

MATH 13187: University Seminar spring 2017 *Practice for Exam 1*

Instructions: Clearly explain the answers to the following questions. If appropriate, you may do this by indicating steps clearly without many words. Showing your work may help you get partial credit. Note: this practice exam is longer than the exam you will take on Monday, February 22

1. Suppose the olympic basketball coach has 1 center, 5 forwards, and 3 guards on her team. Her starting line-up must have 1 center, 2 forwards, and 2 guards. How many different starting line-ups can she choose? (see Unit 4, especially sections 1.4 and 1.5)
2. (a) Find $\gcd(20, 48)$ and $\text{lcm}(20, 48)$. (See Unit 6)
(b) Is it possible to write 4 as a combination of 20 and 48? If yes, show a way of doing it. If no, explain why not. (Unit 7)
(c) Is it possible to write 6 as a combination of 20 and 48? If yes, show a way of doing it. If no, explain why not. (Unit 7)
3. (Unit 8) (a) Give the prime factorization of 198.
(b) Decide whether 199 is prime.
4. (Unit 4, 1.4) There are 4 toppings at the salad bar for you to put (or not put) on the bowl of lettuce you hold in your hands.
(a) Assuming that you like every topping, how many possible salads could you create with 2 toppings, if the order on which you place the toppings does not matter?
5. (Unit 1 and 1.4 of Unit 3) How many numbers are there between 20 and 120 that are not divisible by 6? How many numbers are there between 20 and 120 that are divisible by 6 and not by 15?
6. (Unit 1 and 1.4 of Unit 3)
 - (a) How many numbers from 48 to 237 are multiples of 14?
 - (b) How many numbers from 48 to 237 are multiples of 20?
 - (c) How many numbers from 48 to 237 are common multiples of 14 and 20?
 - (d) How many numbers from 48 to 237 are multiples of 14 or 20?
 - (e) How many numbers from 48 to 237 are not multiples of 14 or 20?
7. (Unit 6) Find $\text{lcm}(271, 620)$. How does it compare to 271×620 ?
8. (Unit 7) Write 1 as a combination of 11 and 29.

9. (Unit 7) Write 1 as a combination of 351 and 854. Write 3 as a combination of 351 and 854.
10. (Unit 8) Give a prime factorization of 844.
11. (Unit 8) Find the smallest prime bigger than 300.
12. (Unit 5) Express $\binom{43}{13} + \binom{43}{14} + \binom{44}{13}$ as a binomial coefficient $\binom{m}{k}$.
13. (Unit 4, 1.4) Suppose Cathy is choosing a committee of four kids from a class with 14 boys and 16 girls.
- (a) How many ways can she choose the committee?
 - (b) How many ways can she choose the committee if it must have 2 boys and 2 girls?
 - (c) How many ways can she choose the committee if it must have 3 boys and 1 girl?
14. (Unit 4, 1.4) Seventeen people are running for the Republican nomination for president. Fred is required to break them into two groups for debates. If the first debate is to have 8 people, how many ways can Fred choose the people?
15. (Unit 1 and Unit 2 and Unit 4)
- (a) How many ways are there to pick four different numbers from between 5 and 43 if the order in which the numbers are chosen matters.
 - (b) How many ways are there to pick four different numbers between 5 and 43 if the order in which they are chosen does not matter?
16. (Unit 2)
- (a) How many license plates are there with 3 letters and 4 numbers?
 - (b) How many license plates are there with 3 letters and 4 numbers, if the last letter cannot be X, and the first number cannot be 0?
17. (Unit 9) How many numbers divide the number 26,000?
18. (Unit 9) How many numbers divide 211?
19. (Unit 9)(a) How many numbers divide $2^4 \cdot 3 \cdot 5^7 \cdot 13$?
- (b) Let $d = \gcd(m, n)$, where $m = 2^2 \cdot 3^2 \cdot 5 \cdot 7$ and $n = 2^4 \cdot 3 \cdot 5^7 \cdot 13$. Find the prime factorization of d . How many numbers divide d ?
 - (c) Find the prime factorization of the least common multiple of $2^2 \cdot 3^2 \cdot 5 \cdot 7$ and $2^4 \cdot 3 \cdot 5^7 \cdot 13$.

20. Does 35 divide $\binom{48}{18}$?

21. Does 19 divide $\binom{48}{18}$?