1. Vectors

- Dot product: $\langle a_1, a_2, a_3\rangle \bullet \langle b_1, b_2, b_3\rangle = a_1b_1 + a_2b_2 + a_3b_3$.
  - Angle between two vectors: $\cos(\theta) = \dfrac{\mathbf{a}\bullet\mathbf{b}}{|\mathbf{a}|\cdot|\mathbf{b}|}$
  - Perpendicular and orthogonal: $\mathbf{a}\bullet\mathbf{b} = 0$
  - Projection of $\mathbf{b}$ onto $\mathbf{a}$: $\dfrac{\mathbf{a}\bullet\mathbf{b}}{\mathbf{a}\bullet\mathbf{a}}\mathbf{a}$
  - Work: Force $\bullet$ Displacement
  - Rules: $\mathbf{a}\bullet\mathbf{a} = |\mathbf{a}|^2$; $\mathbf{a}\bullet\mathbf{b} = \mathbf{b}\bullet\mathbf{a}$; $\mathbf{a}\bullet(\mathbf{b}+\mathbf{c}) = \mathbf{a}\bullet\mathbf{b} + \mathbf{a}\bullet\mathbf{c}$;
    $(c\mathbf{a})\bullet\mathbf{b} = \mathbf{a}\bullet(c\mathbf{b}) = c(\mathbf{a}\bullet\mathbf{b})$; $\mathbf{a}\bullet\mathbf{0} = 0$.

- Cross product: $\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix} = \begin{vmatrix} a_2 & a_3 \\ b_2 & b_3 \end{vmatrix}\mathbf{i} - \begin{vmatrix} a_1 & a_3 \\ b_1 & b_3 \end{vmatrix}\mathbf{j} + \begin{vmatrix} a_1 & a_2 \\ b_1 & b_2 \end{vmatrix}\mathbf{k}$
  - Vector orthogonal to two vectors: $\mathbf{a}\times\mathbf{b}$ is orthogonal to both $\mathbf{a}$ and to $\mathbf{b}$.
  - Area of parallelogram: $A = |\mathbf{a}\times\mathbf{b}|$
  - Angle between two vectors: $|\mathbf{a}\times\mathbf{b}| = |\mathbf{a}|\,|\mathbf{b}|\sin(\theta)$
  - Rules: $\mathbf{a}\times\mathbf{b} = -\mathbf{b}\times\mathbf{a}$; $\mathbf{a}\times(\mathbf{b}+\mathbf{c}) = \mathbf{a}\times\mathbf{b} + \mathbf{a}\times\mathbf{c}$;
    $(c\mathbf{a})\times\mathbf{b} = \mathbf{a}\times(c\mathbf{b}) = c(\mathbf{a}\times\mathbf{b})$;
  - $\mathbf{a}\times(\mathbf{b}\times\mathbf{c}) = \begin{vmatrix} \mathbf{b} & \mathbf{c} \\ \mathbf{a}\bullet\mathbf{b} & \mathbf{a}\bullet\mathbf{c} \end{vmatrix}$ & $(\mathbf{a}\times\mathbf{b})\times\mathbf{c} = \begin{vmatrix} \mathbf{b} & \mathbf{a} \\ \mathbf{b}\bullet\mathbf{c} & \mathbf{a}\bullet\mathbf{c} \end{vmatrix}$

- Triple scalar product: $\mathbf{a}\bullet(\mathbf{b}\times\mathbf{c})$
  - To calculate: $\mathbf{a}\bullet(\mathbf{b}\times\mathbf{c}) = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$
  - Volume of parallelepiped: $|\mathbf{a}\bullet(\mathbf{b}\times\mathbf{c})|$
  - Rule: $\mathbf{a}\bullet(\mathbf{b}\times\mathbf{c}) = (\mathbf{a}\times\mathbf{b})\bullet\mathbf{c}$.

2. Lines and planes

- Equations of a line
  - Vector equation: $t\mathbf{v} + \mathbf{c}$ with $\mathbf{v} \neq \mathbf{0}$.
  - Symmetric equation: $\dfrac{x - c_1}{v_1} = \dfrac{y - c_2}{v_2} = \dfrac{z - c_3}{v_3}$ with adjustments when $v_i = 0$.

- Equation of a plane: normal vector $\mathbf{N} = \langle a, b, c \rangle \neq \mathbf{0}$ and point $\mathbf{p} = \langle p_1, p_2, p_3 \rangle$ gives the equation $\mathbf{N} \bullet \langle x, y, z \rangle = \mathbf{N} \bullet \langle p_1, p_2, p_3 \rangle$ or $ax + by + cz = d$.

- Parametric version of a plane with point $\langle p_1, p_2, p_3 \rangle$ containing vectors $\mathbf{v}_1$ and $\mathbf{v}_2$: $\langle p_1, p_2, p_3 \rangle + \mathbf{v}_1 u + \mathbf{v}_2 w$.

- Given two vectors in a plane, $\mathbf{v}_1$ and $\mathbf{v}_2$, $\mathbf{N} = \mathbf{v}_1 \times \mathbf{v}_2$ is a normal vector.

- Given a normal vector to a plane, $\mathbf{N} = \langle a, b, c \rangle$, at least one of $\langle -b, a, 0 \rangle$, $\langle -c, 0, a \rangle$, $\langle 0, -c, b \rangle$ is a non-zero vector, $\mathbf{v}_1$, in the plane and $\mathbf{N} \times \mathbf{v}_1$ is another.

3. Curves

3.1. **Parametrization.** Vector valued function $\mathbf{r}(t)$, $a \leqslant t \leqslant b$. A parametrization is smooth provided $\mathbf{r}'(t) \neq \mathbf{0}$. A smooth parametrization orients the curve.

3.2. **Arc Length.** Arc length between points $\mathbf{p}_1$ and $\mathbf{p}_2$: find t_1 and t_2 such that $\mathbf{r}(t_i) = \mathbf{p}_i$ and if the parametrization is smooth, the arc length is $\displaystyle\int_{t_1}^{t_2} |\mathbf{r}'(t)|\,dt$ provided $t_1 < t_2$.

$$\frac{ds}{dt} = |\mathbf{r}'(t)|$$

3.3. **Tangent vector(s).** A tangent vector to the curve at t is $\mathbf{r}'(t)$. At a point $\mathbf{p}$ find c such that $\mathbf{r}(c) = \mathbf{p}$ and use $\mathbf{r}'(c)$.

3.4. **Velocity and acceleration.** Velocity $= \mathbf{r}'(t)$, speed $= |\mathbf{r}'(t)|$. Acceleration $= \mathbf{r}''(t)$.

3.5. **Frenet-Serret frame.** Unit tangent $\mathbf{T}(t)$, unit normal $\mathbf{N}(t)$, and unit binormal, $\mathbf{B}(t)$. These three unit vectors are mutually orthogonal and $\mathbf{T}(t)$, $\mathbf{N}(t)$, $\mathbf{B}(t)$ is a right-handed frame.

$$\mathbf{r}''(t) = a_{\mathbf{T}}(t)\mathbf{T}(t) + a_{\mathbf{N}}(t)\mathbf{N}(t) \text{ where } a_{\mathbf{T}}(t) = \frac{d\,|\mathbf{r}'(t)|}{dt} \text{ and } a_{\mathbf{N}}(t) = \kappa(t)\,|\mathbf{r}'(t)|^2.$$

If you actually want to compute $a_{\mathbf{T}}(t)$ use $a_{\mathbf{T}}(t) = \dfrac{\mathbf{r}'(t) \bullet \mathbf{r}''(t)}{|\mathbf{r}'(t)|}$.

If you actually want to compute $\kappa(t)$ use $\kappa(t) = \dfrac{|\mathbf{r}'(t) \times \mathbf{r}''(t)|}{|\mathbf{r}'(t)|^3}$.

If you actually want to compute $a_{\mathbf{N}}(t)$ use $a_{\mathbf{N}}(t) = \dfrac{|\mathbf{r}'(t) \times \mathbf{r}''(t)|}{|\mathbf{r}'(t)|}$.

If you actually want to compute the vectors in the Frenet-Serret frame

- $\mathbf{T}(t)$: take $\mathbf{r}'(t)$ and make it unit length.
- $\mathbf{B}(t)$: take $\mathbf{r}'(t) \times \mathbf{r}''(t)$ and make it unit length.
- $\mathbf{N} = \mathbf{B} \times \mathbf{T}$: take $\big(\mathbf{r}'(t) \times \mathbf{r}''(t)\big) \times \mathbf{r}'(t) = \begin{vmatrix} \mathbf{r}''(t) & \mathbf{r}'(t) \\ \mathbf{r}''(t) \bullet \mathbf{r}'(t) & \mathbf{r}'(t) \bullet \mathbf{r}'(t) \end{vmatrix}$ and make it unit length.

If you want any of these quantities at a single value of t, compute $\mathbf{r}'(t)$ and $\mathbf{r}''(t)$ at the point and then work with vectors of numbers.

The *osculating plane* to a curve at a point is the plane spanned by $\mathbf{T}(t)$ and $\mathbf{N}(t)$. A normal vector to this plane is $\mathbf{B}(t)$. If you actually want to find an equation for the osculating plane, use $\mathbf{r}' \mathbf{\times} \mathbf{r}''$ for the normal vector.

The *normal plane* to a curve at a point is the plane with normal vector $\mathbf{T}$ through the point.

3.6. **Antiderivatives.** Find the equation of a curve given an equation for velocity, $\mathbf{v}(t)$. $\displaystyle\int \mathbf{v}(t)\,dt$ is the family of curves with that velocity. One can also compute the velocity from the acceleration up to a constant vector.

4. Limits and continuity

- If $\lim_{\mathbf{x}\to\mathbf{a}} f(\mathbf{x}) = L$ we can calculate L by taking any path, $\lambda(t)$, with $\lim_{t\to 0}\lambda(t) = \mathbf{a}$ and then compute $\lim_{t\to 0} f\big(\lambda(t)\big) = L$.
- Given two paths $\lambda_i(t)$, with $\lim_{t\to 0}\lambda_i(t) = \mathbf{a}$, $i = 1, 2$, compute $\lim_{t\to 0} f\big(\lambda_i(t)\big) = L_i$. If $L_1 \neq L_2$ then $\lim_{\mathbf{x}\to\mathbf{a}} f(\mathbf{x})$ does not exit.
- f is continuous at $\mathbf{a}$ if and only if $\lim_{\mathbf{x}\to\mathbf{a}} f(\mathbf{x}) = f(\mathbf{a})$.
- Constant functions and coordinate functions are continuous.
- Sums, products, quotients and compositions of continuous functions are continuous everywhere they are defined.

5. Partial derivatives

- $\lim_{t\to 0} \dfrac{f(\mathbf{a}+t\mathbf{e}_i) - f(\mathbf{a})}{t} = \dfrac{\partial f}{\partial x_i}(\mathbf{a})$
- Compute partial derivatives using 1st year calculus.
- Higher partial derivatives.
- Clairaut's Theorem.

6. Gradient

If $f(\mathbf{x}) = f(x_1, \cdots, x_k)$, $\mathrm{grad} f(\mathbf{x}) = \left\langle \dfrac{\partial f}{\partial x_1}, \cdots, \dfrac{\partial f}{\partial x_k} \right\rangle$.

6.1. Chain Rule.

$$\frac{\partial f\big(\mathbf{G}(\mathbf{x})\big)}{\partial x_i} = \mathrm{grad} f\big(\mathbf{G}(\mathbf{x})\big) \bullet \frac{\partial \mathbf{G}}{\partial x_i}(\mathbf{x})$$

6.2. Directional derivative.

If $\mathbf{u}$ is a direction, the directional derivative of f in the direction $\mathbf{u}$ is defined to be

$$D_{\mathbf{u}} f(\mathbf{a}) = \lim_{t\to 0} \frac{f(\mathbf{a}+t\mathbf{u}) - f(\mathbf{a})}{t}$$

which can be computed as

$$D_{\mathbf{u}} f(\mathbf{a}) = \mathrm{grad} f(\mathbf{a}) \bullet \mathbf{u}$$

It is the instantaneous rate of change of f in the direction $\mathbf{u}$.

6.3. Implicit differentiation.

How to do it.

6.4. **Direction of maximal increase.** At a point $\mathbf{a}$ the direction in which f is increasing as fast as possible is the direction of $\mathrm{grad} f(\mathbf{a})$. The directional derivative in this direction is $|\mathrm{grad} f(\mathbf{a})|$.

At a point $\mathbf{a}$ the direction in which f is decreasing as fast as possible is the direction of $-\mathrm{grad} f(\mathbf{a})$. The directional derivative in this direction is $-|\mathrm{grad} f(\mathbf{a})|$.

6.5. **Critical points and their classification.** Critical points are solutions to the vector equation $\mathrm{grad} f(\mathbf{x}) = \mathbf{0}$.

In two dimensions form the 2×2 determinant

$$\mathcal{H}\big(f(x,y)\big) = \begin{vmatrix} \dfrac{\partial^2 f}{\partial x^2} & \dfrac{\partial^2 f}{\partial y \partial x} \\ \dfrac{\partial^2 f}{\partial x \partial y} & \dfrac{\partial^2 f}{\partial y^2} \end{vmatrix}$$

Suppose (x,y) is a critical point.

- If $\mathcal{H}\big(f(x,y)\big) = 0$ you learn nothing about the critical point.
- If $\mathcal{H}\big(f(x,y)\big) < 0$, the critical point is a saddle point.
- If $\mathcal{H}\big(f(x,y)\big) > 0$, the critical point is a local extrema.
  - If $\dfrac{\partial^2 f}{\partial x^2} > 0$ the critical point is a local minimum.
  - If $\dfrac{\partial^2 f}{\partial x^2} < 0$ the critical point is a local maximum.
  - Instead of $\dfrac{\partial^2 f}{\partial x^2}$ you may use $\dfrac{\partial^2 f}{\partial y^2}$

6.6. **Tangent planes and normal lines.** The tangent plane to an implicit surface $f(x,y,z) = C$ at the point (a,b,c) is the plane which contains the point (a,b,c) and which has $\mathrm{grad} f(a,b,c)$ as a normal vector. An equation for the tangent plane is then $\mathrm{grad} f(a,b,c) \bullet \langle x,y,z\rangle = \mathrm{grad} f(a,b,c) \bullet \langle a,b,c\rangle$.

The normal line to an implicit surface $f(x,y,z) = C$ at the point (a,b,c) is the line which contains the point (a,b,c) and which has $\mathrm{grad} f(a,b,c)$ as vector in the line. An equation for the normal line is then $\langle a,b,c\rangle + t\,\mathrm{grad} f(a,b,c)$.

Tangent lines to level curves of $f(x,y)$ are perpendicular to $\mathrm{grad} f$.

6.7. **Lagrange multipliers.**

6.7.1. *One constraint.* We want to maximize/minimize $f(x,y)$ (or $f(x,y,z)$) subject to the constraint $g(x,y)=C$ (or $g(x,y,z)=C$).

Step 1 - the equations.

$$\begin{aligned}\mathrm{grad} f(x,y) &= \lambda \mathrm{grad} g(x,y)\\ g(x,y) &= C\end{aligned}$$

OR

$$\begin{aligned}\frac{\partial f}{\partial x}(x,y) &= \lambda\frac{\partial g}{\partial x}(x,y)\\ \frac{\partial f}{\partial y}(x,y) &= \lambda\frac{\partial g}{\partial y}(x,y)\\ \text{if 3-variables}&\\ \frac{\partial f}{\partial z}(x,y,z) &= \lambda\frac{\partial g}{\partial z}(x,y,z)\\ g(x,y) &= C\end{aligned}$$

Step 2 - find all solutions to these equations.

This is pure algebra. Try to eliminate variables until you get an equation with only one variable; solve it; and see what this forces on the rest of the variables.

Remember to be careful dividing and when you take square roots be sure to consider both solutions.

Step 3 - answer the question. Plug your solutions for 2 and see which values are the biggest and which are the smallest. Were you asked for the points where the minimum or maximum occurred or were you asked for the value?

6.7.2. *Two constraints.* We want to maximize/minimize $f(x,y,z)$ subject to the constraints $g_1(x,y,z)=C_1$ and $g_2(x,y,z)=C_2$.

Step 1 - the equations.

$$\begin{aligned}\mathrm{grad} f(x,y,z) &= \lambda \mathrm{grad} g_1(x,y,z) + \mu \mathrm{grad} g_2(x,y,z)\\ g_1(x,y,z) &= C_1\\ g_2(x,y,z) &= C_2\end{aligned}$$

OR

$$\begin{aligned}\frac{\partial f}{\partial x}(x,y,z) &= \lambda\frac{\partial g_1}{\partial x}(x,y,z)+\mu\frac{\partial g_2}{\partial x}(x,y,z)\\ \frac{\partial f}{\partial y}(x,y,z) &= \lambda\frac{\partial g_1}{\partial y}(x,y,z)+\mu\frac{\partial g_2}{\partial y}(x,y,z)\\ \frac{\partial f}{\partial z}(x,y,z) &= \lambda\frac{\partial g_1}{\partial z}(x,y,z)+\mu\frac{\partial g_2}{\partial z}(x,y,z)\\ g_1(x,y) &= C_1\\ g_2(x,y) &= C_2\end{aligned}$$

Steps 2 and 3 are the same as the one constraint case.

6.8. **Max-min problems.** Maximize or minimize $f(x, y)$ over a closed bounded region D in the plane. Or a solid V in 3-space;

Step 1 - Find the critical points of f.

Step 2 - Find the max/min along the boundary of D. Either use Lagrange multipliers if the boundary of D is a level curve $g(x, y) = C$ or parametrize the boundary and use 1st year claculus.

Step 3 - Solve the problem. Evaluate f at the points from steps 1 and 2. The biggest number you get is the maximum and the smallest is the minimum.

7. Double integrals

Given a closed bounded region D in the plane,

$$\iint_D f(x, y)\, dA$$

is a number given as the limit of Riemann sums.

From this definition we have (so far)

$$\text{Area}(D) = \iint_D 1\, dA$$
$$\text{Volume}(V) = \iint_D f(x, y)\, dA$$

where V is the solid above D in the xy-plane and below the graph $z = f(x, y)$. The interpretation of the double integral as a volume assumes $f(x, y) \geqslant 0$.

More applications will follow later.

7.1. **Iterated integrals in Cartesian coordinates.** To calculate $\iint_D f(x, y)\, dA$ for any continuous f, we can express the answer as an iterated integral (or perhaps a sum of iterated integrals).

First decide if you want to do $dA = dx\, dy$ or $dA = dy\, dx$. Then

$$\iint_D f(x, y)\, dA = \int_a^b \int_{\ell(y)}^{r(y)} f(x, y)\, dx\, dy$$
$$\iint_D f(x, y)\, dA = \int_a^b \int_{b(x)}^{t(x)} f(x, y)\, dy\, dx$$

See here for more details.

8. Riemann sums

All the integrals we studied this year are defined as limits of Riemann sums.

- Double integrals
- Triple integrals
- Line integrals
- Surface integrals

Hence they all have similar applications.

- Mass density ρ, total mass
  - of a thin wire C, $\int_C \rho\, ds$
  - of a thin plate R in the plane or in 3-space, $\iint_R \rho\, dA$
  - of a solid E, $\iiint_E \rho\, dV$
- Charge density q, total charge
  - of a thin wire C, $\int_C q\, ds$
  - of a thin plate R in the plane or in 3-space, $\iint_R q\, dA$
  - of a solid E, $\iiint_E q\, dV$
- Moment
  - of a thin wire C about an axis (2-D) or a coordinate plane (3-D), $\int_C x\rho\, ds$, $\int_C y\rho\, ds$ and maybe $\int_C z\rho\, ds$.
  - of a thin plate R about an axis in the plane or in 3-space, $\iint_R x\rho\, dA$ and $\iint_R y\rho\, dA$
  - of a solid E about a coordinate plane, $\iiint_E x\rho\, dV$, $\iiint_E y\rho\, dV$ and $\iiint_E z\rho\, dV$
- Formula for center of mass.

9. Iterated integrals

- Line integral: $\displaystyle\int_C f\,ds = \int_a^b f\big(\mathbf{r}(t)\big)\cdot|\mathbf{r}'(t)|\;dt$

  Flux integral: $\displaystyle\int_C \mathbf{V}\bullet\mathbf{T}\,ds = \int_a^b \mathbf{V}\big(\mathbf{r}(t)\big)\bullet\mathbf{r}'(t)\,dt$

- Double integral: $\displaystyle\iint_R f\,dA$
  - Associated iterated integrals:

    Cartesian: $\displaystyle\int_\alpha^\omega\int_{\ell(y)}^{u(y)} f(x,y)\,dx\,dy$ or $\displaystyle\int_\alpha^\omega\int_{\ell(x)}^{r(x)} f(x,y)\,dy\,dx$

    Polar: $\displaystyle\int_a^b\int_{s(\theta)}^{e(\theta)} f(r,\theta)\cdot r\,dr\,d\theta$

    Arbitrary, $x=x(u,w)$, $y=y(u,w)$, $\displaystyle\iint_S f(u,w)\cdot\frac{\partial(x,y)}{\partial(u,w)}\,dA$

- Surface integral: $\displaystyle\iint_T f(x,y,z)\,dS = \iint_D f\big(\mathbf{r}(u,w))\big)\cdot|\mathbf{r}_u\times\mathbf{r}_w|\;dA$
- Flux integral: $\displaystyle\iint_T \mathbf{F}\bullet d\mathbf{S} = \iint_D \mathbf{F}\big(\mathbf{r}(u,w))\big)\bullet(\mathbf{r}_u\times\mathbf{r}_w)\,dA$

- Triple integral: $\displaystyle\iiint_E f\,dV$
  - Associated iterated integrals:

    Cartesian: $\displaystyle\int_{R_{xy}}\int_{\ell(x,y)}^{u(x,y)} f(x,y,z)\,dz\,dA$, $\displaystyle\int_{R_{xz}}\int_{\ell(x,z)}^{u(x,z)} f(x,y,z)\,dy\,dA$ or $\displaystyle\int_{R_{yz}}\int_{\ell(y,)}^{u(y,z)} f(x,y,z)\,dx\,dA$

    Cylindrical: $\displaystyle\int_{R_{r,\theta}}\int_{\ell(r,\theta)}^{u(r,\theta)} f(r,\theta,z)\cdot r\,dz\,dA$

    Spherical: $\displaystyle\int_a^b\int_?^?\int_?^? f(\rho,\theta,\phi)\cdot\rho^2\sin(\phi)\,d\phi\,d\theta\,d\rho$

    Arbitrary, $x=x(u,v,w)$, $y=y(u,v,w)$, $z=z(u,v,w)$,
    $\displaystyle\iiint_S f(u,w,v)\cdot\frac{\partial(x,y,z)}{\partial(u,w,v)}\,dV$

10. Vector calculus

10.1. **The curl.** Given $\langle P, Q, R\rangle$, define

$$\operatorname{curl}\langle P, Q, R\rangle = \nabla\times\langle P, Q, R\rangle = \left\langle \frac{\partial R}{\partial y} - \frac{\partial Q}{\partial z}, \frac{\partial P}{\partial z} - \frac{\partial R}{\partial x}, \frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} \right\rangle$$

10.2. **The gradient.**

$$\nabla f = \operatorname{grad} f = \nabla f = \left\langle \frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial z} \right\rangle$$

10.3. **The divergence.**

$$\operatorname{div}\langle P, Q, R\rangle = \nabla\bullet\langle P, Q, R\rangle = \frac{\partial P}{\partial x} + \frac{\partial Q}{\partial y} + \frac{\partial R}{\partial z}$$

10.4. **The (scalar) Laplacian.**

$$\triangle f = \nabla^2 f = \nabla\bullet\nabla f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} + \frac{\partial^2 f}{\partial z^2}$$

10.5. **Curl of a gradient.**

$$\operatorname{curl}\nabla f = \langle 0, 0, 0\rangle$$

10.6. **Divergence of a curl.**

$$\operatorname{div}\big(\operatorname{curl}(\mathbf{v})\big) = 0$$

Given $\operatorname{curl}\mathbf{F} = \langle 0, 0, 0\rangle$ find a potential function p such that $\operatorname{grad} p = \mathbf{F}$.

11. The Stokes'-type theorems

Here are the Stokes'-type theorems.

(1) $p(\mathbf{b}) - p(\mathbf{a}) = \displaystyle\int_C \nabla p \bullet d\mathbf{r}$

(2) $\displaystyle\oint\limits_{\partial T} \mathbf{F} \bullet d\mathbf{r} = \iint\limits_{T} (\mathrm{curl}\mathbf{F}) \bullet d\mathbf{S}$

(3) $\displaystyle\iint\limits_{\partial E} \mathbf{F} \bullet d\mathbf{S} = \iiint\limits_{E} (\mathrm{div}\mathbf{F})\, dV$

Green's Theorem is missing from the list because it is just Stokes' Theorem with $\mathbf{F} = \langle M(x,y), N(x,y), 0\rangle$ and T is parametrized by $\langle x, y, 0\rangle$ with $(x,y) \in T$.