Project 4, due on 04/28.

Problem 1 for undergraduate students. Implementing algorithm for solving the following least square problem using QR factorization.

Fit $f(x) = \frac{4x}{1+10x^2}$ with a polynomial of degree 4. $x \in [0,1]$.

Step 1: Let the polynomial be $p(x) = a_0 + a_1 x + a_2 x^2 + a_3 x^3 + a_4 x^4$. Randomly generate 100 points $x_0, \dots, x_{99}$ between [0, 1]. The least square equation is

$$\begin{gathered} a_0 + a_1 x_0 + a_2 {x_0}^2 + a_3 {x_0}^3 + a_4 {x_0}^4 = f(x_0) \\ \vdots \\ a_0 + a_1 x_{99} + a_2 {x_{99}}^2 + a_3 {x_{99}}^3 + a_4 {x_{99}}^4 = f(x_{99}) \end{gathered}$$

Denote it by $A\boldsymbol{x} = \boldsymbol{b}$.

Step 2: Use householder transformation approach to do QR factorization on $A\boldsymbol{x} = \boldsymbol{b}$. Namely,

$$A\boldsymbol{x} = \boldsymbol{b}$$

$$\Rightarrow QR\boldsymbol{x} = \boldsymbol{b}$$

$$\Rightarrow Q^T QR\boldsymbol{x} = Q^T\boldsymbol{b}$$

$$\Rightarrow R\boldsymbol{x} = Q^T\boldsymbol{b}$$

Step 3: Solve the above equation by backward substitution to obtain a_0, a_1, a_2, a_3 and a_4.

Hand-In.

1. The hardcopy of your source code (Also send the source code to me by email. Please use the email title: acms40212S14-Proj4-your-ND-ID.).

2. A report which contains validation of results by plotting graphs of $f(x)$ and $p(x)$ respectively, and a description of your algorithm using the pseudo code language.

Bonus. Implement the parallel algorithm by using the column decomposition code as the base code.

Problem 2 for graduate students. Implementing Arnoldi algorithm.

Step 1: Generate a 100×100 strictly diagonally dominant matrix A.

Step 2: Implement your stable Arnoldi algorithm for constructing orthnormal bases of $K_{20}(A, \boldsymbol{r})$. You can simply use unit vector $\boldsymbol{e}_1$ as $\boldsymbol{r}$.

Step3: Validation. Since now you have the orthonormal basis V_{20} of the Krylov matrix K_{20} and upper Hessenberg matrix H_{20} of A, Compute L1 norms of columns of AV_{20} and $V_{20}H_{20}$ respectively. Check if they agree.

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3. A report which contains validation of results and a description of your algorithm using the pseudo code language

Remark: for parallel implementation of GMRES method, see

(a) H.F. Walker. Implementation of the GMRES method using Householder transformations, SIAM Journal on Scientific and Statistical Computing, 1988

(b) J. Erhel. A parallel GMRES version for general sparse matrices, Electronic Transactions on Numerical Analysis, 1995

Bonus. Implement the parallel algorithm by using the column decomposition code as the base code.