DYNAMICAL MODELING OF THE LARGE-SCALE MOTION OF A PLANAR TURBULENT JET USING POD MODES.

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The existence and importance of the large-scale structures, that is large regions of coherent fluid motion in fully turbulent flows has been experimentally proven and a substantial body of research has been performed in this area. Research has clearly shown that these structures have a significant effect on the macrocharacteristics of the flow, like mass, heat transfer and mixing. Thus knowledge of dynamics and possible ways of controlling these structures have a great practical importance.

An objective and rigorous way of extracting large-scale structure from turbulent flow is the Proper Orthogonal Decomposition (POD), proposed by Lumley [4]. The idea is to decompose a given turbulent flow into a minimum number of modes by maximizing the energy content in each mode. A few important results from POD theory which make it a powerful tool for investigating turbulent flows are worth mentioning here. First, the POD gives a complete set of orthogonal modes and the flow can be expanded in this series. Second, the series exhibits the most rapid convergence among all possible complete orthogonal sets. Third, the input parameter to calculate the POD modes is a two-point correlation tensor which is relatively straightforward to measure.

The rapid convergence POD modes makes them useful for building low-order dynamical models of different turbulent flows. Example 'success stories' are a turbulent boundary layer model developed by Aubry et al. [1] and the dynamical model of a turbulent mixing layer [7]. Another candidate flow for investigating large-scale structure dynamics is the turbulent planar jet. Two-point correlation measurements across the centerline of the jet suggested the presence of well-defined large-scale structures. The interested reader is referred to [6] for a comprehensive review on structure in jets.

Detailed measurements of the two-point cross-correlation tensor in the self-similar region of the planar turbulent jet in [2] have shown the self-similarity of the underlying large-scale structure, when conventionally scaled by the local mean velocity half-width b and local maximum velocity. The large-scale structure was reconstructed in physical space by a Fourier transform-based projection of measured u, v, and w-component POD eigenmodes onto instantaneous flow field realizations in [3]. Results indicate that a dominant planar component of the large-scale structure in the planar jet consists of two lines of large-scale spanwise vortices arranged approximately asymmetrically with respect to the jet centerline, as shown in Figure 1. This planar component of

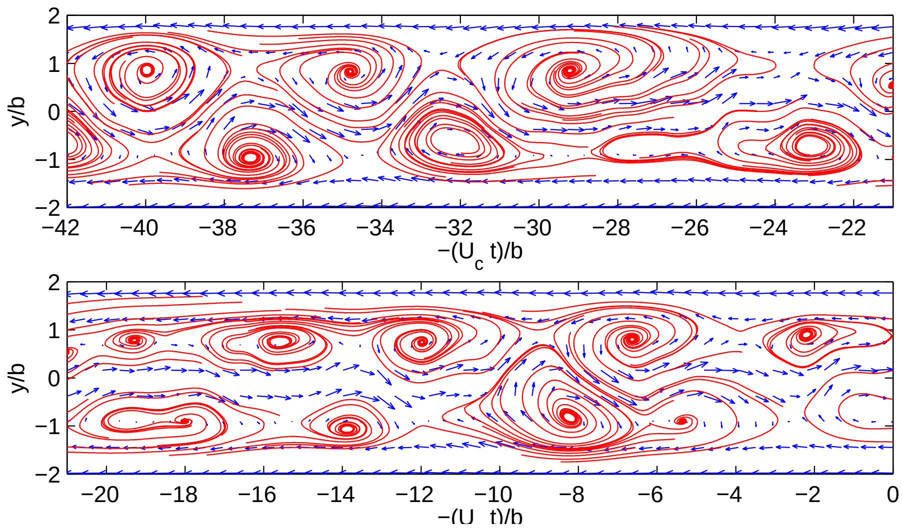

Figure 1: The velocity field (u, v) and instantaneous streamlines of the experimentally reconstructed planar structure $u_\alpha^{planar}(y,t)$ from [3]. y is the lateral spatial coordinate and $x = -U_c t$ is a speudo-spatial streamwise coordinate.

the structure resembles the classic Karman vortex street. There is a strong interaction between structures on opposite sides of the jet in the form of lateral streaming motions that extend well across the flow.

Modeling

In was shown in [4], [5] that POD modes are resolved up to an arbitrary phase in homogeneous directions. An alternative way to reconstruct the large-scale structure is to project the experimentally obtained POD mode onto the governing Navier-Stokes equations and thus obtain the phase information necessary to reconstruct the topology and dynamics of the large-scale structure.

In what follows spatial coordinates in the flow will be denoted $x_\alpha \equiv (x, y, z)$ where x, y, z are the streamwise, lateral (cross-stream) and spanwise spatial coordinates respectively. In a similar manner, the corresponding instantaneous velocity components are designated $\widetilde{u}_\alpha(\mathbf{x}, t) = (\widetilde{u}, \widetilde{v}, \widetilde{w})$. The conventional Reynolds decomposition is used to express the velocity as the sum of a time-mean and zero-mean fluctuating component, $\widetilde{u}_\alpha(\mathbf{x}, t) = U_\alpha(\mathbf{x}) + u_\alpha(\mathbf{x}, t)$. Due to the basic geometry of the turbulent planar jet flow field certain simplifying assumptions can be made and these are next described.

1. The flow is statistically homogeneous in the spanwise z direction.

2. The lateral mean velocity component will be neglected.

3. The flow is considered to be homogeneous in the streamwise direction.

The last assumption allows one to treat the streamwise coordinate through the temporal evolution by using a Taylor's frozen field $x = -U_c t$ with U_c taken as a constant convective speed. This allows one to transform the POD eigenmodes from the frequency to the streamwise wavenumber domain. Physically the last assumption is equivalent to letting the spreading rate of the jet be zero (i.e. imposing wall-like boundary conditions which prevent the jet from spreading). One then expects that the new modified system is *locally* dynamically similar to the original system. Using this approach is clearly inappropriate, however, for addressing issues related to global jet dynamics.

Galerkin Projection

After the spatial orthogonal POD modes $\varphi_\alpha^{(n)}(y; k_x, k_z)$ are determined from experiment [2], and exploiting the rapid energy convergence of the POD modes, the fluctuating velocity can be decomposed in the discrete series as

$$\mathbf{u}(\mathbf{x}, t) = \sum_n^N \sum_{\mathbf{k}} c^{(n)}(t, \mathbf{k}) \varphi^{(n)}(y, \mathbf{k}) \exp(i\mathbf{k}\mathbf{x}) + \text{unresolved modes}, \qquad (1)$$

where $\mathbf{k} = (k_x, k_z)$ and N is the number of POD modes kept in the expansion. We will term these modes as "resolved". The number of resolved modes can be varied to investigate the effect of neglected modes on the jet dynamics.

To get a system of ODE's for the temporal coefficients $c^{(n)}(t; k_x, k_z)$, the Galerkin projection procedure onto Fourier-transformed Navier-Stokes is performed. Details can be found in [7], for instance. The final system of finite ODE's for the temporal coefficients $c^{(n)}(t; k_x, k_z)$ can be symbolically written in the matrix form

$$\frac{dc^{(n)}(\mathbf{k})}{dt} = (C^{(n)} + P^{(n)} + \tilde{\nu} D^{(n)}) c^{(n)}(\mathbf{k}) + \sum_{\mathbf{k}', l, m} T_{\mathbf{k},\mathbf{k}'}^{(n,l,m)} c^{(l)}(\mathbf{k}') c^{(m)}(\mathbf{k} - \mathbf{k}') \quad (2)$$

where convection, production, dissipation and nonlinear transfer terms are denoted as $C^{(n)}$, $P^{(n)}$, $D^{(n)}$ and $T^{(n,l,m)}$, respectively. The modified viscosity $\tilde{\nu}$ will be defined below.

Because of the nonlinear nature of the N-S equations, there exists inertial energy transfer from the large resolved modes to smaller unresolved scales. Consequently, the effect of the unresolved scales may be modeled as a dissipative influence on the large scales. This interaction can be parametrically described by a Heisenberg type of spectral model for isotropic turbulence. The algorithm, proposed in [1] provides a way to estimate the turbulent eddy viscosity ν_T from the POD modes. The model introduces an effective turbulent viscosity ν_T into the system and the molecular viscosity shall be replaced by a modified viscosity $\widetilde{\nu} = \nu + \alpha\nu_T$. The parameter α allows one to vary the eddy viscosity, thus allowing to fine-tune the model in order to match experimentally observed dynamics of the resolved large-scale structures.

In the expansion (1) we will restrict ourselves to the first POD mode only $N_{mode} = 1$. Furthermore, due todominance of the planar part of the underlying large-scale structure [3], only two spanwise wavenumbers are kept in the expansion, $k_z b/2\pi = 0$ and 0.5, so $N_z = 2$. Finally, the number of the resolved streamwise wavenumbers was chosen as $N_x = 50$. Thus the total number of solved complex equations in (2) is $2 \times 50 = 100$, or 200 real equations.

Results

The numerical investigation of the low-order system (2) has revealed several regimes. Above $\alpha = 40$ the system has a trivial solution only. At $\alpha = 34$ the system undergoes a Hopf bifurcation. For $\alpha = 40..34$ the coefficients c have a simple periodic behavior with amplitudes reaching constant values. At $\alpha = 34$ a second Hopf bifurcation occurs. The region $\alpha = 32..2$ corresponds to a situation, where the amplitudes exhibit an apparent chaotic behavior. Below $\alpha \simeq 2$ the system eventually reaches infinite values for mode amplitudes in finite timedue to insufficient dissipation.

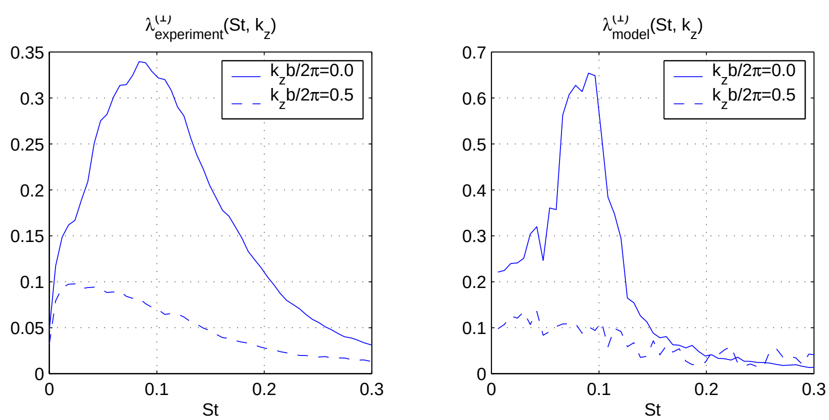

Figure 2: Experimental and model predicted (for $\alpha = 10$) eigenspectrum $\lambda^{(1)}(St, k_z b)$.

One of the criteria to gauge α by comparison of experimental and computational results is to match the eigenvalue spectrum $\lambda^{(1)}(St, k_z) = \overline{|c^{(1)}(St, k_z)|^2}$. For large α, the dissipation effectively damps all modes except only few modes around $St = 0.1$, where production prevails. At small α dissipation is not strong enough and many modes via a nonlinear energy transfer term in (2)

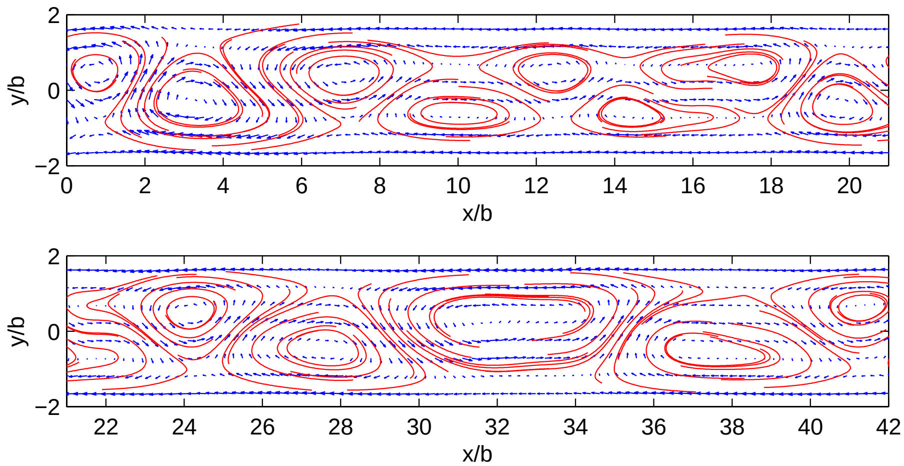

Figure 3: The velocity field (u, v) and instantaneous streamlines of the numerically reconstructed planar structure $u_\alpha(y, t, k_z = 0)$ for $\alpha = 10$.

are excited. Comparison with the experimental spectrum allows one to choose the value of $\alpha \approx 10$ where the shape of the numerical spectrum is close to the experimental one at $k_z b = 0.0$ and 0.5, as shown in Figure 2. Overall, the shape of numerically generated eigenspectrum looks quite similar to the experimental one, although the numerical eigenvalue spectrum is less energetic at high St. The maximum peak is located at $St = 0.1$, where the production term $P^{(1)}$ is essentially non-zero. This matches the characteristic flapping frequency of the planar jet [6].

As can be seen in Figure 2 the model overpredicts the amplitude of the eigenspectrum by a factor of two. The analysis of u_{rms} and v_{rms} also shows the model overshoots the maximum values by a factor of 2..2.5 in the range $\alpha = 10..15$. This may be due to the very crude nature of the model, since it uses only one POD mode and a minimal number of spanwise wavenumber components. Viewed from this point, the model works surprisingly well considering the small number of resolved modes. Additional numerical investigation indicates that the inclusion of more POD modes, especially additional k_z-components will most likely lower the model prediction for velocity amplitudes.

A sample reconstructed velocity field for $\alpha = 10$ is presented in Figure 3. It reveals a striking similarity to the experimentally reconstructed topology of the large-scale structure shown in Figure 1. In fact, many essential features of the structure's topology is correctly predicted by the numerical model, including the y-locations of the vortical structures, as well as the typical size and spacing between them. Thus, the model correctly reconstructs the essential topology of the self-similar jet large-scale structure.

Rigorously speaking, the dynamical model is based on experimental data obtained at one Reynolds number, namely $Re_D = 28,000$ (based on exit velocity and nozzle width). While the set of basis POD functions is still complete, it could become non-optimal for planar jets with different Re. But the viscosity effects appear mostly on small scales. Investigations of large-scale structures in different turbulent flows indicate that the shapes of these structures are macroscale-dependent and are primarily a function of the boundary and initial conditions. So, the coherent structure is mostly governed by inertia forces, rather than viscous ones and the viscosity will not likely change the spatial shape of the first few POD modes. Rather, it will primerily effect the value of the number of involved modes and the dissipation energy rate from the large-scale modes. Hence it is expected that the set of POD modes will still be close to the optimal set and the application of the model can be relatively easily extended for a range of planar turbulent jets with different Re-numbers.

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