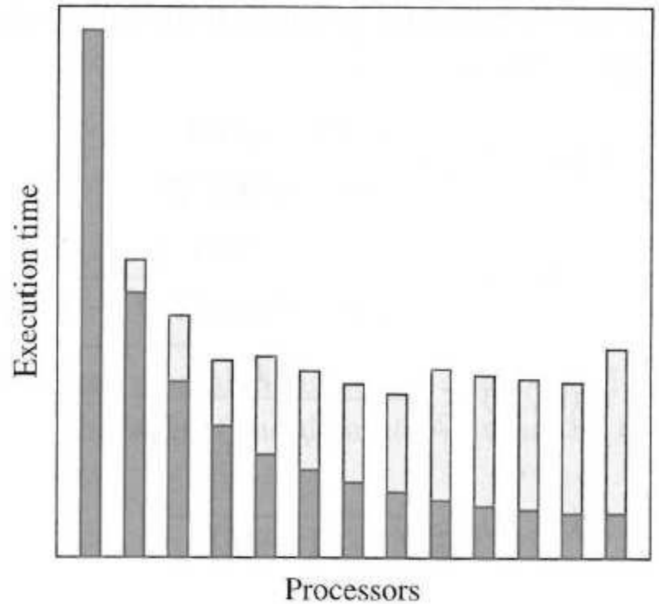
Lecture 5: Performance Analysis (part 1)

Typical Time Measurements



Dark grey: time spent on computation, decreasing with # of processors

White: time spent on communication, increasing with # of processors

Operations in a parallel program:

1. Computation that must be performed sequentially
2. Computations that van be performed in parallel
3. Parallel overhead including communication and redundant computations

Basic Units

- n problem size
- p number of processors
- $\sigma(n)$ inherently sequential portion of computation
- $\varphi(n)$ portion of parallelizable computation
- $\kappa(n,p)$ parallelization overhead
- Speedup $\Psi(n,p) = \frac{sequential\ execution\ time}{parallel\ execution\ time}$
- Efficiency

$$\varepsilon(n,p) = \frac{sequential\ execution\ time}{processors\ used \times parallel\ execution\ time}$$

Amdahl's Law (1)

- Sequential execution time = $\sigma(n) + \varphi(n)$

Assume that the parallel portion of the computation that can be executed in parallel divides up perfectly among p processors

- Parallel execution time $\geq \sigma(n) + \frac{\varphi(n)}{p} + \kappa(n,p)$

Speedup $\Psi(n,p) \leq \frac{\sigma(n)+\varphi(n)}{\sigma(n)+\frac{\varphi(n)}{p}+\kappa(n,p)}$

Efficiency $\varepsilon(n,p) \leq \frac{\sigma(n)+\varphi(n)}{p\sigma(n)+\varphi(n)+p\kappa(n,p)}$

Amdahl's Law (2)

- If the parallel overhead $\kappa(n, p)$ is neglected, then

$$\text{Speedup} \quad \Psi(n, p) \leq \frac{\sigma(n)+\varphi(n)}{\sigma(n)+\frac{\varphi(n)}{p}}$$

Let f be the inherently sequential portion of the computation,

$$f = \frac{\sigma(n)}{\sigma(n)+\varphi(n)}$$

Amdahl's Law (3)

$$\Psi(n,p) \le \frac{\sigma(n)+\varphi(n)}{\sigma(n)+\frac{\varphi(n)}{p}}$$

$$\Psi(n,p) \le \frac{\sigma(n)/f}{\sigma(n)+\sigma(n)(\frac{1}{f}-1)/p}$$

$$\Psi(n,p) \le \frac{1/f}{1+\ (\frac{1}{f}-1)/p}$$

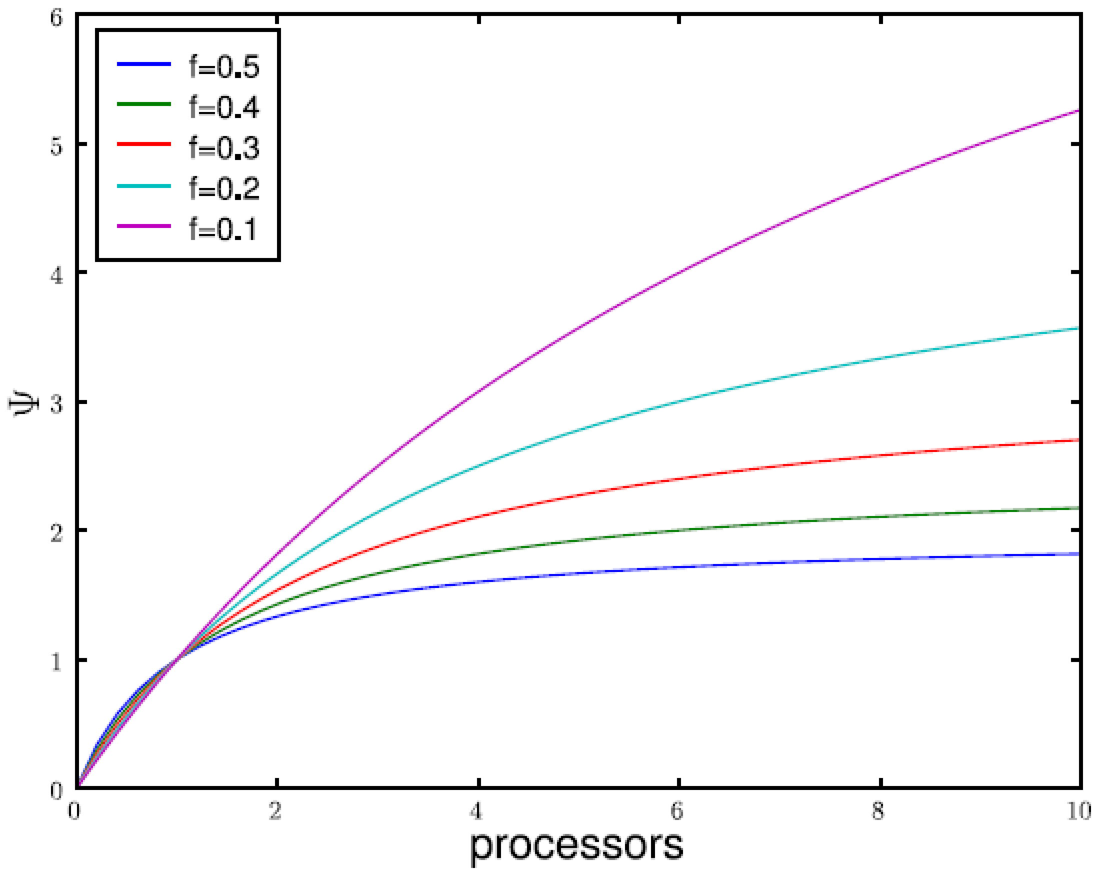
$$\Psi(n,p) \le \frac{1}{f+\ (1-f)/p}$$

Amdahl's Law: Let f be the fraction of operations in a computation that must be performed sequentially, where $0 \le f \le 1$. The maximum speedup $\Psi(n,p)$ achieved by a parallel computer with p processors performing the computation is $\Psi(n,p) \le \frac{1}{f+(1-f)/p}$

Upper limit: as $p \to\ \infty, \Psi(n,p) \le \frac{1}{f+\frac{1-f}{p}} < \frac{1}{f}$

Speedup vs. f

Amdahl's law assumes that the problem size is fixed. It provides an upper bound on the speedup achievable by applying a certain number of processors.



Example 1

If 90% of the computation can be parallelized, what is the max. speedup achievable using 8 processors?

Solution:

$$f = 10\%,$$

$$\Psi(n,p) \leq \frac{1}{0.1+\frac{1-0.1}{8}} \approx 4.7$$

Example 2

Suppose $\sigma(n) = 18000 + n$

$$\varphi(n) = \frac{n^2}{100}$$

What is the max. speedup achievable on a problem of size $n = 10000$?

Solution: $\Psi(n,p) \leq \frac{\sigma(n)+\varphi(n)}{\sigma(n)+\frac{\varphi(n)}{p}} \leq \frac{28000+1000000}{28000+1000000/p}$

Remark

- Parallelization overhead $\kappa(n, p)$ is ignored by Amdahl's law
  - Optimistic estimate of speedup
- The problem size n is constant for various p values
  - Amdahl's law does not consider solving larger problems with more processors
- **Amdahl effect**
  - Typically $\kappa(n, p)$ has lower complexity than $\varphi(n)$. For a fixed number of processors, speedup is usually an increasing function of the problem size.
- The inherently sequential portion f may decrease when n increases
  - Amdahl's law ($\Psi(n, p) < \frac{1}{f}$) can under estimate speedup for large problems

Gustafson-Barsis's Law

- Amdahl's law assumes that the problem size is fixed and show how increasing processors can reduce time.
- Let the problem size increase with the number of processors.
- Let s be the fraction of time spent by a parallel computation using p processors on performing inherently sequential operations.

$$s = \frac{\sigma(n)}{\sigma(n) + \frac{\varphi(n)}{p}}$$

so $1 - s = \frac{\varphi(n)/p}{\sigma(n)+\frac{\varphi(n)}{p}}$

$$\sigma(n) = \left(\sigma(n) + \frac{\varphi(n)}{p}\right) s$$

$$\varphi(n) = \left(\sigma(n) + \frac{\varphi(n)}{p}\right)(1 - s)p$$

$$\begin{aligned}
\Psi(n,p) &\leq \frac{\sigma(n) + \varphi(n)}{\sigma(n) + \frac{\varphi(n)}{p}} \\
&= \frac{(s + (1-s)p)(\sigma(n) + \frac{\varphi(n)}{p})}{\sigma(n) + \frac{\varphi(n)}{p}} \\
&= s + (1 - s)p \\
&= p + (1 - p)s
\end{aligned}$$

Gustafson-Barsis's law: Given a parallel program of size n using p processors, let s be the fraction of total execution time spent in serial code. The maximum speedup $\Psi(n,p)$ achieved by the program is

$$\Psi(n,p) \leq p + (1 - p)s$$

Remark

- Gustafson-Barsis's law allows to solve larger problems using more processors. The speedup is called scaled speedup.
- Since parallelization overhead $\kappa(n, p)$ is ignored, Gustafson-Barsis's law may over estimate the speedup.
- Since $\Psi(n, p) \leq p + (1 - p)s = p - (p - 1)s$, the best achievable speedup is $\Psi(n, p) \leq p$.
- If $s = 1$, then there is no speedup.

Example

An application executing on 64 processors using 5% of the total time on non-parallelizable computations. What is the scaled speedup?

Solution: $s = 0.05,$

$\Psi(n,p) \leq p + (1-p)s = 64 + (1-64)0.05 = 60.85$

Karp-Flatt Metric

- Both Amdahl's law and Gustafson-Barsis's law ignore the parallelization overhead $\kappa(n,p)$, they overestimate the achievable speedup.

Recall:

- Parallel execution time $T(n,p) = \sigma(n) + \frac{\varphi(n)}{p} + \kappa(n,p)$
- Sequential execution time $T(n,1) = \sigma(n) + \varphi(n)$

- Define **experimentally determined serial fraction** e of parallel computation:

$$\mathrm{e}(n,p) = \frac{\sigma(n)+\kappa(n,p)}{\sigma(n)+\varphi(n)}$$

- **experimentally determined serial fraction** e may either stay constant with respect to p (meaning that the parallelization overhead is negligible) or increase with respect to p (meaning that parallelization overhead dominates the speedup)

- Given $\Psi(n, p)$ using p processors, how to determine $\mathrm{e}(n, p)$?

Since $T(n,p) = T(n,1)e + \frac{T(n,1)(1-e)}{p}$ and $\Psi(n,p) = \frac{T(n,1)}{T(n,p)}$

$$\Psi(n,p) = \frac{T(n,1)}{T(n,1)e + \frac{T(n,1)(1-e)}{p}} = \frac{1}{e + \frac{1-e}{p}}$$

Therefore, $\frac{1}{\Psi} = e + \frac{1-e}{p} \rightarrow e = \frac{\frac{1}{\Psi} - \frac{1}{p}}{1 - \frac{1}{p}}$

Example 1

Benchmarking a parallel program on 1, 2, …, 8 processors produces the following speedup results:

p	2	3	4	5	6	7	8
$\Psi(n, p)$	1.82	2.50	3.08	3.57	4.00	4.38	4.71

What is the primary reason for the parallel program achieving a speedup of only 4.71 on 8 processors?

Solution: Compute $\mathrm{e}(n, p)$ corresponding to each data point:

p	2	3	4	5	6	7	8
$\Psi(n, p)$	1.82	2.50	3.08	3.57	4.00	4.38	4.71
$\mathrm{e}(n, p)$	0.1	0.1	0.1	0.1	0.1	0.1	0.1

Since the experimentally determined serial fraction $\mathrm{e}(n, p)$ is not increasing with p, the primary reason for the poor speedup is the 10% of the computation that is inherently sequential. Parallel overhead is not the reason for the poor speedup.

Example 2

Benchmarking a parallel program on 1, 2, …, 8 processors produces the following speedup results:

p	2	3	4	5	6	7	8
$\Psi(n,p)$	1.87	2.61	3.23	3.73	4.14	4.46	4.71

What is the primary reason for the parallel program achieving a speedup of 4.71 on 8 processors?

Solution:

p	2	3	4	5	6	7	8
$\Psi(n,p)$	1.87	2.61	3.23	3.73	4.14	4.46	4.71
e	0.07	0.075	0.08	0.085	0.09	0.095	0.1

Since the experimentally determined serial fraction e is steadily increasing with p, parallel overhead also contributes to the poor speedup.