# EE 30372, Spring 2010 <br> Final Exam <br> 4 May, 2010 

Show all your work and your answers clearly on the test pages. In any plots and sketches, label and include units (if possible) on anything that might be of interest. You are each allowed one two-sided 8.5 by 11 inch page of formulae for reference. Calculators may be used only for simple trigonometric and (complex variable) arithmetic operations. For full credit, simplify your answers as much as possible.

Problem 1 (30) $\qquad$

Problem 2 (15) $\qquad$

Problem 3 (15) $\qquad$

Problem 4 (40) $\qquad$

Problem 5 (30) $\qquad$

Total (130)

Name $\qquad$

1. Shorter exercises (5 pts. each):
(a) A transmission line of impedance $(0.03+j 0.10) \Omega_{p u}$ is carrying current of rms magnitude $0.5 A_{p u}$, with the load having power factor 0.8 lagging. Assume the voltage at the "receiving" end is $1.0 V_{p u}$. Compute "sending" end voltage and sketch an accurate phasor diagram for this state.
(b) A 5 MVA load, with power factor 0.7 lagging, is connected in parallel with a 1 MVA capacitor bank. What is the power factor of the parallel combination?
(c) A transformer, rated at 10 MVA and $600 \mathrm{~V} / 14 \mathrm{kV}$, has internal impedances of $R_{p u}=0.05 \Omega$ and $X_{p u}=0.1 \Omega$. This transformer is to be installed in a system with base power of 20MVA, operating with low-side base voltage of 480 V . Find the pu impedance under the installed system's base values.
(d) The bus admittance matrix of a system has the form

$$
\mathbf{Y}_{\text {bus }}=\left[\begin{array}{ccc}
-j 4 & j 2 & j 2 \\
j 2 & -j 3 & j 1 \\
j 2 & j 1 & -j 3
\end{array}\right]
$$

What is the bus impedance matrix?
(e) If $\mathbf{V}_{\mathbf{A}}=1.0, \mathbf{V}_{\mathbf{B}}=1.5 \angle-90^{\circ}$ and $\mathbf{V}_{\mathbf{C}}=2.0 \angle-180^{\circ}$, find the symmetric component voltages.
(f) Two busses are separated by a transmission line with impedance $j 1$. If both bus 1 and bus 2 have voltage magnitude 1.0, how much would we advance the phase of bus 1 to deliver real power of 0.1 to the second bus?
2. Starting with the definition of symmetric components, find the most compact expression possible for the total three phase power delivered for fixed voltages and currents in an unbalanced system, in terms of its symmetric components.
3. ( 15 pts .) A 100 km long transmission line operating at 75 kV (input) and 60 Hz has parameters $r=0.1 \Omega / \mathrm{km}, x=0.5 \Omega / \mathrm{km}$ and $y=5 \times 10^{-6} \mathrm{~S} / \mathrm{km}$. Find the charging current in this state (with receiving end unloaded).

4. The components of the power system above have the following ratings:

Generator $1\left(G_{1}\right)$ :
Motor $2\left(M_{2}\right)$ :
Motor $3\left(M_{3}\right)$ :
All $Y-\Delta$ transformers:
All $Y-Y$ transformers:
All $\Delta-\Delta$ transformers:
Line 1:
Line 2:
Line 2: $\quad X=20 \Omega$
Regardless of transformer type, their high voltage sides will be connected to the transmission line.
(a) (10 pts.) Using the given numbering of the busses for rows/columns of matrices, find the per-unit admittance matrix for this system, in its balanced state, for power flow calculations. Use a base power of 50 MVA , and base voltage of 10 kV at the generator.
(b) ( 10 pts.) Let bus 1 be the slack bus. Motor 2 is consuming power $0.3+\mathrm{j} 0.1$, and Motor 3 is consuming $0.2+\mathrm{j} 0.1$. With the initial state of the system being $1.0 V_{p u}$ at all busses, write the equation for the first update of the voltage at bus 2 , and find the value of the voltage after that first update.
(c) (10 pts.) Suppose you have run Gauss-Seidel updates on the load bus equations until convergence, and at this point you have $V_{2}^{k}=0.94 \angle-10^{\circ} V_{p u}$ and $V_{3}^{k}=0.90 \angle-21^{\circ} V_{p u}$. Find the power being supplied at the slack bus (at $1.0 V_{p u}$ ). Without further detailed calculations, judge whether this is a possible solution to the problem posed in (b). State your reasoning clearly.
(d) (10 pts.) Find and sketch the per-phase equivalent circuit for the zero-sequence symmetric component of this system, and find the zero-sequence bus impedance matrix.
5. Suppose we have the following per-unit bus admittance and impedance matrices for the three symmetric components of a 3-phase power system. The system has $S_{\text {base }}=10 \mathrm{MVA}$, and $V_{\text {baseLL }}$ of 10 kV at all of the busses. Assume that in normal, balanced operation, the system behaves according to the positive-sequence parameters.

$$
\begin{aligned}
& \mathbf{Y}_{b u s 0}=\left(\begin{array}{ccc}
-j 5 & 0 & 0 \\
0 & -j 10 & j 4 \\
0 & j 4 & -j 4
\end{array}\right) \mathbf{Y}_{\text {bus } 1}=\left(\begin{array}{ccc}
-j 4 & j 2 & j 1 \\
j 2 & -j 4 & j 1 \\
j 1 & j 1 & -j 5
\end{array}\right) \mathbf{Y}_{b u s 2}=\left(\begin{array}{cc}
-j 5 & j 2 \\
j 2 & -j 6 \\
j 1 & j 4 \\
j 1 & -j 10
\end{array}\right) \\
& \mathbf{Z}_{b u s 0}=\left(\begin{array}{ccc}
j 0.2 & 0 & 0 \\
0 & j 0.17 & j 0.17 \\
0 & j 0.17 & j 0.42
\end{array}\right) \mathbf{Z}_{b u s 1}=\left(\begin{array}{ccc}
j 0.40 & j 0.23 & j 0.13 \\
j 0.23 & j 0.40 & j 0.13 \\
j 0.13 & j 0.13 & j 0.25
\end{array}\right) \mathbf{Z}_{b u s 2}=\left(\begin{array}{ccc}
j 0.28 & j 0.15 & j 0.09 \\
j 0.15 & j 0.31 & j 0.14 \\
j 0.09 & j 0.14 & j 0.16
\end{array}\right)
\end{aligned}
$$

(a) (10 pts.) A single line-to-ground fault (line A) occurs at bus 2. Assuming that before the fault, all busses held voltage $1.0 \angle 0^{\circ}$ in the A phase and all of the system was balanced, find the fault currents in all phases at bus 2 .
(b) (10 pts.) Compute the (real) voltage $V_{C}$ at bus 1 during the fault of part (b).
(c) (10 pts.) Sketch a 3-bus power system, with appropriate impedances attached to busses such that $\mathbf{Z}_{b u s 1}$ is its impedance matrix for balanced operation.

