Teaching Note on Factor Model with a View --- A tutorial

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This tutorial demonstrates how to incorporate economic views in optimal asset allocation in the framework of factor model. The tutorial is structured in a question-and-answer format. Please try to solve each question before looking at its answer. All original data and detailed calculation are contained in an excel spreadsheet --- *factorwview.xls*. In addition, MarkowitzII is required for the optimal portfolio calculation.

Basic problem: You are interested in investing in six portfolios, formed by sorting all stocks according to their market capitalization and book-to-market ratio.[1]

	Book-to-market		
Market Cap	Low	Medium	High
Small	Portfolio 1: Small growth	Portfolio 2: Small neutral	Portfolio 3: Small value
Big	Portfolio 4: Big growth	Portfolio 5: Big neutral	Portfolio 6: Big value

You believe that the returns are affected by macro economic variables and the dynamics can be captured by a four-factor model, where the excess return of portfolio (or asset) i can be written as:

$$R_{it} = a_i + b_{1i}F_{1i} + b_{2i}F_{2i} + b_{3i}F_{3i} + b_{4i}F_{4i} + e_{it}. \qquad (1)$$

The first factor is the excess return on the stock market index portfolio. The second factor is the change in the slope of the term structure. The third factor is the change in the yield spread between Baa and Aaa bonds. The fourth factor is Oil inflation (percentage change in Oil price). The quarterly time series of the four factors and excess returns of the six portfolios[2] from 1994 to 2003 are also provided in *factorwview.xls*.

Today is Dec 31, 2003 and you are currently holding a passive portfolio of the six assets. However one energy expert recently told you that the oil price next year is going to be a lot higher than you might expect. Specifically, his forecast of oil inflation for year 2004 is 25% higher than the consensus forecast. You are very confident in his forecast and therefore want to incorporate this view in your asset allocation decision.

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[1] Detailed description on the six portfolios and their returns can be found in Prof Ken French's website: http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html.

[2] The excess return is defined as portfolio return minus risk free rate.

Question 1: *What are the factor loadings for each portfolio?*

Equation (1) can be estimated using regression in Excel. To do that, go to Tools --- Data Analysis --- Regression. In this case, Y corresponds to excess returns of one of the six portfolios and X corresponds to the four factors. The regression coefficients are the factor loadings. Repeat the regression six times to obtain the factor loadings for all six portfolios as reported in Table 1. In general, the market factor is significant for all portfolios. In addition, the term spread factor is significant for the small growth portfolio and the oil factor is significant for the big value portfolio. In another words, the big value portfolio will be affected the most by the oil price shock.

Question 2: *What is the variance of each of the six portfolio returns? What is the variance due to factor exposure, i.e., systematic variance? What is the residual or specific variance? What is the variance of the value-weighted market portfolio? What is the specific and systematic variance of the market portfolio?*

Using the factor loadings, we can decompose the total variance of any asset into systematic variance and residual variance. Let b_i denote a vector containing factor loadings for asset i, i.e., $b_i = [b_{1i}\ b_{2i}\ b_{3i}\ b_{4i}]$. Let Σ denote the covariance matrix of the four factors. Then, we have:

$$Var_i = SysVar_i + \mathrm{Re}\,sVar_i,$$
$$SysVar_i = b_i \Sigma b_i'.$$

We can also do the same variance decomposition for the market portfolio. Let w denote a vector containing the market portfolio's current weights in the six assets and Ω denote the covariance matrix of the six assets. Using the market values of the six portfolios at the end of year 2003, we can compute the weights to be:

	Portfolio 1: Small growth	Portfolio 2: Small neutral	Portfolio 3: Small value	Portfolio 4: Big growth	Portfolio 5: Big neutral	Portfolio 6: Big value
Mkt port weight	2.95%	3.89%	2.23%	58.07%	24.10%	8.76%

Then we can compute the total variance of the market as:

$$Var_{mkt} = w\Omega w'.$$

To compute the systematic variance of the market, we need to find out the factor loadings of the market, which are simply weighted averages of factor loadings of individual asset:

$$b_{mkt,j} = \sum_{i=1}^{6} w_j b_{i,j} \text{ , and}$$
$$SysVar_{mkt} = b_{mkt} \Sigma b_{mkt}'.$$

To compute the covariance matrix Σ and Ω, go to Tools --- Data Analysis --- Covariance. To carry out matrix operation, use excel function MMULT() for matrix multiplication and TRANSPOSE() for matrix transpose.[3] Notice, the market portfolio have a positive residual. In this example, all factors are macro variables (rather than risk factors) which are convenient for converting views on economy to changes in expected returns.

Question 3: *Assume the expected returns of the six portfolios for year 2004 are:*

	Portfolio 1: Small growth	Portfolio 2: Small neutral	Portfolio 3: Small value	Portfolio 4: Big growth	Portfolio 5: Big neutral	Portfolio 6: Big value
Expected return	0.1467	0.1066	0.1016	0.1123	0.0971	0.0931

In addition, we assume every investor (including yourself) has a risk aversion of 2.407 and the risk free rate is 3%. What will be the optimal portfolio weights?

We solve this problem using the MarkowitzII spreadsheet. Compute standard deviations and correlations of the six portfolios using the historical returns. Input expected returns, standard deviations, correlations, risk aversion and risk free rate into the MarkowitzII and use solver to find the optimal portfolio weights that maximize the slope of the CAL.

Number of securities: 6

Fill in Names

No	Name	Fraction 0	Expected Return	Standard Deviation
1	port 1	0.031	0.147	0.306
2	port 2	0.033	0.107	0.208
3	port 3	0.026	0.102	0.210
4	port 4	0.581	0.112	0.197
5	port 5	0.242	0.097	0.174
6	port 6	0.088	0.093	0.169
		1		

Correlations

	1 port 1	2 port 2	3 port 3	4 port 4	5 port 5	6 port 6
port 1	1	0.89	0.80	0.86	0.71	0.68
port 2		1.00	0.97	0.73	0.87	0.82
port 3			1.00	0.65	0.85	0.81
port 4				1.00	0.76	0.74
port 5					1.00	0.94
port 6						1.00

Corr OK? YES

Results:

Portfolio's Expected Return	0.1075
Portfolio's Standard Deviation	0.1795

Risk Free Rate	0.0300	Risk Aversion Coefficient: A=	2.407
Slope of CAL	0.4319	Weight on optimal risky portfolio: x*=	1.00

All investor will hold only the optimal risky portfolio (x*=1) and the weights of the optimal risky portfolio coincide with those of the current market portfolio. In another words, under the expected returns in this questions, all investor chooses to hold the same market portfolio. We call these expected returns --- the consensus expected returns.

Question 4: *You expect the oil inflation to be 25% higher than the consensus forecast. How will your view on the oil factor affect your expectation (measured as deviation from the consensus) on other three factors?*

[3] Remember to press CRTL+SHIFT+ENTER at the same time for matrix operation in Excel.

Since all factors are correlated. If you expect oil inflation to deviate from consensus, you would also expect other factors to deviate. To determine the impact of factor surprise on oil inflation on other factors, we compute the oil beta as:

$$\beta_i^{oil} = \mathrm{cov}(F_i, F_{oil}) / \mathrm{var}(F_{oil}).$$

The oil beta of factor i is just the slope coefficient when we regress factor i on oil inflation factor. Intuitively, it captures the sensitivity of other factor to changes in oil inflation. Therefore we have:

expected change in F_i = β_i^{oil} * expected change in oil inflation.

The factor surprises are computed as:

	Mkt excess return	change in term spread	change in credit spread	Oil inflation
oil beta	-0.1467	-0.0048	-0.0032	1.0000
factor surprise	-3.67%	-0.12%	-0.08%	25.00%

Question 5: *What are the new expected returns of the six portfolios and the market portfolio under your view?*

The new expected returns are just consensus expected returns plus changes due to factor surprises.

$$E(R_1 \mid view) = Consensus\ ER_i + \sum_{i=1}^{4} b_{i,j} \Delta F_j .$$

The market expected return is again is weighted-average of expected returns of the six portfolios.

Question 6: *Suppose you can only invest in the market portfolio and the risk free asset. Given the view on the oil inflation, what will be the fraction invested in the market, and how much will be invested in the risk free asset? Compute the expected return on your portfolio and its standard deviation. What would be the expected return and the standard deviation if you are forced to hold the market portfolio instead? What is the certainty equivalent return on the two portfolios -- the combination of the market and the risk free you chose to hold given the view, and if you are forced to hold the market?*

Input the new expected returns under the view into MarkowitzII and read off the weight on the optimal risky portfolio (x*) to be 0.45. This is the fraction you should invest in the market portfolio given your view.

Number of securities: 6

Fill in Names

No	Name	Fraction 0	Expected Return	Standard Deviation
1	port 1	0.031	0.098	0.306
2	port 2	0.033	0.066	0.208
3	port 3	0.026	0.058	0.210
4	port 4	0.581	0.071	0.197
5	port 5	0.242	0.056	0.174
6	port 6	0.088	0.042	0.169
		1		

Correlations	1 port 1	2 port 2	3 port 3	4 port 4	5 port 5	6 port 6
port 1	1	0.89	0.80	0.86	0.71	0.68
port 2		1.00	0.97	0.73	0.87	0.82
port 3			1.00	0.65	0.85	0.81
port 4				1.00	0.76	0.74
port 5					1.00	0.94
port 6						1.00

Corr OK? YES

Results:

Portfolio's Expected Return	0.0652
Portfolio's Standard Deviation	0.1795

Expected return	0.045935
Std dev	0.081364
Certainty Equivalent return	0.037967

Risk Free Rate	0.0300	Risk Aversion Coefficient: A=	2.407
Slope of CAL	0.1958	Weight on optimal risky portfolio: x*=	0.45

Table 2 summarizes the main results for various cases. First thing to note is that the Sharpe ratio comes down significantly under your view on oil inflation since the oil shock has a substantial negative impact on expected returns. Consequently, the risky asset becomes less favorably against the risk free asset. That's why you choose to invest more than half of your wealth in risk free asset when the risk free asset is allowed. The certainty equivalent return is 3.80%. However if you are forced to hold the market portfolio only (x=1), the certainty equivalent return drops to 2.64%. This means that simply by the risk free asset, there is an improvement of 1.16% (3.80%-2.64%) in terms of certainty equivalent return. For a total investment of $500,000, this is the same as an increase in value at the end of the year of $500,000*1.16% = $5,782, which is not a small amount.

Question 7: *Suppose you can now in addition choose a different combination of the six portfolios to construct the optimal risky asset: what would be the weights assigned to each portfolio? What are the expected return and the standard deviation of this portfolio given the view? What fraction will you hold in the risk free and this risky asset? What are the expected return and the standard deviation of your portfolio? What is the certainty equivalent? How much do you gain by being able to use a different risky asset than being forced to choose a combination of the market and the risk free?*

In this case, investor will form a new optimal risky portfolio by optimally investing in the six assets to maximize the slope of CAL under the new expected returns.

Number of securities: 6

Fill in Names

No	Name	Fraction 0	Expected Return	Standard Deviation
1	port 1	3.418	0.098	0.306
2	port 2	-1.234	0.066	0.208
3	port 3	-2.010	0.058	0.210
4	port 4	0.395	0.071	0.197
5	port 5	12.641	0.056	0.174
6	port 6	-12.209	0.042	0.169
		1		

Correlations	1 port 1	2 port 2	3 port 3	4 port 4	5 port 5	6 port 6
port 1	1	0.89	0.80	0.86	0.71	0.68
port 2		1.00	0.97	0.73	0.87	0.82
port 3			1.00	0.65	0.85	0.81
port 4				1.00	0.76	0.74
port 5					1.00	0.94
port 6						1.00

Corr OK? YES

Results:

Portfolio's Expected Return	0.3600
Portfolio's Standard Deviation	1.0261

Expected return	0.072964
Std dev	0.133602
Certainty Equivalent return	0.051482

Risk Free Rate 0.0300 Risk Aversion Coefficient: A= 2.407

Slope of CAL 0.32 Weight on optimal risky portfolio: x*= 0.13

Both Sharpe ratio and certainty equivalent return increase as a result of re-optimization after changes in expected returns (Table 2). Compare to the case when you can only hold the market and risk free asset (in question 6), the certainty equivalent return increases by 1.35% (5.14%-3.80%). However, you observe extreme positions in some assets which may not be desirable.[4]

Question 8: *Show that it is possible to think of the new optimal risky portfolio as a portfolio of the current market portfolio and an ACTIVE portfolio that is managed using the view. What is the composition of the ACTIVE portfolio; its expected return; standard deviation and Sharpe Ratio?*

Since the market portfolio weight in asset i and the ACTIVE portfolio weight in asset i should add up to be the optimal risky portfolio's weight in asset i, or

$$w_{mkt,i} + w_{act,i} = w_{opt,i},$$

we can solve for the ACTIVE portfolio weight as:

$$w_{act,i} = w_{opt,i} - w_{mkt,i}.$$

In addition, we know:

$$\sum_{i=1}^{6} w_{mkt,i} = \sum_{i=1}^{6} w_{opt,i} = 1,$$

which implies:

$$\sum_{i=1}^{6} w_{act,i} = 0.$$

In another words, the ACTIVE portfolio is a zero-investment strategy where the long and short position net out and the net investment is zero.

Once we have the weights of the ACTIVE portfolio, we can compute the expected return and standard deviation and Sharpe ratio in the usual way:

[4] In fact, now the optimal weights solver calculates are more sensitive to the precision level setting in solver. You might want to choose a high precision level. To do that, go to solver --- options and choose a very small number in the "precision" field.

$$E(R_{act} \mid view) = \sum_{i=1}^{6} w_{act,i} E(R_i \mid view),\ std.dev = \sqrt{w_{act} \Omega w_{act}^{'}}\ \ and\ \ SR = \frac{E(R_{act} \mid view) - Rf}{std.dev}.$$

Question 9: *What is the alpha with respect to the CAPM of the ACTIVE portfolio from the perspective of the consensus view? What is the alpha (given the expected return on the market given your view)? What is the beta?*

The CAPM alpha is defined as:

$$\alpha = ER_{act} - Rf - \beta_{act}(ER_{mkt} - Rf),$$
$$\alpha \mid view = E(R_{act} \mid view) - Rf - \beta_{act} E[(R_{mkt} \mid view) - Rf].$$

All the expected returns can be computed as usual. To determine the CAPM beta, make use of its definition:

$$\beta_{act} = \frac{\operatorname{cov}(R_{act}, R_{mkt})}{\operatorname{var}(R_{mkt})} = \frac{w_{act} \Omega w_{mkt}^{'}}{w_{mkt} \Omega w_{mkt}^{'}}.$$

Under the consensus view, CAPM holds with respect to the current market portfolio. Therefore all asset should have zero alpha including the ACTIVE portfolio. However, the current marker portfolio is no longer the “market portfolio” under the oil view (as seen in Q7, you would rather hold a different optimal risky portfolio), therefore CAPM does not hold with the current market portfolio and the ACTIVE portfolio has a huge positive alpha of more than 20%.

Question 10: *Suppose you can not short sell any asset. What would be the new optimal risky portfolio? Again show that this can be decomposed into the current market portfolio plus an active portfolio. What is the expected return, standard deviation, Sharpe Ratio of this active portfolio? What is the alpha? What is the beta? What is the composition of the investor's portfolio? What is the certainty equivalent return? By how much has the certainty equivalent of the investor's portfolio come down by the no short sale constraint given the view?*

This question is similar to Q7 with additional constraints that all weights must be greater or equal to 0. Using solver, we obtain the weights of the constrained optimal risky portfolio which invests only in two assets. Alpha, beta and Sharpe ratio of the ACTIVE portfolio in this case can be computed in a similar fashion as in Q8 and Q9.

Number of securities: 6

Fill in Names

No	Name	Fraction 0	Expected Return	Standard Deviation
1	port 1	0.614	0.098	0.306
2	port 2	0.000	0.066	0.208
3	port 3	0.000	0.058	0.210
4	port 4	0.386	0.071	0.197
5	port 5	0.000	0.056	0.174
6	port 6	0.000	0.042	0.169
		1		

0.101634 0.11231881 0.097063 0.093092

Correlations

	1 port 1	2 port 2	3 port 3	4 port 4	5 port 5	6 port 6
port 1	1	0.89	0.80	0.86	0.71	0.68
port 2		1.00	0.97	0.73	0.87	0.82
port 3			1.00	0.65	0.85	0.81
port 4				1.00	0.76	0.74
port 5					1.00	0.94
port 6						1.00

Corr OK? YES

Results:

Portfolio's Expected Return	0.0876
Portfolio's Standard Deviation	0.2564

Expected return	0.05094
Std dev	0.093271
Certainty Equivalent return	0.04047

Risk Free Rate	0.0300	**Risk Aversion Coefficient: A=**	2.407
Slope of CAL	0.22	**Weight on optimal risky portfolio: x*=**	0.36

The no-short-sale constraint reduces the certainty equivalent return from 5.14% in Q7 to 4.05%. However, it is still slightly higher than that in Q6 where investor is restricted to only the current market portfolio and the risk free asset.

Table 1: Factor Loadings and T-values for the six portfolios

	Factor Loadings			
port	**mkt**	**term spread**	**credit spread**	**oil**
1	1.439	5.487	0.279	0.043
2	0.872	3.419	-0.018	-0.019
3	0.786	3.925	1.250	-0.036
4	1.022	-0.602	2.617	-0.010
5	0.749	-0.221	-2.134	-0.062
6	0.672	-0.795	-1.202	-0.113

	T-values			
port	**mkt**	**term spread**	**credit spread**	**oil**
1	13.122	3.004	0.028	0.634
2	8.469	1.993	-0.002	-0.298
3	6.472	1.941	0.113	-0.480
4	30.962	-1.096	0.873	-0.469
5	8.614	-0.152	-0.270	-1.153
6	7.639	-0.542	-0.150	-2.070

Table 2: Optimal weights, expected returns, standard deviations, certainty equivalent returns and Sharpe ratios under various cases

	hold market under consensus view (Q3)	**hold market + rf under the oil view (Q6)**	**hold market only under the oil view (Q6)**	**hold optimal risky port + rf under the oil view (Q7)**	**hold optimal risky port (with no short sale constraint + rf under the oil view (Q10)**
weight on risky port	100.00%	45.37%	100.00%	13.01%	36.37%
weight on riskfree	0.00%	54.63%	0.00%	86.99%	63.63%
Expected return	10.75%	4.59%	6.52%	7.29%	5.09%
std-dev	17.94%	8.14%	17.94%	13.34%	9.33%
certainty equ ret	6.88%	3.80%	2.64%	5.14%	4.05%
Sharpe ratio	0.43	0.20	0.20	0.32	0.22