The diamond symbols are encounters that occurred prior to the introduction of RVSM, the square symbols are for encounters that occurred after the introduction of RVSM and the asterisk symbols denote encounters while flying at RVSM altitude. Woodfield concluded RVSM is not likely to increase the occurrence of hazardous wake encounters but recommended that climb and descending encounters need to be monitored.

![Figure 2. Vortex Encounters at cruise altitude](Numbers are \( b_c/b_f \)).

While none of the encounters reported resulted in injury or loss of life these events certainly were of concern to the flight crew and passengers. One encounter described in Reference 27 is for a regional jet flying 1000 feet below a Boeing 747. The pilot’s remarks about the vortex encounter are reproduced below.

> “The aircraft violently banked 30 degrees to the left and the nose raised approximately 6-7 degrees (nose up) and the autopilot did automatically disconnect. At that point the a/c very violently banked to the left (?) with a nose down attitude approximately 45 degrees bank to the right (?). At most I think we were approximately at 32,200-33,300 ft. Lot spilled (no injuries).”

Another study conducted in the United Kingdom to examine clear air turbulence encounters at cruise altitude determined that approximately 5% of the reported clear air turbulence incidences were more likely wake vortex encounters. Information on unexpected roll excursions was collected as part of a study conducted by the Civil Aircraft Airworthiness Data Recording Program. In this program aircraft from collaborating airlines were equipped with additional sensors and recording equipment. Britton describes one such encounter. The incident occurred when a medium size jet transport encountered the wake of a large wide body transport.

> “Both aircraft were cruising at the same altitude and were between 16 and 20 miles apart when the wake encounter occurred. The medium size transport aircraft rapidly rolled to a roll angle of approximately 70 degrees and experienced rapid excursions in normal accelerations of..."
up to ± 0.6 g. The pilots regained control of the aircraft after losing altitude and having a significant heading change.\textsuperscript{32}

Although the cruise encounters have been rare and no loss of life or serious injury has occurred this could change. During the past decade there have been changes in both the type of aircraft and operational procedures at cruise altitude that in the author’s opinion warrant a more in depth study of aircraft/wake encounters at cruise conditions. In the past decade we have witnessed a rapid growth in the number of regional jet transports. While the regional jets fly short commuter routes they are also used to fly distances of 700-1200 miles. When operated in this manner they fly at the same altitudes as the larger commercial fleet. Other factors that warrant further examination of cruise altitude encounters are the implementation of (RVSM) in the Continental USA in 2005, and the possibility of introducing something like the free flight concept that has been studied by various research groups. Under free flight operation, pilots would be free to select the most direct flight path to their planned destination. Implementation of the free flight concept would increase the number flight path crossings than is possible by current operational procedures. These concepts all are initiatives to increase airspace capacity and to allow aircraft to fly more efficiently.

Based upon the current understanding of how wakes descend from flight test observation and analytical predictive models the following observations can be made.

### Wake Movement and Upsets at Cruise Altitude

<table>
<thead>
<tr>
<th>Point # 3.</th>
<th>Flight test experiments and analysis indicate trailing vortex wakes created under calm atmospheric conditions will descend to an altitude approximately 700 to 900 feet below the altitude of the generating aircraft. Therefore in calm atmospheric conditions the trailing vortices would not be a hazard to following aircraft at either the same altitude or the next lower RVSM altitude.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Point # 4.</td>
<td>At cruise altitude aircraft wake encounters will most likely occur when either the generating or following aircraft are ascending or descending through the designated RVSM altitudes.</td>
</tr>
<tr>
<td>Point # 5.</td>
<td>Under some atmospheric conditions it may be possible to have greater movement of the trailing vortex wake. Downdrafts or updrafts created by mountain waves could cause the wake to descend or ascend. In a downdraft the wake may descend well beyond the next RVSM altitude while in an updraft the wake could end up near or above the altitude where it was generated.</td>
</tr>
<tr>
<td>Point # 6.</td>
<td>Factors contributing to increased likelihood of a trailing vortex wake encounter at cruise altitudes are RVSM (doubles the number of aircraft), growing disparity in aircraft size, and potential of implementation of the free flight concept.</td>
</tr>
<tr>
<td>Point # 7.</td>
<td>Cruise encounters have been reported by pilots and the study being conducted in Europe should help in assessing the severity of cruise altitude encounters.</td>
</tr>
</tbody>
</table>

### IV. Wake Decay

The next issue that needs to be examined is how fast the trailing vortex wake decays. The trailing vortex wake can be divided into near-field and far-field regions. In the near-field region, the wake is characterized by the roll-up of the vorticity shed from the wing trailing-edge and the merging of co-rotating vortices. In the near-field region the wake experiences very little decay. Early experiments to understand the decay process of the trailing vortex wake showed several interesting features. Ciffone and Orloff\textsuperscript{33} found that the maximum circumferential velocity within the vortex at first remained relatively constant with time and then decayed inversely with the square root of time.

These experimental findings prompted J. D. Iversen to develop a correlation function based on a self-similar variable-eddy viscosity decay model. Iversen used the derived correlation function with a series of wind tunnel, towing tank and flight test measurements. All the experimental data were obtained under calm atmospheric conditions or in a low turbulence wind or water tunnel facilities. The scaling parameters were found to collapse the data from flight test and ground based experiments as illustrated by Figure 3. As indicated by this figure the decay of the trailing vortex wake is quite slow under low turbulence conditions.

Later, Nelson and Tartaglione used Iversen’s approach to correlate a series of experiments conducted by the Federal Aviation Administration (FAA) at the National Aviation Facilities Experimental Centre (NAFEC) in the 1970s. The FAA measured the wake behind various transport aircraft using an instrumented tower. Nelson and
Tartaglione found that Iversen’s scaling parameters were effective in reducing the NAFEC wake data to a single curve as shown in Figure 4.

Figure 3. Maximum circumferential velocity as a function of downstream distance by Iversen.

Figure 4. Iversen’s correlation applied to NAFEC, Nelson and Tartaglione.
At low altitudes, atmospheric conditions such as turbulence influence the trajectory and rate of decay of the vortices. As indicated by the work of Iversen, the trailing vortex wake decays quite slowly. However the trailing vortex wake can undergo a significant distortion due to atmospheric disturbances. The perturbations created by the atmospheric gusts and turbulence are known to excite instabilities in the trailing vortex wake that can cause the wake to decay more rapidly than that which would occur by viscous diffusion.

The existence of vortex instabilities in aircraft wakes has been the subject of numerous investigations. The most notable is the landmark analysis conducted by Crow. He showed that a perturbed vortex pair would exhibit a long wave length instability that is amplified by the mutual induction of one vortex on another. Crow suggests, through the use of a two-vortex model, that wake vortices exhibit sinusoidal motions that are generally confined to fixed planes, inclined to the horizontal. He also found that the spacing between the vortices was an important factor in the growth of the instability. Crow and Bate extended Crow’s theory to include perturbations caused by atmospheric conditions. It was further suggested that the time to vortex linking is a function of the effective span of the vortices. Today this type of instability is called the Crow instability.

The Crow instability can be observed in the contrails of jet transports at cruise altitude on a clear day. At some distance behind the aircraft the wake begins to oscillate in a sinusoidal manner. The amplitude grows with increasing time (distance behind the generating aircraft) until the vortices pinch together to form a series of vortex rings. The vortex rings are the last stage of the instability and it is often suggested that the rings decay rapidly. This argument is largely due to the disappearance of the rings in the contrail pattern. Unfortunately, there is no theoretical or experimental proof to support this observation. However, from flight test experiments of aircraft penetrating a trailing vortex system in the final stages of the Crow instability, i.e. vortex rings, the vortex induced rolling moment was found to be markedly reduced in magnitude.

The time it takes for the Crow instability to grow so that the vortices pinch together or link to form vortex rings was found to be proportional to the square of the vortex span and inversely proportional to the circulation. Mathematically the time to linking is given below.

\[ t^* \sim \frac{b_v}{\Gamma_{\infty}} \]  

where \( b_v \) is the vortex span and \( \Gamma_{\infty} \) is the circulation of the generating aircraft. Condit and Tracy use experimental data shown in Figure 5 to establish the constant of proportionality for Equation (8). Using the constant of proportionality and with some rearranging of equation (8) one can obtain the following relationship for the nondimensional time to link.

\[ t^* = k_t \frac{\rho_{\infty} V_{\infty} b_v^3}{W} \]  

Equation 9 is a function of \( \rho_{\infty} \) the air density at the altitude of the vortices and \( V_{\infty}, b_v, W \) are the velocity, wing span, and weight of the generating aircraft, respectively.

Using Equation (9) and the weight and span data for heavy transport aircraft, an estimate of the time for the trailing vortices to link and create vortex rings is shown in Table 2. A weight range was selected for each airplane to represent weights that might be realistic for the start and end of cruise flight. Each airplane was assumed to be flying at a Mach number of 0.85 and 35,000 ft. The table includes the time to link as well as the distance behind the aircraft in nautical miles. Note that the longer times and distances occur at the end of cruise because of the lower weight.

### Wake Decay at Cruise Altitude

| Point # 8. | Wake vortex decay can be divided into a near field region (very little decay) and far field region. The near field region of the wake exhibits very little decay and is called the plateau region. At cruise altitudes this region lasts for approximately 50 spans behind the wake generating aircraft. |
| Point # 9. | After the plateau region the wake decay follows the wake decay model as developed by Iversen. |

American Institute of Aeronautics and Astronautics
Point # 10. The Crow instability at cruise altitude occurs much farther aft of the generating aircraft than at low speed in the terminal area

Figure 5. Experimental data on the time to vortex linking.

V. Simulation Studies.

Much of what we know about the severity of vortex wake hazard was determine from flight test experiments conducted by NASA\textsuperscript{14-16} and the RAE\textsuperscript{17}. However, before reviewing the flight test results some discussion of the difficulties in interpreting the response data is in order. This is most easily accomplished by reviewing some of the finding from computer simulation of wake aircraft encounters.

There are several factors that influence the maximum roll excursion. The relative span of the following wing to the generating wing, $b/b_G$, and the lateral location of the following wing to vortex core are important factors. A simple presented by Nelson and McCormick\textsuperscript{18} is used to demonstrate the influence of aircraft size and lateral location of the vortex on the magnitude of the vortex induced rolling moment. The analysis was conducted using two geometrically similar rectangular wings. Figure 6 shows the maximum vortex induced roll moment versus the relative span of the following wing to that of the wake generating wing. The strength of the wake generating wing was assumed to be one minute old. Here we see that wings of similar span have the control authority to overcome the vortex induced rolling moment. However, for wings that are less that $2/3$ the generating wing span...
would experience a vortex induced roll moment greater than the control capability of the following airplane for this separation distance. The rectangular planform shape was selected for the ease of calculating the induced roll moment. A more realistic planform shape representative of a commercial transport would be a swept tapered wing. However, similar trends as shown in figures 6 and 7 would be expected.

<table>
<thead>
<tr>
<th>Aircraft Type</th>
<th>Weight Range W lbs (kg)</th>
<th>Span b ft (m)</th>
<th>Circulation Γ ft²/sec (m²/sec)</th>
<th>t* Time to Link sec</th>
<th>Distance Nautical Miles (km)</th>
</tr>
</thead>
<tbody>
<tr>
<td>B747</td>
<td>875,000 to 520,000</td>
<td>211.4 (64.4)</td>
<td>8313 to 5132 (772 to 477)</td>
<td>97.1 to 155.2</td>
<td>13.2 to 21.1 (24.4 to 39.0)</td>
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<tr>
<td></td>
<td>(396,890 to 235,935)</td>
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<td></td>
<td></td>
<td></td>
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<tr>
<td>B777</td>
<td>660,000 to 375,115</td>
<td>200 (60.9)</td>
<td>6540 to 3913 (608 to 364)</td>
<td>109 to 182.2</td>
<td>14.8 to 24.8 (27.4 to 45.9)</td>
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<td></td>
<td>(297,560 to 170197)</td>
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<tr>
<td>B767</td>
<td>412,000 to 261,000</td>
<td>180.25 (54.9)</td>
<td>4767 to 3020 (443 to 280)</td>
<td>127.8 to 191.7</td>
<td>17.4 to 26.1 (32.2 to 48.3)</td>
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<td></td>
<td>(186,880 to 118,390)</td>
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<tr>
<td>B757</td>
<td>272,500 to 200,264</td>
<td>124.8 (38.05)</td>
<td>4327 to 3348 (402 to 311)</td>
<td>64.2 to 82.9</td>
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<td></td>
<td>(123,600 to 90,863)</td>
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<tr>
<td>A380</td>
<td>1,239,000 to 723,000</td>
<td>261 (79.8)</td>
<td>9383 to 5,764 (872 to 535)</td>
<td>130 to 212</td>
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<td></td>
<td>(561905 to 327891)</td>
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<tr>
<td>A340</td>
<td>840,400 to 470,930</td>
<td>208.2 (63.45)</td>
<td>5742 to 4176 (533 to 388)</td>
<td>121.8 167.4</td>
<td>16.7 to 23.0 (30.9 to 42.6)</td>
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<td>(387734 to 213573)</td>
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<tr>
<td>A330</td>
<td>515,700 to 353,979</td>
<td>197.8 (60.3)</td>
<td>5167 to 3733 (480 to 347)</td>
<td>135 to 187</td>
<td>18.4 to 25.4 (34 to 47.0)</td>
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<td></td>
<td>(233,878 to 160,535)</td>
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<tr>
<td>A300</td>
<td>380,500 to 267,100</td>
<td>147.1 (44.84)</td>
<td>5124 to 3788 (476 to 352)</td>
<td>75.2 to 101.8</td>
<td>10.2 to 13.2 (18.9 to 24.4)</td>
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<tr>
<td></td>
<td>(172,562 to 121134)</td>
<td></td>
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</tbody>
</table>

Table 2 Estimates of the time to link for large jet transports at cruise altitude. (Based on a cruise altitude of 35,000 ft and Mach number of 0.85)
Figure 6: Influence of the relative span on vortex induced rolling moment coefficients.\textsuperscript{18}

Figure 7 shows the effect of the location of the vortex to the centerline of the penetrating wing. The induced rolling moment drops off and changes sign as the vortex moves out toward the wing tip of the following wing.

Figure 7. Influence of lateral displacement on vortex induced rolling moment coefficient \textsuperscript{18}.

References 19-24 used computer simulations that included; a model of the velocity field created by two straight line vortex filaments, a method to predict the vortex-induced loads on the wing and tail surfaces, the aircraft equations of motion, and a pilot (math model) and/or autopilot control system. Most of the simulation studies focused on in-track wake penetrations. An in-track penetration is one where the penetrating aircraft is roughly aligned with one of the trailing vortex filaments. To assess the vortex hazard, one would want to determine the maximum roll excursion or maximum roll acceleration as a function of the age of the wake vortex.

Some typical results from a six-degree of freedom computer simulation from References 11-15 are presented below. In this simulation the mathematical model of the pilot tries to maintain a wings level attitude. Figure 8
shows the penetrating angles under consideration. \( \theta_p \) is the angle between the penetrating aircraft’s flight path and the plane of the vortices. The angle \( \psi_p \) is defined as the angle between the flight path and the vortex axis when viewed from above. For the cases considered here the separation distance was 8.5 km (5.3 miles) or approximately 2 minutes behind the generating aircraft. Also it should be pointed out that the flight path of the penetrating aircraft was selected so that it would intersect the vortex core if the aircraft were not influenced by the vortex system.

For an aircraft descending into the vortex system at the angle \( \theta_p \approx -3.0^\circ \), Figures 3-5 illustrate the effect of the oblique penetration angle \( \psi_p \) on the response of the penetrating aircraft. The penetrating aircraft is a business jet and the wake generating aircraft is a light jet transport. In Figure 9 notice that the aircraft descending into the vortex with \( \psi_p \approx 3^\circ \) rolls away from the vortex system. The aircraft is seen to roll slightly and climbs away from the vortex. The aircraft appears to be pushed away from the vortex. The aircraft climbs since it is flying into the upwash region of the port vortex. Upon increasing the penetration angle to \( \psi_p \approx 6^\circ \) we see a significant difference in the aircraft’s response. Figure 10 shows the aircraft rolling sharply to the right and at the same time its rate of descent is increased as the aircraft passes into the strong downwash region between the vortices. Finally, Figure 11 illustrates the effect of including pilot input in the simulation.
Without pilot control the aircraft rolls to an angle greater than 100 degrees whereas with the predicted pilot control input the aircraft rolls to approximately 50° before roll recovery takes place. The time history plot of the yaw angle reveals the influence that the vertical tail plays in the aircraft’s dynamic behavior. As the aircraft enters the vortex field the aircraft is seen to roll in a counterclockwise direction. The simplified aerodynamic analysis shows that the induced rolling moment coefficient reverses sign as the vortex moves toward the wing tip. Thus as
the aircraft approaches the port vortex it initially rolls in the opposite direction with respect to the vortex field. The pilot’s reaction to the disturbance would be to apply aileron control to roll the aircraft in a clockwise direction. Now as the aircraft nears the vortex core the aircraft is rolled sharply in the clockwise direction.

![Figure 11. Yaw and Roll Response](image)

Thus we see that the pilot’s initial reaction would momentarily be out of phase with the roll disturbance. Also during the vortex encounter the vertical tail experiences an induced velocity from the left which causes the aircraft to yaw to the left by approximately 12°. As the aircraft passes into the field of influence of the starboard vortex the upsetting roll moment tends to aid the pilot in regaining a wings level attitude. The yaw excursion to the right is due primarily to the roll orientation.

Dynamic simulations of an aircraft/ trailing vortex encounter clearly show how sensitive the response is to the entry angle into the vortex system and control input by the pilot or autopilot. If the entry angle is shallow the airplane is rolled away from the vortex. However as the entry angle is increased the aircraft penetrates through the vortex core resulting in large roll and yaw excursions. Pilot or autopilot control input can increase magnitude of the potential upset by forcing the airplane into the vortex core region. Similar results were found by Vasatka (reference 15) for a twin engine jet transport in a trailing vortex wake of a large jet transport.

While the simulations shown here were for a business jet flying into wake created by a light jet transport, similar results were obtained for a light jet transport flying into the wake of a larger transport.

**Simulation Observations**

Point # 11. Simulation has shown that the magnitude of the roll and heading excursions are very sensitive to the elevation and heading entry angles $\theta_p$ and $\psi_p$, respectively. If the entry angle is too shallow the airplane is pushed away from the vortex, however, a larger entry angle can result in a large roll and heading excursion. The maximum excursion occurs when the airplane penetrates nearly centered in one of the trailing vortices.

Point # 12. The sensitive nature of the entry angle to the magnitude of the aircraft response helps explain the scatter that is typically observed in flight test measurements of aircraft/wake encounters.

**VI. Flight Test Results**

Figure 12 shows the vortex induced roll acceleration of three different aircraft encountering the trailing vortex wake of a C-5A in a landing configuration. Clearly safe separation distances at low altitudes would require separation distance where the ratio of the induced acceleration to the maximum acceleration capability of the roll control system needs to be less than 1. Safe separation obviously depends on the size of the following airplane.
Earlier we showed that the induced roll moments at cruise altitude where essentially the same as during landing approach if the generating aircraft weight and vortex age were similar at altitude. This implies that separation distances at altitude would be at least 3.5 times greater than in landing approach.

Flight test experiments also showed that the most hazardous wakes were created by aircraft flown in a clean configuration (i.e. wing flaps and landing gear in the retracted position). When the wing flaps are down multiple pairs of vortices can exist in the wake that may lead to a more rapid decay of the vortex velocity field that is possible with the clean configuration. Another possibility is the multiple vortex pairs create a more benign velocity field which results in lower induced rolling moments.

![Figure 12. Ratio of vortex induced roll acceleration to the maximum lateral control power versus separation distance. The generating aircraft was aC-5A in landing configuration.](image)

**Flight Test Observations**

**Point # 13.** The wakes generated by the clean configuration (gear up and flaps retracted) produced larger roll excursions than the landing approach configuration.

**Point # 14.** Flight test data indicated that wake separations of 5-8 nautical miles were required before the following aircraft had sufficient control power greater than the induced roll of the generating aircraft at approach speeds. At cruise altitude the separation distances for the same age wakes would be 17.5 to 28.0 nautical miles.

**VII. Conclusions**

Based upon the discussion and observations presented in this paper it is the author’s position that additional studies need to be conducted to assess the potential hazard of wake vortex encounters at cruise conditions. The study needs to focus first on identification of hazardous flight conditions. If hazardous conditions are uncovered then it may be possible to develop appropriate operational strategies to minimize or eliminate the safety concerns.

A summary of the issues with regard to wake vortex encounters at cruise altitude is presented below.

1. The vortex induced roll moment at cruise altitude is comparable to that during take-off or landing when the generating aircraft has the same weight and the wake is of the same age.
2. While atmospheric turbulence is known to hasten vortex decay, the turbulence levels at cruise altitudes...
are generally quite low and therefore the wake will decay rather slowly. The time for the Crow instability to occur is the same at both sea level and cruise altitude for the same turbulence level. Therefore the time to linking is the same.

3. The wake of an aircraft in a clean configuration (flaps and landing gear up) produced larger upsets than when the gear and flaps were deployed. Flight tests conducted at low altitudes in calm atmospheric conditions provide some insight into problems at cruise altitudes.

4. The reduced vertical separation minimums (RVSM), increased traffic and disparity in the size of aircraft flying at cruise altitudes warrants additional study of wake encounters during cruise.

5. The most likely condition for an encounter is when either the wake generating aircraft or the penetrating aircraft are ascending or descending through a cruise altitude.

References