

Answer Key 1

Math 20-550: Calculus

Name:_____

Exam III November 28, 2006

Class time (MWF):_____

Please sign the honor statement if you agree:

"I strictly followed the Notre Dame Honor Code during this test."

Your Signature _____

number right times 6 = _____

11.

12.

13.

You start with: 10 points

Total Score _____

1. • b c d e

6. • b c d e

2. • b c d e

7. • b c d e

3. • b c d e

8. • b c d e

4. • b c d e

9. • b c d e

5. • b c d e

10. • b c d e

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1. Use a triple integral to find the volume of tetrahedron enclosed by coordinate planes and the planes $x + y + 2z = 4$

(a) $\frac{16}{3}$

(b) $\frac{8}{3}$

(c) $\frac{1}{2}$

(d) $\frac{4}{3}$

(e) $\frac{2}{3}$

2. Use cylindrical coordinates to evaluate

$$\int_{-1}^1 \int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} \int_{\sqrt{x^2+y^2}}^1 (x^2 + y^2) dz dy dx.$$

(a) $\frac{\pi}{10}$

(b) 2π

(c) $\frac{\pi}{4}$

(d) $\frac{\pi}{2}$

(e) π

3. Find the work $\int_C \mathbf{F} \cdot d\mathbf{r}$ done by the force $\mathbf{F}(x, y) = \langle 3x^2, -3xy \rangle$ in moving a particle along the quarter-circle $C = \{(x, y) \mid x^2 + y^2 = 1, x \geq 0, y \geq 0\}$ from the starting point $(1, 0)$ to the ending point $(0, 1)$.

- (a) -2 (b) $-\frac{2}{3}$ (c) -4 (d) 3 (e) 6

4. Let $\mathbf{F} = \langle 100, x + yz, xy - e^z \rangle$. Find $\text{curl } \mathbf{F}$.

- (a) $\langle x - y, -y, 1 \rangle$ (b) $\langle x + y, -y, 1 \rangle$ (c) $\langle x - y, y, 1 \rangle$
(d) $\langle x - y, -y, 0 \rangle$ (e) $\langle y - x, -y, 1 \rangle$

5. Evaluate $\int_C (1 + \frac{5}{64}x^4y)ds$, where C is the upper half of the circle $x^2 + y^2 = 4$ of radius 2.

- (a) $2\pi + 2$ (b) $\pi + 1$ (c) $2\pi + \frac{5}{2}$ (d) $\pi + \frac{5}{64}$ (e) $2\pi + \frac{1}{4}$

6. Evaluate $\int_C \mathbf{F} \cdot d\mathbf{r}$, where $\mathbf{F} = \langle yz, xz, xy \rangle$ and $\mathbf{r}(t) = \langle t, t^2, t^5 \rangle$, $0 \leq t \leq 1$.

- (a) 1 (b) 3 (c) $\frac{1}{3}$ (d) $\frac{1}{4}$ (e) 0

7. Use the transformation $x = 2u$ and $y = 3v$ to find the area of the domain enclosed by ellipse $\frac{x^2}{4} + \frac{y^2}{9} = 1$.

- (a) 6π (b) 36π (c) 4π (d) 9π (e) 6.5π

8. Use Green's Theorem to evaluate $\oint_C [(y^2 + 100x)dx + (5xy + y^{99})dy]$, where C is the boundary of the region D in the upper half-plane between circle $x^2+y^2 = 1$ and $x^2+y^2 = 4$.

- (a) 14 (b) 6 (c) 5 (d) 100 (e) 0

9. Let $\mathbf{r}(u, v) = \langle u, u^2 + v^2, v \rangle$ with $0 \leq u \leq 1, 0 \leq v \leq 1$ be a parametrization of the surface $S = \{(x, y, z) \mid y = x^2 + z^2, 0 \leq x \leq 1, 0 \leq z \leq 1\}$. Suppose that $\mathbf{F} = \langle 0, 0, -z \rangle$ and $d\mathbf{S} = (\mathbf{r}_u \times \mathbf{r}_v)dudv$. Find the flux integral $\int \int_S \mathbf{F} \cdot d\mathbf{S}$.

- (a) $-\frac{2}{3}$ (b) -2 (c) 0 (d) 2 (e) $\frac{2}{3}$

10. Let $S = \{(x, y, z) \mid x^2 + y^2 = 1, 0 \leq z \leq 1\}$ be a cylinder of height 1. Find $\int \int_S zdS$.

- (a) π (b) 2π (c) $\frac{\pi}{2}$ (d) $-\pi$ (e) 0

11. Find the center of mass of a solid of the density $\rho = 1$ that is bounded by parabolic cylinder $y = x^2$ and the planes $y = z$, $z = 0$ and $y = 1$. Please set up in terms of **iterated integrals** but do NOT solve.

12. Let $\mathbf{F} = \langle y^2 + 1, 2xy + e^{3z} - 1, 3ye^{3z} + z^2 \rangle$. Find a function f such that $\nabla f = \mathbf{F}$ if such a function exists.

13. Use spherical coordinates to find the volume of the ice-cream cone that lies above the cone $z = \sqrt{x^2 + y^2}$ and lies inside the ball $x^2 + y^2 + (z - 1)^2 \leq 1$.