Department of Mathematics
University of Notre Dame
MATH 20550 - Calculus III
Summer 2015

Name ____

Final Exam

July 24, 2015

This exam has 13 problems worth a total of **145 points**. You will have 105 minutes to work on it. All answers should be given as exact, closed form numbers as opposed to decimal approximations. **For full credit, you must show all work.** Calculators and all other electronic devices are strictly forbidden.

Please read and sign the Honor Pledge:

Honor Pledge: As a member of the Notre Dame community, I will not participate in or tolerate academic dishonesty.

Signature: ____

Question	Points	Score
1	10	
2	10	
3	10	
4	15	
5	10	
6	10	
7	10	
8	10	
9	10	
10	10	
11	15	
12	10	
13	15	
Total:	145	

1. (10 points) Find the Jacobian of the transformation

$$x = \frac{u}{v} \qquad y = \frac{v}{w} \qquad \frac{w}{u}$$

2. (10 points) Is the vector field

$$\vec{F} = (3x^2 - 2y^2)\hat{\imath} + (4xy + 3)\hat{\jmath}$$

conservative? If so, find a potential function.

3. (10 points) Compute

$$\iint \text{curl}\vec{F} \cdot d\vec{S}$$

where S is the sphere of radius 16 oriented outward, and $\vec{F} = \langle xyz, x^2y^2z^2, x^3y^3z^3 \rangle$.

4. (15 points) Compute the flux of $\vec{F} = \langle xye^z, xy^2z^3, -ye^z \rangle$ through the box bound by the coordinate planes and the planes $x = 3$, $y = 2$, $z = 1$, where the box has outward orientation.

5. (10 points) Find and classify all the critical points of

$$f(x,y) = xy - 2x - 2y - x^2 - y^2$$

6. (10 points) Give a vector function which represents the curve of intersection between the hyperboloid $z = x^2 - y^2$ and the cylinder $x^2 + y^2 = 1$.

7. (10 points) Is there a vector field $\vec{G}$ on $\mathbb{R}^3$ such that

$$\mathrm{curl}\vec{G} = \langle xyz, -y^2 z, yz^2 \rangle$$

? Explain.

8. (10 points) The plane $2x+y+2z=9$ intersects the sphere $x^2+y^2+z^2=9$ tangentially at exactly one point. Find the point of intersection.

9. (10 points) Find an equation for the sphere which has $(2, 1, 4)$ and $(4, 3, 10)$ as antipodal points (the are connected by a line through the center of the sphere).

10. Compute the limits or show they do not exist

(a) (5 points)

$$\lim_{(x,y)\to(0,0)} \frac{x^2 + 2y^2}{xy}$$

(b) (5 points)

$$\lim_{(x,y)\to(0,0)} \frac{xy + \cos(x)}{\frac{x+e^y}{x^2y+\cos(x)e^y}}$$

11. (15 points) Compute the line integral

$$\int_C \vec{F} \cdot d\vec{r}$$

where $\vec{F} = \langle y - \cos y, x \sin y \rangle$ and C is the circle $(x-3)^2 + (y+4)^2 = 4$, oriented clockwise.

12. (10 points) Find $\frac{\partial z}{\partial s}$ and $\frac{\partial z}{\partial t}$ where $z = x^2 - 3y^2$, $x = st$, $y = s + t^2$.

13. (15 points) Compute

$$\iint_S \text{curl}\vec{F} \cdot d\vec{S}$$

where S is the top and four sides of the faces of the box with verticies $(\pm 1, \pm 1 \pm 1)$ (the box without the bottom), given outward orientation, and $\vec{F}(x, y, z) = \langle xyz, xy, x^2yz \rangle$.