Gaussian Elimination for solving:

$$\begin{aligned} E_1 \quad & a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n = a_{1,n+1} \\ E_2 \quad & a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n = a_{2,n+1} \\ & \vdots \\ E_n \quad & a_{n1}x_1 + a_{n2}x_2 + \cdots + a_{nn}x_n = a_{n,n+1} \end{aligned}$$

INPUT: number of equations n; augmented matrix $A = [a_{ij}]$. Here $1 \le i \le N;\ 1 \le j \le N+1$

OUTPUT: solution $x_1,\ x_2,\ \ldots,\ x_n$

STEP 1 For $i = 1, \ldots, n-1$ do **STEP**s 2 - 4

STEP 2 Let p be the smallest integer with $i \le p \le n$ and $a_{pi} \neq 0$.

If no integer p can be found

Then OUTPUT('no solution exists');

STOP.

STEP 3 If $p \neq i$ then perform $(E_p) \Leftrightarrow (E_i)$

STEP 4 For $j = i+1, \ldots, n$ do **STEP**s 5 - 6

STEP 5 Set $m_{ji} = a_{ji}/a_{ii}$

STEP 6 Perform $(E_j - m_{ji}E_i) \Leftrightarrow (E_j)$

STEP 7 If $a_{nn} = 0$, then OUTPUT('no unique solution exists');

STOP.

STEP 8 Set $x_n = a_{n,n+1}/a_{nn}$ // *Start backward substitution*

STEP 9 For $i = n-1, \ldots, 1$ set $x_i = (a_{i,n+1} - \sum_{j=i+1}^{n} a_{ij}x_j)/a_{ii}$

STEP 10 OUTPUT($x_1,\ x_2, \ldots,\ x_n$);

STOP.