Department of Mathematics
University of Notre Dame
MATH 20550 - Calculus III
Summer 2015

Name ____

Exam 2

July 2, 2015

This exam has 12 problems worth a total of **110 points**. You will have 90 minutes to work on it. All answers should be given as exact, closed form numbers as opposed to decimal approximations. **For full credit, you must show all work.** Calculators and all other electronic devices are strictly forbidden.

Please read and sign the Honor Pledge:

Honor Pledge: As a member of the Notre Dame community, I will not participate in or tolerate academic dishonesty.

Signature: ____

Question	Points	Score
1	5	
2	15	
3	10	
4	5	
5	10	
6	10	
7	10	
8	10	
9	10	
10	10	
11	5	
12	10	
Total:	110	

1. (5 points) Sketch the three contours for z where $z^2 - 1 = 4x^2 + y^2$

2. Consider the curve parametrized by

$$r(t) = \left\langle 2t, t^2, \frac{1}{3}t^3 \right\rangle$$

where $0 \le t \le 2$.

(a) (5 points) Find the arc length of the curve.

(b) (5 points) Find the curvature at the point $(2, 1, \frac{1}{3})$.

(c) (5 points) Find an equation for the osculating plane at the point $(2, 1, \frac{1}{3})$.

3. (10 points) Let $z = f(x, y)$, where f is a differentiable function and suppose that $x = x(s, t)$ and $y = y(s, t)$. Given that

$$\begin{array}{lllll} x(1,0) = 2, & x_s(1,0) = -2, & x_t(1,0) = 6, & f_x(2,3) = -1, & f(2,3) = 4 \\ y(1,0) = 3 & y_s(1,0) = 5, & y_t(1,0) = 4, & f_y(2,3) = 10 & \end{array}$$

Find $\frac{\partial z}{\partial t}(1, 0)$.

4. (5 points) Compute

$$\int \vec{r}(t)\,dt$$

where $\vec{r}(t) = \sec^2 t\,\hat{j} + te^{t^2}\,\hat{k}$

5. (10 points) Suppose z is defined implicitly by

$$z = e^x \sin(y + z) \ .$$

Find all first partials of z.

6. Compute the following limits or show they do not exist. Be sure to explain your reasoning fully in either case.

   (a) (5 points)

$$\lim_{(x,y)\to(0,0)} \frac{xy^2}{x^3+y^3}$$

   (b) (5 points)

$$\lim_{(x,y)\to(0,0)} \frac{x+y}{(x+1)^2+(y+1)^2}$$

7. Consider the function $f(x, y) = x \tan y$.

   (a) (5 points) Compute the derivative of f at $(2, \frac{\pi}{4})$ in the direction of $\vec{v} = \hat{\imath} + \hat{\jmath}$.

   (b) (5 points) Compute the direction where f is *decreasing* the fastest.

8. (10 points) Find three numbers whose sum is 12 and the sum of whose squares is as small as possible.

9. (10 points) Find the tangential and normal components of acceleration for a particle traveling along the trajectory

$$\vec{r}(t) = (3t - t^3)\hat{\imath} + 3t^2\hat{\jmath}$$

10. (10 points) Find an equation for the tangent plane to $x^4 + y^4 + z^4 = 3x^2y^2z^2$ at the point $(1, 1, 1)$.

11. (5 points) Sketch the curve represented by

$$\vec{r}(\theta) = \langle 3\sec\theta, 2\tan\theta\rangle$$

Make sure to indicate the orientation of the curve (direction of increasing θ).

12. (10 points) A particle with mass 2 starts at the origin with an initial velocity $\vec{v}_0 = \hat{\imath} - \hat{\jmath} + 3\hat{k}$. Its acceleration is $\vec{a}(t) = 6t\hat{\imath} + 12t^2\hat{\jmath} - 6t\hat{k}$. Find the position and momentum functions. (Recall that its momentum, $\vec{p}(t)$, is mass times velocity.)