Mean-Variance Tradeoff in Trade Distortions:
Analytical Results and Evidence from China

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Trade models typically assume trade distortions (such as tariffs or trade costs) are constant across firms. We consider distortions that vary across firms and measure the gains from reducing dispersion in trade distortions. First we use a standard model to solve for the reduction in the mean that is welfare-equivalent to any given reduction in variance, and show that the mean-variance tradeoff is linear. Second we show how changes in the variance of trade distortions affect real income using a two step extension of the procedure from Arkolakis, Costinot and Rodriguez-Clare (2012). We apply these results to Chinese firm-level data and solve for equivalent mean reductions and for welfare gains in three counterfactuals: 1) reducing variance among all firms to that of private firms in China, 2) reducing variance to the level of French firms, and 3) removing all variance.

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Recent work in the study of firms in developing countries has focused not just on the level of distortions that firms face, but also the distribution of implied distortions. Hsieh and Klenow (2009) uses a simple model of firms facing heterogeneous distortions to capital, labor and productivity and use it to measure the difference in firm distortions across countries. They also measure the implied gains from reducing the distribution of distortions. Our approach is to apply similar methodology to a case they did not consider: firm-level distortions to exports.

We develop a simple model of monopolistically competitive firms that may sell both domestically and in foreign markets. Firms face heterogenous trade distortions that affect the extent to which they sell in both markets. The model implies that the variance of this distortion is proportional to the variance of the natural log of the ratio of export sales to domestic sales, which is easily measured in firm-level data. We develop two versions of the model. In the first, the set of markets in which all firms operate is exogenous, and the distortion is only relevant for those that operate in both the foreign and domestic markets. The second is a Melitz (2003) model with Pareto-distributed productivity, as in Chaney (2008), so that more firms may change the set of markets in which they operate in response to changes in the distribution of trade distortions.

We use data from Chinese manufacturing firms to document several facts about this measure. First, it is uncorrelated with firm productivity. Second, the natural log of the ratio of export sales to domestic sales is normally distributed. Third, breaking it up by ownership type, the variance of this implied distortion is greater in the set of firms overall than it is for privately owned Chinese firm. This is driven by state-owned enterprises, which export
much less intensively than private firms, and the special category for Taiwan, Macau and Hong Kong owned firms, which export much more intensively than private firms. All of these facts are true after controlling for a variety of observables about Chinese firms, such as their location, industry and the composition of countries to which they export.

We use these data facts to develop two useful analytical results. First, we derive a mean-variance tradeoff implied by the model. Using the fact that distortions are independent of productivity and are log-normal, we show that the set of combinations of mean and variance over which the representative household is indifferent is linear, and depends only on one parameter: the elasticity of substitution. Therefore, for any reduction in the variance of distortions, we can easily solve for a reduction in mean distortions for which the representative household would be indifferent. This allows us to relate the magnitude of variance reductions to existing literature that studies reductions in level distortions.

Our second analytical result is to measure the increase in real income associated with any reduction in the variance of trade distortions. We implement this via a two step extension of the Arkolakis, Costinot and Rodriguez-Clare (2012) result (hereafter referred to as ACR). The first step is to solve for changes in one minus the import penetration ratio given the change in the distribution of domestic trade costs using an implicit function that we derive. The implicit function takes only the initial value of the import penetration ratio, the trade elasticity, and the ratio of domestic to foreign income as arguments. The second step is to apply the solution to that implicit function into the ACR formula to measure the increase in real income. Both of these results hold in both the models with and without an extensive margin.
We apply these analytical results to three counterfactuals.\textsuperscript{1} The results from these exercises are given in Table 1.\textsuperscript{2} The first counterfactual is to reduce the variance of trade distortions for the set of Chinese firms overall to the level of Chinese private firms. The second uses French firm-level data showing that the variance of trade distortions is lower in France than in China. We consider a reduction of variance in China to the French level. Third, we consider the extreme case of reducing the variance of distortions to zero. Each counterfactual has a range of equivalent mean reductions and increases in real income, which correspond to the range of trade elasticities considered. These results for welfare-equivalent reductions in mean distortions are given in Table 2, and for increase in real income in Table 3. We parameterize the model to imply a range of trade elasticities in the set $(-5, -10)$ as in Anderson and Van Wincoop (2004). In the model without an extensive margin this directly implies an elasticity of substitution between 6 and 11.\textsuperscript{3} In the model with an extensive margin, we also specify a shape parameter for the productivity distribution of firms and consider the same set of trade elasticities. Together these imply lower levels of the elasticity of substitution between 1.23 and 1.53.\textsuperscript{4} The fact that the model with the extensive margin has higher equivalent reductions in mean distortions,

\textsuperscript{1}In all three counterfactuals we change the set of trade distortions that domestic firms face with no changes to those that foreign firms face. Increases in real income are realized through improvements to the terms of trade.

\textsuperscript{2}To put the real income results in context, note that the ACR formula implies that the real income lost from moving the Chinese economy to autarky are $1.14 - 2.29\%$.

\textsuperscript{3}In that version of the model, the relationship between the trade elasticity $\varepsilon$ and the elasticity of substitution $\sigma$ is $\varepsilon = 1 - \sigma$.

\textsuperscript{4}The shape parameter $\theta$ is set to 2.07 to match the coefficient of variation for estimated firm productivity, then we solve for the elasticity of substitution $\sigma$ by setting the trade elasticity $\varepsilon$ between -5 and -10 using the formula:

$$\varepsilon = 1 - \frac{\sigma \theta}{\sigma - 1}$$

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and higher real income gains is due to the fact that the implied elasticity of substitution is necessarily lower.\textsuperscript{5}

These results provide a simple means of calculating the potential gains from reductions in variance, and shows that the set of parameters needed to solve for them is small. Like the ACR result, the analytical solutions allow for easy comparison across models. However, in our case the gains from any given reduction in variance also depend on elasticity of substitution. Hence, models with the same trade elasticity but different elasticities of substitution imply different gains from variance reduction.\textsuperscript{6}

I. Model

We begin by setting up a standard static household’s problem with constant elasticity of substitution preferences. There are two countries, and the representative household in country \( n \) solves the following problem:

\[
U_n = \max_{\{c_{nj}(i)\}} \sum_{j \in \{0, 1\}} \int_{\Omega_{nj}} \frac{c_{nj}(i)^{1-1/\sigma}}{1-1/\sigma} di
\]

subject to:

\[
\sum_{j \in \{0, 1\}} \int_{\Omega_{nj}} \tau_{nj}(i) p_{nj}(i) c_{nj}(i) di \leq w_n L_n + \Pi_n
\]

Here, \( \Omega_{nj} \) is the set of firms that produce in country \( j \) that sell to country

\textsuperscript{5}The model imposes the parameter restriction \( \theta > \sigma - 1 \). Therefore, if both models match the value of \( \varepsilon \) implied by the data, then \( \sigma \) must be lower in the model with an extensive margin.

\textsuperscript{6}All versions of the model considered in this paper are subject to the ACR result and, therefore, the same change in import penetration implies the same change in real income across models. However, the relationship between the variance of trade distortions and import penetration does vary across models. Therefore, different models can have different predictions for gains from reducing trade distortions, even if they have the same trade elasticity.
n. The trade cost \( \tau_{nj}(i) \) is the cost paid on good \( i \) produced in country \( j \) purchased by country \( n \). We assume that for all \( i \) and \( m \) \( \tau_{mm}(i) = 1 \), so that there are no domestic trade costs. \( L_n \) is the number of units of labor endowed to country \( n \), \( \Pi_n \) is the sum of profits earned by firms in country \( n \) that are rebated to households as a lump sum, \( w_n \) is the wage rate in country \( n \), and \( p_{nj}(i) \) is the price of good \( i \) produced in country \( j \) and sold in country \( n \). Here we assume that the elasticity of substitution \( \sigma \) is greater than one so that markups are finite.

We assume that the trade distortion \( \tau_{nj}(i) \) has two parts. The first is a level iceberg trade cost that affects all firms in the same way, \( \mu_{nj} > 1 \), such that \( \mu_{nj} \) units must be shipped from country \( j \) in order for country \( n \) to receive one unit. In additional, there is an idiosyncratic policy distortion \( t_{nj}(i) \) that we model as a tax charged by country \( j \) on exports to country \( n \), which varies at the firm level. Therefore, the trade distortion \( \tau_{nj}(i) = \mu_{nj}t_{nj}(i) \). This policy distortion is similar to the cross-firm distortions introduced in Buera and Shin (2011) and Restuccia and Rogerson (2008), but applied to exports.

While inelegant, we use this specific formulation for two reasons. First, in order to make use of the ACR result in our aggregate welfare calculations we need that aggregate tariff revenue should be equal to zero. Imposing this would make it impossible to do mean-variance comparisons, since changes in mean tariffs (keeping variance the same) would necessarily change aggregate tariff revenues. Therefore we rule out considering \( \tau_{nj}(i) \) to be a pure tariff. Second, an iceberg trade cost less than one implies that new goods are

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7To be clear, formally the only difference between the policy distortion and the iceberg transportation cost is that the policy distortions enter a government budget constraint and represent a transfer across firms, while iceberg transportation costs are lost when goods are shipped.
created through the process of shipping, which we think is unreasonable. This formulation allows us to avoid both of these issues. As discussed in Section III, we impose a parameter restriction that implies tariff budget balance, and the iceberg cost allows us to change the mean. Moreover, since all variation across firms is coming from tariffs, it is possible that some firms face a trade distortion $\tau_{nj}(i)$ less than one with the interpretation that their exports that subsidized.

Taking first order conditions from the household’s problem implies that the inverse demand function for good $i$ in country $n$ is:

$$p_{nj}(i) = \frac{D_n}{\tau_{nj}(i)} c_{nj}(i)^{-1/\sigma}$$

where we define

$$D_n \equiv \frac{w_n L_n + \Pi_n}{U_n}$$

Firms are monopolistically competitive and maximize profits in every market in which they operate. Production is constant returns to scale, so firms can maximize profits separately by market. The profit firm $i$ from country $j$ makes operating in market $n$ is:

$$\pi_{nj}(i) = \max_{y_{nj}(i), l_{nj}(i)} \frac{D_n}{\tau_{nj}(i)} y_{nj}(i)^{1-1/\sigma} - w_j l_{nj}(i)$$

subject to: $y_{nj}(i) \leq z_j(i) l_{nj}(i)$

Then the first order conditions imply:

$$\frac{\sigma - 1}{\sigma} p_{nj}(i) = \frac{w_j}{z_j(i)}$$
Revenues by market can be written as:

\[ p_{nj}(i)y_{nj}(i) = p_{nj}(i)^{1-\sigma} \left( \frac{D_n}{\tau_{nj}(i)} \right)^\sigma = \left( \frac{\sigma}{\sigma - 1} \frac{w_j}{z_j(i)} \right)^{1-\sigma} \left( \frac{D_n}{\tau_{nj}(i)} \right)^\sigma \]

Notice that only \( D_n \) and \( \tau_{nj}(i) \) vary by market. Then for a firm in country 0 that operates in both markets we get that the firm’s ratio of exports to domestic sales is:

\[ \frac{p_{10}(i)y_{10}(i)}{p_{00}(i)y_{00}(i)} = \left( \frac{D_1}{D_0} \right)^\sigma \tau_{10}(i)^{-\sigma} \]

Taking the natural logarithm of this equation implies:

\[ \log \left( \frac{p_{10}(i)y_{10}(i)}{p_{00}(i)y_{00}(i)} \right) = \sigma \log \left( \frac{D_1}{D_0} \right) - \sigma \log(\tau_{10}(i)) \]

Notice that the first term in equation (8) does not vary across firms. In the model, variation in \( \tau_{nj}(i) \) determines all variation in the ratio of exports to domestic sales. That is:

\[ \text{Var} \left( \log \left( \frac{p_1(i)y_1(i)}{p_0(i)y_0(i)} \right) \right) = \sigma^2 \text{Var} \left( \log(\tau_{10}(i)) \right) \]

Finally, we define the parameter \( \xi_{nj}(i) \) as:

\[ \xi_{nj}(i) \equiv \sigma^2 \tau_{nj}(i) \]

In the model, all variation in the ratio of exports to domestic sales is driven by differences in distortions across firms. Therefore, studying the distribution of export to domestic sales ratios is, through the lens of the model, informative about the distribution of trade distortions.
We finish our description of the model by considering two environments that we will contrast throughout the paper. In the first the set of markets in which firms operate is fixed, so there are only intensive margin adjustments to change in trade distortions. In the second, firms pay fixed costs to enter each market, as in Melitz (2003), so that more or fewer firms may operate in other market in response to a trade reform.

A. No Extensive Margin: Fixed Set of Markets

Suppose there is an exogenous measure of firms in each country, and those firms are exogenously assigned a set of markets to which they sell.\(^8\) Then an equilibrium is the solution to the household’s problem (1) and firms’ problem (4) together with wages in each country and labor market clearing conditions:

\[
\forall j: L_j = \int_{\Omega_{0j}} \mu_{0j} l_{0j}(i) di + \int_{\Omega_{1j}} \mu_{1j} l_{1j}(i) di
\]

In this environment we need not specify the distribution of productivity from which \(z_j(i)\) is drawn.

B. Extensive Margin of Exporters

As an alternative, suppose that firms choose which markets they operate in by paying fixed costs to access them, as in Melitz (2003). A mass of firms is born at the beginning of the period. Given that draw, a firm in country \(j\) may pay fixed cost \(f_{0j}\) to operate in market 0, and they may pay a fixed cost \(f_{1j}\) to operate in market 1. All fixed costs are paid in terms of labor.

\(^8\)We allow for some firms to be assigned to sell to the export market and not the domestic market, which is a potentially important group to consider as emphasized in Lu (2010).
In this environment we assume that exporting firms only observe their idiosyncratic $\tau_{nj}(i)$ for the export market after they have paid the fixed cost to enter the export market.\(^9\)

The pattern of entry decisions for firms follows a cutoff rule, where all firms that are above a productivity cutoff $\bar{z}_{nj}$ in country $j$ choose to pay the fixed cost to enter market $n$. Let $F$ be the joint distribution of productivity $z$ and trade distortions $\tau$ across firms. In this case the labor market clearing condition is:

$$\forall j : L_j = \int_T \int_{\bar{z}_0j}^{\infty} f_{0j} + \mu_{0j} l_{0j}(z, \tau) dF(z, \tau)$$

$$+ \int_T \int_{\bar{z}_1j}^{\infty} f_{1j} + \mu_{1j} l_{1j}(z, \tau) dF(z, \tau)$$

(12)

II. Evidence on Variation in Trade Distortions

We follow an approach similar to Hsieh and Klenow (2009) in that we use the model to infer the model-implied trade distortions that firms face on their exports. The model implies that variation in trade distortions show up in the data as variation in the ratio of exports to domestic sales, as shown in equation (9). In this section, we will describe the empirical properties of these implied distortions.

Our primary data source is the Annual Survey of Chinese Industrial Enterprises from 2006. This survey is conducted by the Chinese government and covers all manufacturing firms with at least 5 million RMB in sales.

\(^9\)Our reason for making this modeling choice is empirical. As an alternative, suppose that firms drew their productivity and export trade cost simultaneously at birth. Then the decision to export would introduce a positive correlation between trade costs and the scale of exports within the set of exporters driven by selection. As we show in the next section, this correlation is not observed in the data, so we set the timing such that the trade cost draw is observed after the export fixed cost is paid.
This is the most comprehensive data set on Chinese firms and has been used in many previous studies, such as Hsieh and Klenow (2009) and Song, Storesletten and Zilibotti (2011). We augment this firm-level data with customs form data to match exports to particular Chinese exporters. This allows us to measure exports country-by-country at the firm level.

First we control for obvious explanations for why the ratio of exports to domestic sales varies across firms within the same country. We consider three possibilities. First, the location of the firm within China may affect access to outside markets. It is less costly for firms located in coastal areas or near ports to access outside markets than for firms located in inland provinces. Therefore, we control for firm location at the province-level. Second, industries vary considerably in their export intensity, so we will control for industry. Finally, we have customs data that allows us to match customs forms to individual producers. Hence we know the set of countries to whom each firm is exporting, and the relative value of exports to each destination. Therefore, we will control for intensity of exports to each destination.

To implement this, we regress the natural log of export sales divided by domestic sales on industry fixed effects, province fixed effects, and the fraction of exports to each of the 189 export destinations that appear in the customs data. We then apply the exponential function to the residuals from this regression, and we call the result the residual trade distortion.\textsuperscript{10}

We document three facts about these residual distortions that will inform our analytical results. First, the residual distortions are log-normally distributed as shown in Figure 1a, where a histogram of the log of the residual distortions are shown.

\textsuperscript{10}The average of these is necessarily 0, so it is a distortion to the extent to which it differs from zero.
Second, firm-level productivity is uncorrelated with the residual trade distortion. The raw correlation of the residual trade distortions with number of employees is 0.008, with output per worker is -0.002 and with output per unit of capital is 0.006. We also estimate firm-level productivity using Levinsohn and Petrin (2003) using raw material usage to control for unobservables, and we find the correlation of that productivity measure and the residual trade distortion is 0.004. Therefore, we find no evidence that any measure of size or productivity is related to residual distortions.

Third, the value of the residual trade distortions varies considerably across ownership types. State-owned enterprises exhibit low export intensity, while other those of Taiwan, Macau and Hong Kong owned firms are highly export intensive. Table 1 shows mean values of the ratio of exports to domestic sales by ownership type.\textsuperscript{11} This variation by ownership type is a source of variation for the residual trade distortion. The variation of the log of residual distortions in the set of firms overall is 6.20, but among private firms it is 5.85. The higher variance in the full set of firms is driven by the presence of low export intensity state-owned enterprises as well as high export intensity foreign-owned firms. These firm ownership laws, therefore, introduce policy distortions across firms that affect export intensity.

For comparison, we also consider data from French exporting firms. This data comes from the Amadeus database, which collects data on European firms. We use data from 2006 and have firms in the data set.\textsuperscript{12} We do not have as much data on these firms as we do on the Chinese firms, so we

\textsuperscript{11} Our first and second empirical results are unaffected by introducing ownership type as a control. We do not do that in the main exercise because of our first counterfactual exercise: to reduce the variance of distortions to that of private firms.

\textsuperscript{12} This data was collected in 2008, which eliminates the selection problem generated by the well-known problem in this dataset of back-deleting firms that do not appear in later years of the dataset.
cannot control for location within France or country of export.\textsuperscript{13} Therefore, the residual dispersion for French firms controls only for industry.

The French data also exhibits a log-normal distribution of residual trade distortions, as shown in Figure 1b. Moreover, we also find in this data that there is no evidence of correlation between the residual trade distortions and size or productivity. The correlation of the residual trade distortion with number of employees is 0.004, with output per worker is -0.003, with output per unit of capital is -0.009, and with the Levinsohn and Petrin (2003) measure it is 0.001. We will now introduce these empirical results as assumptions into our model and show what they imply for gains from reduction in variance.

\textbf{III. Mean-Variance Tradeoff}

In this section we derive the mean-variance tradeoff in trade costs that is implied by the model. To do this, we make two assumptions about the distribution of trade costs across firms that follow from the empirical results presented in Section II.

\textbf{ASSUMPTION 1:} Draws of productivity $z$ are independent of draws of trade costs $\tau$.

\textbf{ASSUMPTION 2:} Trade costs $\tau$ are log-normally distributed.

We begin by solving the household’s problem for economic aggregates as a function of the trade costs and productivities of operating firms. Using the

\textsuperscript{13}Because of this, our measure of residual dispersion for France has more variation than it would if we could control for those things. Therefore, our results in Sections III and IV, if anything, understate the gains from reducing the variation in distortions from the level of China to the level of France.
results from Section I, note that the price index in country 0 can be written as:

\[(13) \quad P_0^{1-\sigma} = \int_{\Omega_{00}} p_0(i)^{1-\sigma} \, di + \int_{\Omega_{01}} (\tau_0(i)p_0(i))^{1-\sigma} \, di \]

Labor market clearing and the budget constraints in both countries together imply trade balance, which in turn implies that the price index can be written as:14

\[(14) \quad P_0^{1-\sigma} = \int_{\Omega_{00}} p_0(i)^{1-\sigma} \, di + \int_{\Omega_{10}} (\tau_1(i)p_1(i))^{1-\sigma} \, di \]

From the solution to the firm’s problem given above, prices are the product of firm-level marginal cost and a constant markup. Substituting this result in for prices implies:

\[(15) \quad P_0^{1-\sigma} = \left( \frac{w_0\sigma}{\sigma - 1} \right)^{1-\sigma} \left( \int_{\Omega_{00}} z_0(i)^{\sigma-1} \, di + \int_{\Omega_{10}} \tau_1(i)^{1-\sigma} z_0(i)^{\sigma-1} \, di \right) \]

Then by Assumption 1, productivity and trade costs are independent, which implies this can be written as:

\[(16) \quad P_0^{1-\sigma} = \left( \frac{w_0\sigma}{\sigma - 1} \right)^{1-\sigma} \left( \int_{\Omega_{00}} z_0(i)^{\sigma-1} \, di + \int_{\Omega_{10}} \tau_1(i)^{1-\sigma} \, di \int_{\Omega_{10}} z_0(i)^{\sigma-1} \, di \right) \]

In this environment, real income varies one-for-one with the perfect price index. Therefore, if the mean or variance of \( \tau_1 \) is changed, that may affect welfare directly, since an aggregation of \( \tau_1 \) appears in the price index, and indirectly by affecting prices \( w_0 \) or, in the extensive margin version, the sets

\[14\text{See the appendix for the derivation of this claim.}\]
of operating firms $\Omega_{10}$ and $\Omega_{10}$. However, our first result shows that when we analyze the mean-variance tradeoff these indirect effects are absent.

**PROPOSITION 1:** Consider two economies that are identical except that in one $\tau_1 \sim F$ and in the other $\tilde{\tau}_1 \sim \tilde{F}$. If:

$$\int_{\Omega_{10}} \tau_1^{1-\sigma} \, d\mu = \int_{\Omega_{10}} \tilde{\tau}_1^{1-\sigma} \, d\mu$$

Then all equilibrium prices and sets of operating firms are the same in both economies.

**PROOF:**

See appendix.

Given this result, then the mean-variance tradeoff can be characterized for general distributions from which trade costs are drawn. We will proceed using Assumption 2, that the trade costs are exactly log-normal, to give exact results.

As derived above, if $\log(\tau) \sim Normal(\mu + (\sigma - 1/2)s^2, s^2)$ then we will analyze the set of $(\mu, s^2)$ pairs between which the representative household is indifferent. Using the derived perfect price index above, and the result from Proposition 1, we can characterize this set completely in the log-normal case by noting that $\log(\tau^{1-\sigma}) = (1 - \sigma) \log(\tau) \sim Normal((1 - \sigma)(\mu + (\sigma - 1/2)s^2), (1 - \sigma)^2 s^2)$. Therefore using the properties of the log-normal distribution:

$$\int_{\Omega_{10}} \tau_1^{1-\sigma} \, d\mu = e^{(1-\sigma)(\mu+(\sigma-1/2)s^2)+(1-\sigma)^2s^2/2} = e^{(1-\sigma)(\mu+s^2)}$$

Then the implied mean-variance tradeoff is given by Proposition 2.
PROPOSITION 2: Consider two economies that are identical except that in the first the parameters on the trade cost distribution are \((\mu_1, s_1^2)\) and in the second they are \((\mu_2, s_2^2)\). Then real income is the same in these two economies as long as:

\[
\mu_1 - \mu_2 = -\frac{\sigma}{2} (s_1^2 - s_2^2)
\]

PROOF:

Follows immediately from Proposition 1, and the derivation in the text above.

Proposition 2 demonstrates that in the log-normal case, the tradeoff between the mean and variance in trade costs is linear and depends only on the elasticity of substitution parameter \(\sigma\). The simple interpretation of this result is that the representative household would be willing to accept greater variation in trade costs only in exchange for a lower mean. In the next subsection we show how this simple result is useful for measuring the importance of dispersion in trade costs in terms of the equivalent reduction in mean trade costs.

An immediate question is how the parameters of the trade cost distribution are translated into observables. Given that the trade costs are log-normally distributed, then the ratio of export and domestic sales is also log-normal in the model. In fact, we know that:

\[
(18) \quad \log \left( \frac{p_1(i)y_1(i)}{p_0(i)y_0(i)} \right) \sim \text{Normal} \left( \sigma \log \left( \frac{D_1}{D_0} \right) - \sigma \mu, \sigma^2 s^2 \right)
\]
The empirical mean is given by:

\begin{equation}
    m \equiv \sigma \log \left( \frac{D_1}{D_0} \right) - \sigma \mu + \frac{1}{2} \sigma^2 s^2
\end{equation}

So that:

\begin{equation}
    e^m = E \left( \frac{p_1(i)y_1(i)}{p_0(i)y_0(i)} \right)
\end{equation}

**PROPOSITION 3:** In any two economies with parameters \((\mu_1, s_1^2)\) and \((\mu_2, s_2^2)\) such that real income is equal in both, the empirical mean \(m\) and variance \(\xi^2\) of the export to domestic sales ratio satisfy the following equation:

\[
m_1 - m_2 = \frac{1}{2\sigma} (\xi_1^2 - \xi_2^2)\]

**PROOF:**

Follows immediately from Proposition 2 and derivation in the text.

Propositions 3 is analogous to Proposition 2, but is stated in terms of empirical moments rather than model parameters. Propositions 2 and 3 together allow us to make statements about how changes in the empirical dispersion in trade costs are traded off with both changes in mean trade costs and with empirical export behavior. We first discuss how to choose the parameter \(\sigma\) that appears in these relationships, then we look at some counterfactuals.

**A. Parameterization**

The parameter that controls the degree of the tradeoff between the mean and variance is the elasticity of substitution \(\sigma\). To analyze the effect of changes in the dispersion of trade costs, we must first choose a value of \(\sigma\).
In both cases, we rely on the results from the empirical gravity literature. In that literature, the trade elasticity $\varepsilon$ is estimated and can be related back to the elasticity of substitution. As reviewed in (Arkolakis, Costinot and Rodriguez-Clare, 2012), the formulas that relate $\varepsilon$ to $\sigma$ depend on the underlying structural model. In the model with only an intensive margin, the formula is:

$$\varepsilon = 1 - \sigma$$

In the model with an extensive margin and Pareto distribution with shape parameter $\theta$, the relationship is:

$$\varepsilon = 1 - \frac{\sigma \theta}{\sigma - 1}$$

Following Anderson and Van Wincoop (2004), we consider values of $\varepsilon$ that are in line with empirical estimates, which fall in the interval between $(-5, -10)$. In the model without an extensive margin, this immediately implies that $\sigma \in (6, 11)$.

In the model with an extensive margin, we must settle on a value of $\theta$ as well. To do this, we first use the method of Levinsohn and Petrin (2003) to estimate firm-level productivity sector-by-sector in the Chinese data. We find that the coefficient of variation in firm-level productivity is 2.67. We set $\theta = 2.07$ to match this moment.\(^{15}\) Therefore, using the formula above, if $\varepsilon \in (-5, -10)$, then $\sigma \in (1.21, 1.53)$. Notice that the two models imply very different ranges of the value of $\sigma$, and will therefore have very different

\(^{15}\)If productivity is Pareto distributed, then the coefficient of variation $CV$ is:

$$CV = \frac{1}{\sqrt{\theta} \sqrt{\theta - 2}}$$
implied effects of dispersion. This difference derives directly from how the elasticity of substitution $\sigma$ is parameterized.

B. Application to Chinese Firm-level Data

Here we apply Propositions 2 and 3 to do counterfactuals to measure the importance of trade cost dispersion among Chinese manufacturing firms. In particular we will examine three counterfactuals: 1) lowering the dispersion level of the entire set of Chinese firms to the level of the set of private Chinese firms, 2) lowering the dispersion level of Chinese firms to the level of French firms, and 3) eliminating all dispersion.

Mean-Variance Tradeoff in Three Counterfactuals

China is known for its variety of ownership types, which play a variety of roles. These various ownership types exist to address a variety of economic objectives and exist under widely varying regulatory regimes. Several of these ownership types face special implicit or explicit barriers or subsidies to international trade relative to Chinese private firms. Therefore, the existence of these classifications acts as a type of trade distortion.

State Owned Enterprises (SOEs) are perhaps the most well-known example of a special ownership classification. These firms are typically very large and operate in industries intended to supply Chinese firms. As they are owned by the government, it is not clear what objective function individual SOEs are trying to maximize. However, it is clear that these firms export much less intensively than other firms, and that they have advantaged access to finance and some inputs. By and large, SOEs are in the left tail of export intensity.
On the other hand, another special ownership classification exists for Chinese firms operated by foreign partnerships based in Taiwan, Macau and Hong Kong. These firms exist as Chinese export platforms for firms based in those localities and export much more intensively than private Chinese firms. Firms with this classification face much less regulation in exporting than similarly situated private Chinese firms, and could be safely classified as the recipients of implicit subsidies to exporting.

In the data, we first measure the dispersion of the measure of trade costs implied by the model: the ratio of export sales to domestic sales. In the set of firms with positive foreign and domestic sales, the variance of the log of this ratio is 6.35 for the whole set of Chinese firms (all ownership types together), and is 5.90 for private firms. Our first exercise is to measure the equivalent change in mean tariffs and export intensity implied by that change in dispersion.

As explained above, we consider values of $\sigma$ between 6 and 11 for the model without an extensive margin, and between 1.21 and 1.53 for the extensive margin model. For this exercise, we label variables in the observed economy with 1 and in the counterfactual economy with 2. Therefore, $\xi_1^2 - \xi_2^2 = 0.45$. Then by Proposition 3, we know that this implies that:

$$m_1 - m_2 = \frac{1}{2\sigma} \cdot 0.45$$

Therefore, decreasing trade cost dispersion by that much would have the same effect on welfare as if all exporting firms would increase their ratio of export to domestic sales by between 2.1% and 3.8% in the model without an extensive margin, or between 15.9% and 20.5% in the model with an extensive margin.
Using Proposition 2, we can also make these statements in terms of the equivalent reduction in average trade costs, noting that:

\[ s_1^2 - s_2^2 = \frac{1}{\sigma^2} 0.45 \]

Then the representative household would be indifferent between this reduction in trade cost dispersion and a reduction in mean trade costs of between 2.0% and 3.7% in the intensive margin model, or between 13.7% and 17.0% in the extensive margin model.16

Next, we consider the case where the dispersion of trade costs is reduced from the level in the Chinese data to what it is in the French data. The variance of export sales to domestic sales among French exporters is 4.75, so this represents is a reduction of 1.60. Proceeding as in the previous counterfactual, the representative household would be indifferent between the decrease in dispersion and all firms exporting between 7.5% and 14.2% more in the intensive margin model, or between 68.7% and 93.7% more in the extensive marginal model. In terms of trade costs, the representative household would be indifferent between the reduction in dispersion and a reduction in trade costs for all firms of between 7.0% and 12.5% in the intensive margin model, and between 40.7% and 48.3% in the extensive margin model.

Last, we consider the case of reducing the dispersion of trade costs all the way to zero. We interpret this as the most extreme possible gain from a reduction in dispersion, but we do not consider this to be the likely result.

16 These values correspond to different values of \( \sigma \) as discussed in the previous subsection. We are considering \( \sigma \in (6,11) \) for the model without an extensive margin, and \( \sigma \in (1.23,1.52) \) in the model with an extensive margin.
of any feasible policy (such as equating tariff rates or removing all export
subsidies). With that in mind, we proceed as above to compute the gains
from this extreme case. The representative household would be indifferent
between removing all distortions in trade costs and all exporting firms in-
creasing their export to domestic sales ratios by between 33% and 70% in
the intensive margin model, and between 800% and 1384% in the extensive
margin model. Likewise, they would be indifferent between this decrease
in dispersion and a reduction in trade costs for all firms of between 25%
and 41% in the intensive margin model, and between 87% and 93% in the
extensive margin model.

IV. Welfare Gains from Variance Reduction

In the previous section we focus on quantifying the gains from reducing
trade cost dispersion by finding the equivalent level decrease in tariffs or
level increase in exports. In this section we instead directly measure the
increase in real income associated with reductions in trade cost dispersion.
To do this, we develop a two step procedure. The first is to relate changes in
the dispersion in trade costs to changes in import penetration (taking into
account general equilibrium effects). Second, we use the formula derived
in Arkolakis, Costinot and Rodriguez-Clare (2012) to relate the change in
import penetration to changes in real income.

For the first step, we first solve for the relationship between changes in the
distribution of domestic trade distortions and changes in import penetration.
First we define some notation:

\[ \lambda_{mn} = \frac{\int_{\Omega_{mn}} \tau_{mn}(i)p_{mn}(i)y_{mn}(i)}{\frac{\sigma}{\sigma-1}w_mL_m} \]
This term is the share of country \( m \) expenditure on goods from country \( n \). In this two country model, note that \( \lambda_m0 + \lambda_m1 = 1 \). Also our measure of the magnitude of trade distortions is:

\[
T_{mn} = \int_{\Omega_{mn}} \tau_{mn}(i)^{1-\sigma} \, di
\]

Note that \( \forall m, T_{mm} = 1 \).

PROPOSITION 4: Consider two economies that differ only in the trade distortion term \( T_{01} \). Then:

\[
\frac{T_{01}}{T'_{01}} = \frac{\frac{1}{1-\lambda'_{00}} \frac{w_1 L_1}{w_0 L_0} \left( \frac{\lambda'_{00}}{1-\lambda'_{00}} \frac{\lambda_{00}}{1-\lambda_{00}} \right)^{\frac{1}{\sigma-1}} - \frac{\lambda'_{00}}{1-\lambda'_{00}} \frac{\lambda_{00}}{1-\lambda_{00}}}{\frac{1}{1-\lambda_{00}} \frac{w_1 L_1}{w_0 L_0} - 1}
\]

where variables written as \( x \) are from the first economy and \( x' \) are from the second.

PROOF: See Appendix.

For any given change in \( T_{01} \), this demonstrates that the only information you need to solve for \( \lambda'_{00} \) is the initial value of \( \lambda_{00} \), the elasticity of substitution \( \sigma \), and the ratio of initial output in the two countries. Moreover, the implicit function of \( \lambda'_{00} \) has several properties that make it easy to solve.

LEMMA 5: Equation (23) is strictly convex in \( \lambda'_{00} \) and has a unique solution in \( \lambda'_{00} \).

PROOF: INSERT PROOF
Given this result, solving (23) is simple using Newton’s method. This allows for us to solve for $\lambda'_{00}$ and therefore to measure the welfare gains from changes in $T_{01}$, coming from changes in $\mu$ or $s$, using the main result from Arkolakis, Costinot and Rodriguez-Clare (2012), which implies:

$$
\frac{W'}{W} = \left( \frac{\lambda'_{00}}{\lambda_{00}} \right)^{1/\varepsilon}
$$

where $W$ is real income and $\varepsilon$ is the trade elasticity. We now revisit our three examples discussed above and give the welfare gains from each reduction in the variance of trade distortions.

To do this, the only additional information that we need relative to before is the ratio of output in the two countries and the initial import penetration ratio. Here we assume the two countries are China and the rest of the world, so the relevant ratio of incomes is Chinese GDP divided by world GDP less China. In 2007, this ratio is 0.065.\(^\text{17}\) For the initial value of $\lambda_{00}$ we use Chinese import penetration of 10.7%, which implies that $\lambda_{00}$ is 0.893.\(^\text{18}\) We then use values of the trade elasticity $\varepsilon$ and the elasticity of substitution $\sigma$ as discussed above.

**Welfare Gains in Three Counterfactuals**

Just as in the case of the mean-variance tradeoff considered above, we consider three counterfactual reductions the variance of all Chinese firms: reducing variance to the level of private Chinese firms, reducing variance to the French level, and reducing variance to zero.

\(^{17}\)We use data from the World Bank’s *World Economic Outlook Database*. World GDP in this calculation is the sum of GDP in the 189 countries that appear in the database.

\(^{18}\)This value comes from the *OECD Structural Analysis Database*, and is the ratio of total imports to gross output for China in the “mid-2000s”.
Reducing the variance of all Chinese exporters to the level of Chinese private firms, as before, reduces the variance of the log of the ratio of exports to domestic sales from 6.20 to 5.86. We now apply Proposition 4 to compute the new import penetration ratio, then measure the increase in real income associated with that change in import penetration. Following the same parameterization as before, this implies an increase in real income for China between 0.09% and 0.16% in the model without an extensive margin, and between 0.17% and 0.26% in the model with an extensive margin. In is worth noting that the overall gains from moving from autarky to free trade is between 1.1% and 2.3% in both versions of the model.\(^{19}\)

Second, we apply the variance in trade distortions implied by the French data to Chinese firms, which reduces the variance of the log of export to domestic sales from 6.20 to 4.75. In the model without an extensive margin this implies increases in real income between 0.42% and 0.71%, while in the model with an extensive margin it is between 0.88% and 1.28%.

Finally, we consider the extreme case of removing all variance in trade distortions. In the model without an extensive margin the increase in real income is between 3.15% and 5.02%, and with an extensive margin it is between 11.60% and 12.80%. In this case the gains are extreme because the implied equivalent reduction in mean trade costs is very large. In the extensive margin case, for example, this implies that the import penetration ratio increases from 10.7% to between 50.1% and 70.2%.

\(^{19}\)Again, we are considering the range of trade elasticities to be between \((-5,-10)\), which is what gives a range of real income increases in all cases.
V. Conclusion

We provide a framework for evaluating the effects of changes in the dispersion of trade distortions across firms, rather than just changes in the level distortion for all firms. We provide two reasonable counterfactuals for China: reducing the dispersion across Chinese firms to that of private Chinese firms, and reducing the dispersion in Chinese firms to that of France. Both of these suggest that the gains in dispersion are sizable. Moreover, in all cases the gains with an extensive margin are around twice as large as those without, suggesting that the extensive margin plays an important role. Finally, we consider the counterfactual in which all dispersion is removed, which implies very high gains. Although we include this example as a first pass at estimating the overall losses due to dispersion, we do not believe that exactly equating the ratio of export to domestic sales across firms is a reasonable policy objective.

In this paper we took an approach similar to Hsieh and Klenow (2009) in that we assume that if the variation in the ratio of export to domestic sales is greater in China than in France, then the difference is, at least potentially, due to policy distortions. Future work should focus on what policies generate distortions to trade that vary across firms, and how they can be measured. Then the results developed in this paper can translate changes in variance to welfare gains.

REFERENCES


Figure 1.: Histograms of the Log of Residual Trade Dispersion

(a) China

(b) France

Table 1—: Average Ratio of Exports to Domestic Sales by Ownership Type

<table>
<thead>
<tr>
<th>Ownership Type</th>
<th>Avg. Ratio of Exports to Domestic Sales</th>
</tr>
</thead>
<tbody>
<tr>
<td>Private Firms</td>
<td></td>
</tr>
<tr>
<td>State-Owned Enterprises</td>
<td></td>
</tr>
<tr>
<td>Collective-Owned Enterprises</td>
<td></td>
</tr>
<tr>
<td>HMT Cooperative Enterprises</td>
<td></td>
</tr>
<tr>
<td>Foreign Firms</td>
<td></td>
</tr>
<tr>
<td>Other Ownership Types</td>
<td></td>
</tr>
</tbody>
</table>

Table 2—: Counterfactuals: Equivalent Reductions in Mean

<table>
<thead>
<tr>
<th>Variance reduced to:</th>
<th>No Extensive Margin</th>
<th>With Extensive Margin</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\varepsilon = -5$</td>
<td>$\varepsilon = -10$</td>
</tr>
<tr>
<td>Private Firms</td>
<td>2.82%</td>
<td>1.55%</td>
</tr>
<tr>
<td>French Firms</td>
<td>11.36%</td>
<td>6.37%</td>
</tr>
<tr>
<td>Zero</td>
<td>40.34%</td>
<td>24.56%</td>
</tr>
</tbody>
</table>

Welfare-equivalent percentage reduction in mean trade distortions.
Table 3—: Counterfactuals: Increase in Real Income

<table>
<thead>
<tr>
<th>Variance reduced to:</th>
<th>No Extensive Margin</th>
<th>With Extensive Margin</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\varepsilon = -5$</td>
<td>$\varepsilon = -10$</td>
</tr>
<tr>
<td>Private Firms</td>
<td>0.16%</td>
<td>0.09%</td>
</tr>
<tr>
<td></td>
<td>0.27%</td>
<td>0.17%</td>
</tr>
<tr>
<td>French Firms</td>
<td>0.71%</td>
<td>0.42%</td>
</tr>
<tr>
<td></td>
<td>1.28%</td>
<td>0.88%</td>
</tr>
<tr>
<td>Zero</td>
<td>5.02%</td>
<td>3.15%</td>
</tr>
<tr>
<td></td>
<td>12.90%</td>
<td>11.60%</td>
</tr>
</tbody>
</table>

Real Income Losses of Autarky Relative to Observed Trade

| Gains from Trade     | 2.29%               | 1.14%                 |
|                      | 2.29%               | 1.14%                 |