Department of Mathematics
University of Notre Dame
Name $\qquad$
MATH 20550-Calculus III
Summer 2015

## Final Exam

July 24, 2015

This exam has ? problems worth a total of ? points. You will have 105 minutes to work on it. All answers should be given as exact, closed form numbers as opposed to decimal approximations. For full credit, you must show all work. Calculators and all other electronic devices are strictly forbidden.

Please read and sign the Honor Pledge:

Honor Pledge: As a member of the Notre Dame community, I will not participate in or tolerate academic dishonesty.

## Signature:

$\qquad$

| Question | Points | Score |
| :---: | :---: | :---: |
| 1 | 0 |  |
| 2 | 0 |  |
| 3 | 0 |  |
| 4 | 0 |  |
| 5 | 0 |  |
| 6 | 0 |  |
| 7 | 0 |  |
| Total: | 0 |  |

1. Compute

$$
\int_{C} \vec{F} \cdot d \vec{r}
$$

where $\vec{F}(x, y, z)=\left\langle x^{2} y, \frac{1}{3} x^{3}, x y\right\rangle$ and $C$ is the curve of intersection of the hyperbolic paraboloid $z=y^{2}-x^{2}$ and the cylinder $x^{2}+y^{2}=1$ oriented counterclockwise as viewed from above.
2. Compute the surface integral

$$
\iint_{S}\left(x^{2} y+y^{2} z\right) d S
$$

where $S$ is the hemisphere $x^{2}+y^{2}+z^{2}=4 z \geq 0$.
3. Compute the center of mass of the hemisphere $x^{2}+y^{2}+z^{2}=a^{2}$ and $z \geq 0$, if it has constant density.
4. Compute

$$
\iint_{S} \vec{F} \cdot d \vec{S}
$$

where $\vec{F}=\langle z, y, z x\rangle$ and $S$ is the surface of the tetrahedron enclosed by the doordinate plances and the plane

$$
\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1
$$

where $a, b, c$ are positive numbers.
5. Let $C$ be a simple closed smooth curve that lies in the plane $x+y+z=1$. Show that the line integral

$$
\int_{C} z d x-2 x d y+3 y d z
$$

depends only on the area of the region enclosed by $C$ and not on the shape of $C$ or its location in the plane.
6. Compute

$$
\iint_{S} \vec{F} \cdot d \vec{S}
$$

where $\vec{F}=\left\langle z^{2} x, \frac{1}{3} y^{3}+\tan z, x^{2} z+y^{2}\right\rangle$ where $S$ is the top half of the sphere $x^{2}+y^{2}+z^{2}=1$.
7. Compute

$$
\iint_{S} \vec{F} \cdot d \vec{S}
$$

where $\vec{F}=x y \hat{i}+4 x^{2} \hat{j}+y z \hat{k}$ and $S$ is the surface $z=x e^{y}, 0 \leq x \leq 1,0 \leq y \leq 1$ with the upward orientation.

