DOT AND CROSS PRODUCTS
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LAURENCE R. TAYLOR

Properties of the determinant

- If two rows of a determinant are interchanged, the sign of the determinant changes.
- If two rows of a determinant are equal, the determinant is 0.

A way to remember formula 6 in the book for the cross product is

$$\vec{a} \times (\vec{b} \times \vec{c}\,) = \begin{vmatrix} \vec{b} & \vec{c} \\ \vec{a}\bullet\vec{b} & \vec{a}\bullet\vec{c} \end{vmatrix}$$

The missing analogue of 6 is

$$(\vec{a} \times \vec{b}\,) \times \vec{c} = \begin{vmatrix} \vec{b} & \vec{a} \\ \vec{b}\bullet\vec{c} & \vec{a}\bullet\vec{c} \end{vmatrix}$$

Both can be remembered in English:

The first row of the matrix starts with the middle vector; the second entry is the other vector inside the parenthesis. The bottom row consists of the dot products of the two vectors in the first row with the third vector in the product.

Another useful formula is the one in problem 52 from the book

$$(\vec{a} \times \vec{b}\,)\bullet(\vec{c} \times \vec{d}\,) = \begin{vmatrix} \vec{a}\bullet\vec{c} & \vec{b}\bullet\vec{c} \\ \vec{a}\bullet\vec{d} & \vec{b}\bullet\vec{d} \end{vmatrix}$$

A corollary is

$$(\vec{a} \times \vec{b}\,)\bullet(\vec{a} \times \vec{b}\,) = (\vec{a}\bullet\vec{a}\,)(\vec{b}\bullet\vec{b}\,) - (\vec{a}\bullet\vec{b}\,)^2$$

which, if you unravel it, is a version of $\sin^2(\theta) + \cos^2(\theta) = 1$.

Solve

$$\begin{array}{ccccc} a_1x & + & b_1y & = & d_1 \\ a_2x & + & b_2y & = & d_2 \end{array}$$

$$x = \frac{\begin{vmatrix} d_1 & b_1 \\ d_2 & b_2 \end{vmatrix}}{\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}} \qquad y = \frac{\begin{vmatrix} a_1 & d_1 \\ a_2 & d_2 \end{vmatrix}}{\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}}$$

$$\begin{array}{ccccccc} a_1x & + & b_1y & + & c_1z & = & d_1 \\ a_2x & + & b_2y & + & c_2z & = & d_2 \end{array}$$

Set one of x, y or z equal to 0 and solve the resulting two variable equation. If there is a solution, one of

$$X = \begin{vmatrix} b_1 & c_1 \\ b_2 & c_2 \end{vmatrix} \qquad Y = \begin{vmatrix} a_1 & c_1 \\ a_2 & c_2 \end{vmatrix} \qquad Z = \begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}$$

will be non-zero. If $X \neq 0$ set $x = 0$ and solve; if $Y \neq 0$ set $y = 0$ and solve; if $Z \neq 0$ set $z = 0$ and solve;

Department of Mathematics
University of Notre Dame
Notre Dame, IN 46556
U.S.A.
E-mail address: taylor.2@nd.edu