Department of Mathematics
University of Notre Dame
MATH 20550 - Calculus III
Summer 2015

Name
Name

## Final Exam

July 24, 2015

This exam has 13 problems worth a total of **145 points**. You will have 105 minutes to work on it. All answers should be given as exact, closed form numbers as opposed to decimal approximations. For full credit, you must show all work. Calculators and all other electronic devices are strictly forbidden.

Please read and sign the Honor Pledge:

Honor	Pledge:	As a	member	of the	Notre	Dame	community,	I will	not	participate	in	or	tolerate
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Question	Points	Score
1	10	
2	10	
3	10	
4	15	
5	10	
6	10	
7	10	
8	10	
9	10	
10	10	
11	15	
12	10	
13	15	
Total:	145	

 $1.\ (10\ \mathrm{points})\ \mathrm{Find}\ \mathrm{the}\ \mathrm{Jacobian}\ \mathrm{of}\ \mathrm{the}\ \mathrm{transformation}$ 

$$x = \frac{u}{v}$$

$$x = \frac{u}{v}$$
  $y = \frac{v}{w}$   $\frac{w}{u}$ 

$$\frac{w}{u}$$

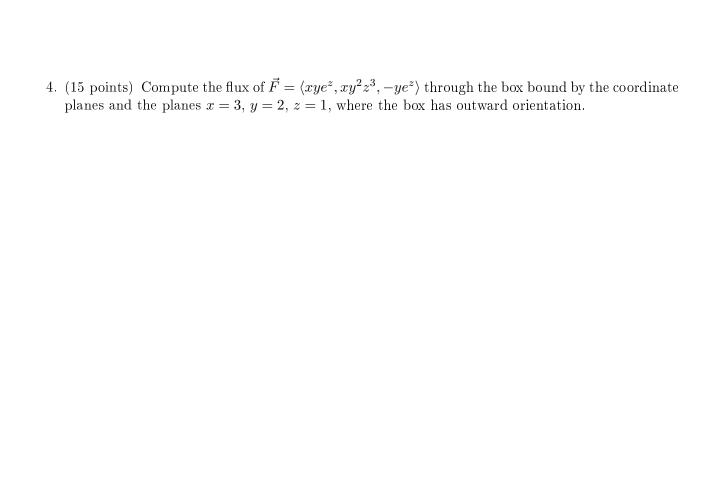
## 2. (10 points) Is the vector field

$$\vec{F} = (3x^2 - 2y^2)\hat{i} + (4xy + 3)\hat{j}$$

conservative? If so, find a potential function.

$$\iint {\rm curl} \vec{F} \cdot d\vec{S}$$

where S is the sphere of radius 16 oriented outward, and  $\vec{F} = \langle xyz, x^2y^2z^2, x^3y^3z^3 \rangle$ .



 $5.\ (10\ \mathrm{points})\ \mathrm{Find}$  and classify all the critical points of

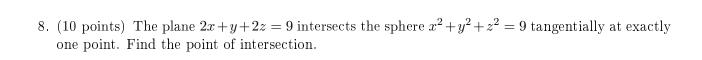
$$f(x,y) = xy - 2x - 2y - x^2 - y^2$$



7. (10 points) Is there a vector field  $\vec{G}$  on  $\mathbb{R}^3$  such that

$$\operatorname{curl} \vec{G} = \langle xyz, -y^2z, yz^2 \rangle$$

? Explain.



9.	(10 points) Find an equation for the sphere whi (the are connected by a line through the center of	ch has $(2,1,4)$ and $(4,3,10)$ as a of the sphere).	antipodal points

10. Compute the limits or show they do not exist

$$\lim_{(x,y)\to(0,0)} \frac{x^2 + 2y^2}{xy}$$

$$\lim_{(x,y)\to(0,0)} \frac{xy + \cos(x)}{\frac{x+e^y}{x^2y + \cos(x)e^y}}$$

11. (15 points) Compute the line integral

$$\int_C \vec{F} \cdot d\vec{r}$$

where  $\vec{F} = \langle y - \cos y, x \sin y \rangle$  and C is the circle  $(x-3)^2 + (y+4)^2 = 4$ , oriented clockwise.

12. (10 points) Find  $\frac{\partial z}{\partial s}$  and  $\frac{\partial z}{\partial t}$  where  $z=x^2-3y^2,\,x=st,\,y=s+t^2.$ 

## 13. (15 points) Compute

$$\iint_{S} \operatorname{curl} \vec{F} \cdot d\vec{S}$$

where S is the top and four sides of the faces of the box with vertices  $(\pm 1, \pm 1 \pm 1)$  (the box without the bottom), given outward orientation, and  $\vec{F}(x,y,z) = \langle xyz, xy, x^2yz \rangle$ .