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Control of Multiple Networked Passive Plants With Delays and Data Dropouts

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***Abstract*— This paper provides a framework to synthesize l_2^m-*stable* and L_2^m-*stable* control networks in which m *strictly-output passive* controllers can control $n-m$ *strictly-output passive* plants. The communication between the plants and controllers can tolerate time varying delay and data dropouts. In particular, we introduce a *power junction* which allows even a single controller (typically designed to control a single plant) to accurately control the position of multiple plants even if the dynamics of the plants are different. An illustrative simulated example shows the position tracking performance of the system. We conclude the discussion with two questions for future research.**

I. INTRODUCTION

The primary goal of this research is to develop reliable wireless control networks [1], [2]. These networks typically consist of distributed-wireless sensors, actuators and controllers which communicate with low cost devices such as the MICA2 and MICAz motes [3]. Specifically we have shown how to create a l_2^m-*stable* control network for a continuous *passive* plant [4, Theorem 4]. In particular the control and sensor data is transmitted over the network using *wave variables*. The use of *wave variables* allows the network [4, Fig. 2] to remain l_2^m-*stable* when subject to fixed time delays and data dropouts [4, Lemma 2]. Furthermore if the data is appropriately handled, then the network will remain l_2^m-*stable* in spite of time varying delays [4, Lemma 3]. One apparent limitation with the use of *wave variables* in *passive* control theory is that *passive* plants and controllers are connected in a series configuration in order to preserve a *passive* mapping (i.e. [4, Fig. 2]). However, in this paper we show how to use a *power junction* 1 in order to allow m controllers to control up to $n-m$ plants. We prove that such a network can be shown to be l_2^m/L_2^m-*stable* if all the interconnected plants and controllers are *strictly-output passive*. In particular simulated results are provided which show that either 1 or 2 controller(s) can accurately control the position of 3 perturbed plants in spite of time varying delays. We have also simulated and found similar results for other values of m and n. Although, we were not surprised to find the network to be stable, we were pleasantly surprised to discover how well a single controller could control the angular position of multiple motors in-spite of differences between the plants controlled.

Besides using the *power junction*, there do exist other ways to interconnect wave variables for *LTI* systems as is done with the design of wave digital filters [5]. The manner in which wave ports are interconnected in order to realize a digital filter is done differently than is done in control implementations. For example in [4, Fig. 2] the waves $u_{op} \in \mathbb{R}^m$ and $u_{oc} \in \mathbb{R}^m$ are each computed in a manner similar to a voltage *incident* wave (a), and the waves $v_{op} \in \mathbb{R}^m$ and $v_{oc} \in \mathbb{R}^m$ are each computed in a manner similar to a voltage *reflective* wave (b) [5]. For wave digital filters a voltage *incident* waves can be thought of as a wave traveling into a two port junction, likewise a *reflective* wave travels out of a two port junction. When interconnecting two port elements for a wave digital filter, a voltage *incident* wave should connect to a voltage *reflective* wave or vice-versa [5, Section IV-A-2)]. If we denote u_{op} and v_{oc} as a *reflective* waves (with outgoing arrows) and denote u_{oc} and v_{op} as *incident* waves (with incoming arrows), then the interconnection rules appear to be in agreement. Clearly, if we can straighten out these differences, then we can discuss various wave interconnections, such as series and parallel adapters, etc. For example the *unit element* can be used to represent identical fixed delays $p=c$, and the *quasi-reciprocal line* (*QUARL*) can represent different fixed delays such that $p \neq c$ [5, Table 2].

The rest of the paper is as follows: in Section II the *power junction* is defined and discussed. In Section III a *power junction* control network is presented and shown to be l_2^m/L_2^m-*stable*. In Section IV a detailed simulation is discussed in which continuous motors are digitally controlled over a wireless ring network. Section V provides our conclusions and summary of results.

II. THE POWER JUNCTION

Networks of a *passive* plant and controller are typically interconnected using *power variables*. *Power variables* are generally denoted with an *effort* and *flow* pair (e_*,f_*) whose product is power. They are typically used to show the exchange of energy between two systems using *bond graphs* [6], [7]. However, when these *power variables* are subject to communication delays the communication channel ceases to be *passive* which leads to network instabilities. Wave variables allow *effort* and *flow* variables to be transmitted over a network while remaining *passive* when subject to arbitrary fixed time delays and data dropouts [8].

$$u_{pk}(x) = \frac{1}{\sqrt{2b}}(bf_{opk}(x) + e_{dock}(x)),\ k \in \{m+1,\dots,n\} \quad (1)$$

$$v_{cj}(x) = \frac{1}{\sqrt{2b}}(bf_{opdj}(x) - e_{ocj}(x)),\ j \in \{1,\dots,m\} \quad (2)$$

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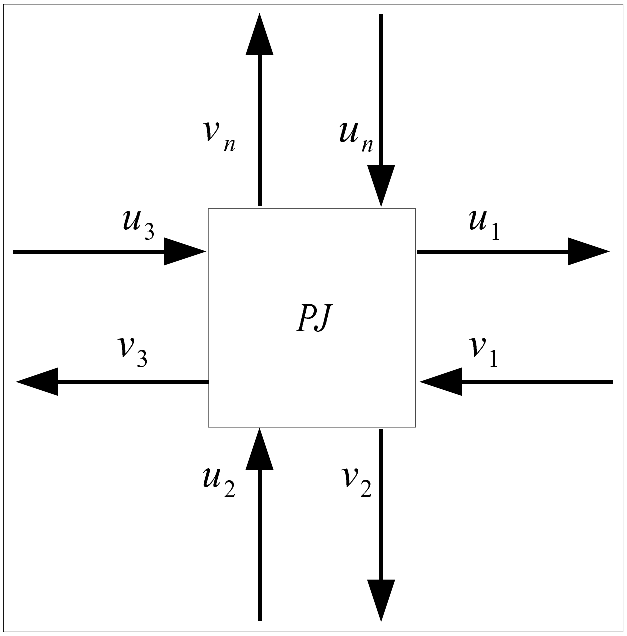

Fig. 1. The *power junction*.

(1) can be thought of as each sensor output in a *wave variable* form for each plant $G_{pk},\ k \in \{m+1, \ldots, n\}$ depicted in Fig. 2. Likewise, (2) can be thought of as each command output in a *wave variable* form for each controller $G_{cj},\ j \in \{1, \ldots, m\}$ depicted in Fig. 2. The symbol x depicts either continuous time t or discrete time k. Denote $I \in \mathbb{R}^{m \times m}$ as the identity matrix. When actually implementing the wave variable transformation the "outputs" (u_{pk}, e_{dock}) are related to the corresponding "inputs" (v_{pk}, f_{opk}) as follows (see [9, Figure 2.2]):

$$\begin{bmatrix} u_{pk}(x) \\ e_{dock}(x) \end{bmatrix} = \begin{bmatrix} -I & \sqrt{2b}I \\ -\sqrt{2b}I & bI \end{bmatrix} \begin{bmatrix} v_{pk}(x) \\ f_{opk}(x) \end{bmatrix} \tag{3}$$

likewise the "outputs" (v_{cj}, f_{opdj}) are related to the corresponding "inputs" (u_{cj}, e_{ocj}) as follows:

$$\begin{bmatrix} v_{cj}(x) \\ f_{opdj}(x) \end{bmatrix} = \begin{bmatrix} I & -\sqrt{\frac{2}{b}}I \\ \sqrt{\frac{2}{b}}I & -\frac{1}{b}I \end{bmatrix} \begin{bmatrix} u_{cj}(x) \\ e_{ocj}(x) \end{bmatrix} \tag{4}$$

The *power junction* indicated in Fig. 1 and Fig. 2 by the symbol **PJ** has waves entering and leaving the junction as indicated by the arrows. In particular the input wave to the plant $v_{pk}(x)$ is a delayed version of the outgoing wave from the *power junction* $v_k,\ k \in \{m+1, \ldots, n\}$ such that

$$v_{pk}(x) = v_k(x - pk(x)),\ k \in \{m+1, \ldots, n\} \tag{5}$$

in which $pk(x)$ is denoted $\tau_{pk}(t)$ for a continuous time varying delay or $pk(i)$ for a discrete time delay. In Fig. 2 the delays are represented as fixed for the discrete time case (i.e. z^{-pk}). Next, the input wave to the controller $u_{cj}(x)$ is a delayed version of the outgoing wave from the *power junction* $v_j,\ j \in \{1, \ldots, m\}$ such that

$$u_{cj}(x) = u_j(x - cj(x)),\ j \in \{1, \ldots, m\} \tag{6}$$

in which $cj(x)$ is denoted $\tau_{cj}(t)$ for a continuous time varying delay or $cj(i)$ for a discrete time delay. In Fig. 2 the delays are represented as fixed for the discrete time case (i.e. z^{-cj}). An analogous statement can be made in regards to relating the waves into the *power junction* ($v_j(x)$, $u_k(x)$) are delayed inputs of the corresponding controller waves $v_{cj}(x)$ and plant waves $u_{pk}(x)$.

$$v_j(x) = v_{cj}(x - pj(x)),\ j \in \{1, \ldots, m\} \tag{7}$$

$$u_k(x) = u_{pk}(x - ck(x)),\ k \in \{m+1, \ldots, n\} \tag{8}$$

From the extensive literature search we have done in this area, we have yet to see wave variables connected as follows:

Definition 1: A "power junction" is implemented as follows (see Fig. 1 for the case when $m = 1$, $n = 4$): n systems with the corresponding wave variable pairs $(u_1, v_1), (u_2, v_2), \ldots, (u_n, v_n)$ are interconnected such that the first $(u_j, v_j),\ j \in \{1, \ldots, m\}$ pairs consist of a corresponding (power output, power input) pair and the remaining $(u_k, v_k),\ k \in \{m+1, \ldots, n\}$ are (power input, power output) pairs. The power junction is *passive* and lossless as long as

$$\sum_{j=1}^{m}(u_j^2 - v_j^2) = \sum_{k=m+1}^{n}(u_k^2 - v_k^2) \tag{9}$$

always holds. The following is sufficient to satisfy (9):

$$\sum_{j=1}^{m} u_j^2 = \sum_{k=m+1}^{n} u_k^2 \tag{10}$$

$$\sum_{j=1}^{m} v_j^2 = \sum_{k=m+1}^{n} v_k^2 \tag{11}$$

Remark 1: Clearly this can be generalized to satisfy the case for vectors. There are numerous ways such a "power junction" can be implemented, for example:

1. Let there be $m = 1$ controllers G_1 with the corresponding wave variables (u_1, v_1) in which v_1 is the control output, and u_1 represents the "weighted" feedback from the remaining $n-1$ plants $G_k, k \in \{2, \ldots, n\}$.
2. Each plant G_k has the corresponding wave variables (u_k, v_k) in which u_k is the corresponding plant sensor output and v_k is the corresponding "distributed" command from the controller to each individual plant (see Fig. 2).
3. A basic "average" power distribution can be implemented as follows:

$$\begin{aligned} v_k &= \mathtt{sgn}(\sum_{j=1}^{m} v_j)\frac{\sqrt{\sum_{j=1}^{m} v_j^2}}{\sqrt{n-m}},\ k\ \in \{m+1, \ldots, n\} \\ &= \frac{v_1}{\sqrt{n-1}} \text{ when } m = 1 \end{aligned} \tag{12}$$

$$\begin{aligned} u_j &= \mathtt{sgn}(\sum_{k=m+1}^{n} u_k)\frac{\sqrt{\sum_{k=2}^{n} u_k^2}}{\sqrt{m}},\ j\ \in \{1, \ldots, m\} \\ &= \mathtt{sgn}(\sum_{k=2}^{n} u_k)\sqrt{\sum_{k=2}^{n} u_k^2} \text{ when } m = 1 \end{aligned} \tag{13}$$

4. The effective output command from the controllers are attenuated by $\sqrt{n-m}$ while the feedback signals from the plants are attenuated by $\sqrt{m}$ as indicated by (12) and (13) respectively. Therefore, the set-point to each controller $r_{cj},\ j \in \{1,\ldots,m\}$ should be pre-multiplied by k_s in which

$$k_s = \sqrt{\frac{n-m}{m}}\ . \tag{14}$$

Remark 2: For simplicity we will consider the case in which $r_{opk} = 0$ and all plants G_{pk} are single-input single-output satisfying:

$$f_{opk}(x) = -k_{pk} e_{dock}(x),\ k_{pk} > 0 \tag{15}$$

from (3) we see that:

$$e_{dock}(x) = -\sqrt{2b} v_{pk}(x) - b k_{pk} e_{dock}(x) \tag{16}$$

therefore,

$$f_{opk}(x) = -k_{pk}(x) e_{dock}(x) = \frac{k_{pk}\sqrt{2b}}{1 + b k_{pk}} v_{pk}(x). \tag{17}$$

If $(b k_{pk} >> 1),\ \forall k\ \in \{m+1, m+2, \ldots, n\}$ then

$$f_{opk}(x) \approx \sqrt{\frac{2}{b}} v_{pk}(x). \tag{18}$$

This implies that as long as each plant processes the average wave commands from the controllers satisfying (12) for example, then as the system reaches a steady state $v_{pk}(x) = 0,\ \forall x > x_S$ and the delays are fixed then the following will approximately hold for some real constant C:

$$\sqrt{\frac{b}{2}} \int_0^{x_s} f_{opk}(t) dt \approx \quad \textstyle\int_0^{x_s} v_{pk}(t) dt = C \tag{19}$$

$$\sqrt{\frac{b}{2}} \sum_{i=0}^{x_s} f_{opk}(i) \approx \quad \textstyle\sum_{i=0}^{x_s} v_{pk}(i) = C. \tag{20}$$

Furthermore this tracking of each system using the *power junction* can be extended for *LTI* systems with out a constant gain as long as the frequency content of $v_{pk}(j\omega)$ is band width limited such that

$$v_{pk}(j\omega) = 0, \forall \omega > \omega_M \tag{21}$$

$$b H_{pk}(j\omega) >> 1, \forall \omega \le \omega_M, \tag{22}$$

or for the discrete time case

$$v_{pk}(e^{j\omega}) = 0, \text{ when } \omega_M < \omega \le \pi \tag{23}$$

$$b H_{pk}(j\omega) >> 1, \text{ when } 0 \le \omega \le \omega_M. \tag{24}$$

Remark 3: The *power junction* provides a nice addition to the initial work done by Niemeyer and Slotine as summarized in [8, Section 6.4]. In which a method is described showing how to augment potential position drift by modifying one of the waves u_m in a *passive* manner [8, Fig. 9].

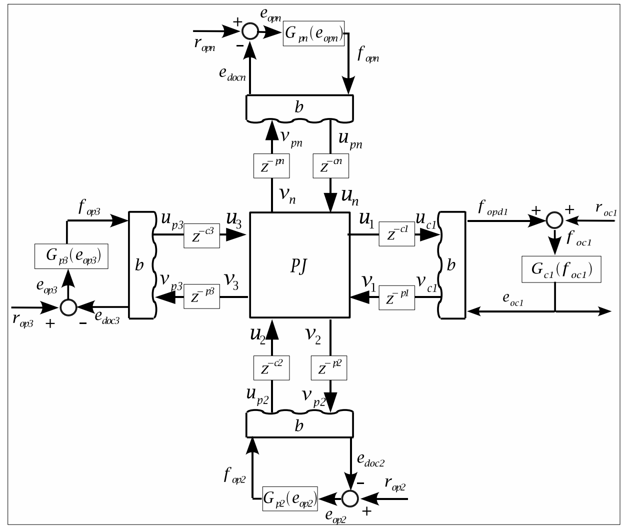

Fig. 2. An example of a *power junction* control network.

III. L_2^m/l_2^m STABLE NETWORKS WITH THE POWER JUNCTION

Fig. 2 depicts m controllers interconnected to $n-m$ plants using a *power junction*. It can be shown that this network will remain l_2^m/L_2^m*-stable* when subject to either fixed delays and/or data dropouts. For the discrete time case we can show how to safely handle time varying delays by dropping duplicate transmissions from the *power junction*. Please refer to Appendix I for corresponding definitions or nomenclature.

Theorem 1: The system depicted in Fig. 2 is l_2^m*-stable* if all plants $G_{pk}(e_{opk}(i)),\ k \in \{m+1,\ldots,n\}$ and all controllers $G_{cj}(f_{ocj}(i)), j \in \{1,\ldots,m\}$ are *strictly-output passive* and

$$\sum_{k=m+1}^{n} \langle f_{opk}, e_{dock} \rangle_N \ge \sum_{j=1}^{m} \langle e_{ocj}, f_{opdj} \rangle_N \tag{25}$$

holds for all $N \ge 1$.

Proof: Each *strictly-output passive* plant for $k \in \{m+1,\ldots,n\}$ satisfies

$$\langle f_{opk}, e_{opk} \rangle_N \ge \epsilon_{opk} \|(f_{opk})_N\|_2^2 - \beta_{opk} \tag{26}$$

while each *strictly-output passive* controller for $j \in \{1,\ldots,m\}$ satisfies (27).

$$\langle e_{ocj}, f_{ocj} \rangle_N \ge \epsilon_{ocj} \|(e_{ocj})_N\|_2^2 - \beta_{ocj} \tag{27}$$

Substituting, $e_{dock} = r_{opk} - e_{opk}$ and $f_{opdj} = f_{ocj} - r_{ocj}$ into (25) yields

$$\sum_{k=m+1}^{n} \langle f_{opk}, r_{opk} - e_{opk} \rangle_N \ge \sum_{j=1}^{m} \langle e_{ocj}, f_{ocj} - r_{ocj} \rangle_N$$

which can be rewritten as

$$\sum_{k=m+1}^{n}\langle f_{opk}, r_{opk}\rangle_N + \sum_{j=1}^{m}\langle e_{ocj}, r_{ocj}\rangle_N \geq \sum_{k=m+1}^{n}\langle f_{opk}, e_{opk}\rangle_N + \sum_{j=1}^{m}\langle e_{ocj}, f_{ocj}\rangle_N \qquad (28)$$

so that we can then substitute (26) and (27) into (28) to yield

$$\sum_{k=m+1}^{n}\langle f_{opk}, r_{opk}\rangle_N + \sum_{j=1}^{m}\langle e_{ocj}, r_{ocj}\rangle_N \geq \epsilon[\sum_{k=m+1}^{n}\|(f_{opk})_N\|_2^2 + \sum_{j=1}^{m}\|(e_{ocj})_N\|_2^2] - \beta \qquad (29)$$

in which $\epsilon = \min(\epsilon_{opk}, \epsilon_{ocj}),\ k \in \{m+1,\ldots,n\}\ j \in \{1,\ldots,m\}$ and $\beta = \sum_{k=m+1}^{n}\beta_{opk} + \sum_{j=1}^{m}\beta_{ocj}$. Thus (29) satisfies [4, (12) in Definition 3.II.] for *strictly-output passivity* in which the input is the row vector of all controller and plant inputs $[r_{oc1},\ldots,r_{ocm},r_{op(m+1)},\ldots,r_{opn}]$, and the output is the row vector of all controller and plant outputs $[e_{oc1},\ldots,e_{ocm},f_{op(m+1)},\ldots,f_{opn}]$. ■

Theorem 2: The system depicted in Fig. 2 is L_2^m*-stable* if all plants $G_{pk}(e_{opk}(t)),\ k \in \{m+1,\ldots,n\}$ and all controllers $G_{cj}(f_{ocj}(t)), j \in \{1,\ldots,m\}$ are *strictly-output passive* and

$$\sum_{k=m+1}^{n}\langle f_{opk}, e_{dock}\rangle_\tau \geq \sum_{j=1}^{m}\langle e_{ocj}, f_{opdj}\rangle_\tau \qquad (30)$$

holds for all $\tau \geq 0$.

Proof: The proof is completely analogous to the proof given for Theorem 1, the differences being that the discrete time delays are replaced with continuous time delays in Fig. 2. ■

Remark 4: When we let $\epsilon_{opk} = \epsilon_{ocj} = 0$ we see that all the plants and controllers are *passive*, therefore the system depicted in Fig. 2 is *passive* if it satisfies either (25) for the discrete time case or (30) for the continuous time case.

With these proofs complete, it is a fairly simple exercise to use Definition 1 and use the techniques shown in the proof for [4, Lemma 2] in order to prove the following:

Corollary 1: If all of the discrete time varying delays in the network depicted in Fig. 2 are fixed $pl(i) = pl, cl(i) = cl,\ l \in \{1,\ldots,n\}$ and/or data packets are dropped then (25) holds.

Corollary 2: The discrete time varying delays $pl(i), cl(i),\ l \in \{1,\ldots,n\}$ depicted in Fig. 2 can vary arbitrarily as long as (25) holds. The main assumption (25) will hold if duplicate transmissions to the *power junction* are dropped when received, and duplicate transmissions from the *power junction* to the receivers are dropped. This can be accomplished for example by transmitting the tuple $(i, u_{pk}(i))$ to the *power junction*, if $i \in \{$ the set of received indexes $\}$ then set $u_{pk}(i) = 0$ before computing $u_j(i)$ to transmit to the controllers, etc.

IV. SIMULATION

Using a *power junction*, we shall control $n-m$ motors using an ideal current source for each motor, which will allow us to neglect the effects of the motor inductance and resistance for simplicity. The fact that the current source is non-ideal, leads to a non-*passive* relationship between the desired motor current and motor velocity [10]. There are ways to address this problem using *passive* control techniques by controlling the motors velocity indirectly with a switched voltage source and a minimum phase current feedback technique [11], and more recently incorporating the motors back voltage measurement which provides an exact tracking error dynamics *passive* output feedback controller [12].

Each motor is characterized by its torque constant, $K_m > 0$, back-emf constant K_e, rotor inertia, $J_m > 0$, and damping coefficient $B_m > 0$. The *strictly-output passive* dynamics are described by

$$\dot{\omega} = -\delta\frac{B_m}{J_m}\omega + k_\delta\delta\frac{K_m}{J_m}i \qquad (31)$$

in which $\epsilon = \frac{B_m}{k_\delta K_m}$ as determined by Theorem 3 given in the Appendix. The additional terms $\delta > 0$, and $k_\delta > 0$ are used to perturb the nominal plant for simulation. The sensor output for each motor will be the angular velocity ω, therefore the state matrices are $A = -\delta\frac{B_m}{J_m},\ B = k_\delta\delta\frac{K_m}{J_m},\ C = 1, D = 0$. (32) describes the following controller we propose to use. A velocity error signal is typically sent to the controller described by (32) and the integral of velocity is position, therefore (32) is typically referred to as a *passive* "proportional-derivative" controller [8, Fig. 6].

$$K_{PD}(s) = K\frac{\tau s + 1}{s} \qquad (32)$$

Using loop-shaping techniques we choose $\tau = \frac{J_m}{B_m}$ and choose $K = \frac{J_m\pi}{10K_mT}$. This will provide a reasonable crossover frequency at roughly a tenth the Nyquist frequency and maintain a 90 degree phase margin. We choose to use the same motor parameter values given in [12] in which $K_m = 49.13$ (mV×rad ×sec), $J_m = 7.95\times10^{-3}$ (kg × m^2), and $B_m = 41(\mu$N × sec/meter). With $T = .05$ seconds, we use [13, Corollaries 4,5] to synthesize a *strictly-output passive* plant and controller from our continuous model (32). We also use [13, Corollary 3] in order to compute the appropriate gains for both the controller $K_{s_c} = 1$ and the *strictly-output passive* plant $K_{s_p} = 20$. Note that by arbitrarily choosing $K_{s_c} = \frac{1}{T} = 20$ would have led to an incorrectly scaled system in which the crossover frequency would essentially equal the Nyquist frequency (since a zero is extremely close to -1 in the z-plane). Since the plant is *strictly-output passive* we chose $K_p = 0$. For the controller we chose $K_c = 0.001$ in order to make it *strictly-output passive*. Fig. 8 shows the step response to a desired position set-point $\theta_d(k)$ which generates an approximate velocity reference for $r_{c1}(z) = -H_t(z)\theta_d(z)$. $H_t(z)$ is a zero-order

TABLE I

SIMULATION SUMMARY WHEN PLANTS ARE PERTURBED.

Plant/Controller	Assumptions
$G_{p(m+1)}$	$\delta = 0.2,\ k_\delta = 0.5,\ -p(m+1)(k) = 0,$ $-c(m+1)(k) =$ ring model.
$G_{p(m+2)}$	$\delta = 1.0,\ k_\delta = 1.0,\ -p(m+2)(k) = 0,$ $-c(m+2)(k) =$ ring model.
$G_{p(m+3)}$	$\delta = 1.8,\ k_\delta = 1.5,\ -p(m+3)(k) = 0,$ $-c(m+3)(k) =$ ring model.
G_{cj}	$j \in \{1, \cdots, m\},\ -cj(k) = 0,$ $-pj(k) =$ ring model.

hold equivalent of $H_t(s)$, in which $\omega_{traj} = 2\pi$, $\zeta = .9$, and k_s is the appropriate scaling gain given by (14).

$$H_t(s) = k_s \frac{\omega_{traj}^2 s}{s^2 + 2\zeta\omega_{traj} + \omega_{traj}^2} \tag{33}$$

In our simulation we assume that the clocks for the plants and controllers are synchronized. However, the controllers only had to run when data was present in their receive buffer [9, Section 4.3.2.2].

Next we assume that each of the n plants and controllers (*system*) is considered a station in a wireless ring network as depicted in [9, Figure 4.1]. We use the same CC2420 radio and channel model in which the path loss exponent is $n = 3.3$ and free transmission length $d_o = 8$ meters. Additional assumptions, such as limited buffer size which results in random data dropouts, are discussed in greater detail in [9, Section 4.2] in order to simulate the *ring model* delay as denoted in Table I. We assume for simplicity that each *system* takes a turn sending a corresponding *wave variable* (u_{pk} / v_{cj}) around the network until it reaches the corresponding set of controllers/ plants. Every T seconds all pending controller data $v_j(x),\ j \in \{1, \dots, m\}$ and sensor data $u_{pk}(x),\ k \in \{m+1, \dots, n\}$ which has successfully arrived at the power junction (successfully arrived at their corresponding plants or controllers) is used to compute their corresponding $v_k(x)$ with (12) and $u_j(x)$ with (13). Then these values are sent directly to their corresponding plants/ controllers, hence why Table I indicates that certain delays are set equal to zero for the simulation. A simpler way to implement the power junction would be to have the plants collect their data and compute their corresponding $u_j(x)$ and deliver it to the controllers to be processed when necessary. Likewise the controllers should collect their data and compute their corresponding $v_k(x)$ to be delivered to the plants. In this case all delays in Figure 2 will be time varying, however the network will remain l_2^m*-stable*. In fact, an efficient and distributed way of sending the data around the network is as follows:

1) Let $S_{u1} = u_{m+1}$ and $S_{u1^2} = u_{m+1}^2$ and send the tuple $(S_{u1},\ S_{u1^2})$ from plant G_{m+1} to plant G_{m+2} if $(m+1) \neq n$ or to G_{c1} otherwise.
2) For remaining $n - m - 1$ plants denoted by index $Gm+k$, let $S_{uk} = S_{u(k-1)} + u_{m+k}$ and $S_{uk^2} = S_{u(k-1)^2} + u_{m+k}^2$ and send the tuple $(S_{uk},\ S_{uk^2})$ from plant G_{m+k} to plant G_{m+k+1} if $(m+k) \neq n$ or to G_{c1} otherwise.
3) When $(S_{un},\ S_{un^2})$ arrives to controller G_{c1} compute $u_j(x) = \mathtt{sgn}(S_{un})\sqrt{\frac{S_{un2}}{m}}$, store and process when clock fires and continue to relay $u_j(x)$ to G_{c2} if $m \neq 1$.
4) For remaining $m-1$ controllers store $u_j(x)$ and process when clock fires and continue to relay $u_j(x)$ to $G_{c(j+1)}$ if $m \neq j$.
5) In a similar manner, we can modify the above steps to also compute and relay the control *wave variables* $v_k(x)$ from each $v_j(x)$ of each controller.

V. CONCLUSIONS

We have shown how to interconnect multiple *passive* plants and controllers (*systems*) using a *passive power junction* (Definition 1). We have shown how to implement this *power junction* in a practical manner (Remark 1). If the *systems* are *strictly-output passive* then a l_2^m*-stable* network is created, see Fig. 2, Theorem 1, Corollaries 1,2. Remark 2 states the conditions required for a set of different *LTI passive* plants G_{pk} to track each other when interconnected by a *power junction*. Lastly, we simulated a network in which the plants and controllers communicated with each other over a wireless ring network verifying:

1) A set-point to a controller should be scaled by (14). Fig. 3 shows the system response when not scaling the set-point with (14) whereas Fig. 4 used (14).
2) All the plants tracked each other at steady state, *even when perturbed* (see Fig. 3-8, and Remark 2).
3) All plants, with an appropriately scaled set-point approximately track the desired set-point as long as *wave variable* data is not dropped (see Fig. 4, Fig. 5, Fig. 6, and Remark 2).
4) When more than one plant and one controller are interconnected with a *power junction* and data is dropped, the system no longer tracks the desired set point (compare Fig. 6 to Fig. 7 and note how tracking is maintained for the single plant and controller case in Fig. 8).

It still remains to be shown if non-linear plants will track each other as accurately in spite of the time varying delays and plant perturbations even though we have shown the system to be l_2^m*-stable*. Another, open question is whether or not *wave variables* can be used in conjunction with a sensor isolated from the actuator of a plant. In order to separate the sensor from the actuator knowledge of the power entering the plant is required in order to make the wave variable transform. Finally, we emphasize the fact that our requirement for synchronization is a fairly weak form in which we have shown previously that a *passive* asynchronous transfer unit can be used such that a controller is only run when data is available in the pending receive buffer [9, Section 4.3.2.2]. In order to make any reasonable statement about *passivity* or stability a common synchronized time index is still required [9, Theorem 11].

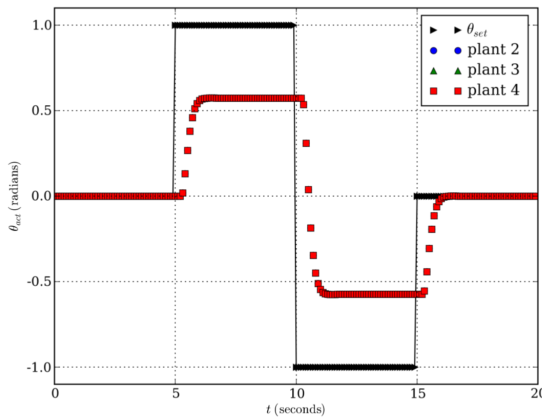

Fig. 3. Step response $m = 1$ controller, $(n-m) = (4-1) = 3$ *identical* plants, $k_s = 1 \neq \sqrt{\frac{n-m}{m}}$, 70 meter spacing between radios.

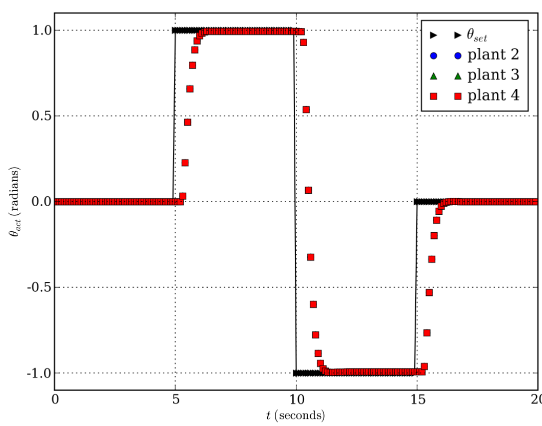

Fig. 4. Step response $m = 1$ controller, $(n-m) = (4-1) = 3$ *identical* plants, $k_s = \sqrt{\frac{4-1}{1}}$, 70 meter spacing between radios.

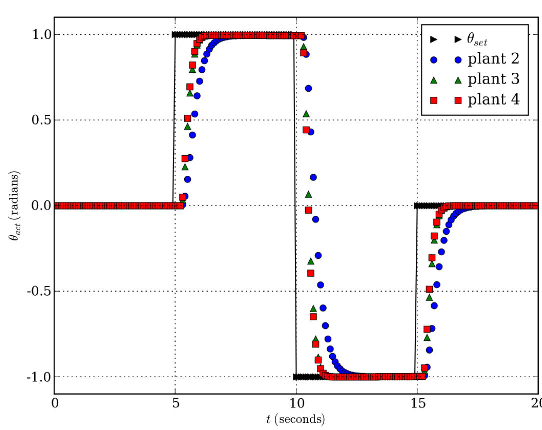

Fig. 5. Step response $m = 1$ controller, $(n-m) = (4-1) = 3$ *perturbed* plants, $k_s = \sqrt{\frac{4-1}{1}}$, 70 meter spacing between radios.

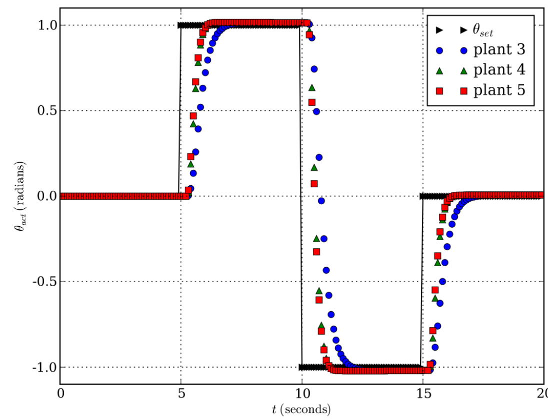

Fig. 6. Step response $m = 2$ controllers, $(n-m) = (5-2) = 3$ *perturbed* plants, $k_s = \sqrt{\frac{5-2}{2}}$, 70 meter spacing between radios.

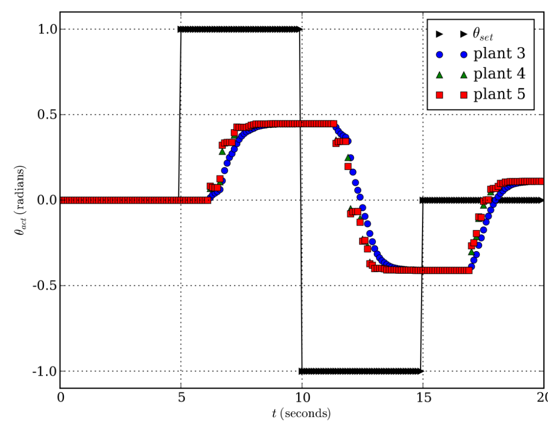

Fig. 7. Step response $m = 2$ controllers, $(n-m) = (5-2) = 3$ *perturbed* plants, $k_s = \sqrt{\frac{5-2}{2}}$, 74 meter spacing (high data dropout rate).

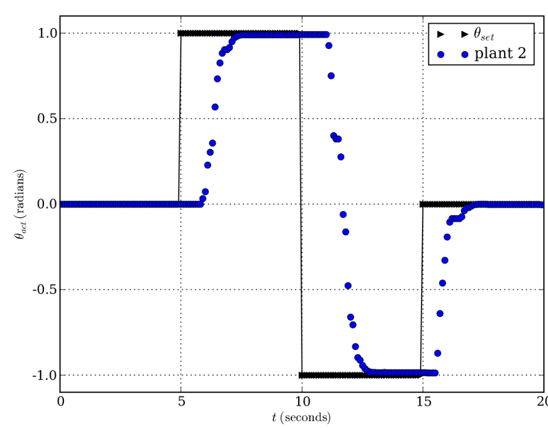

Fig. 8. Step response $m = 1$ controller, $(n-m) = (2-1) = 1$ *perturbed* plant, $k_s = 1$, 76 meter spacing (high data dropout rate).

APPENDIX I
Passive SYSTEMS

The following is a brief summary on *passive* systems. The interested reader is referred to [14]–[16] for additional information. Let $\mathcal{T}$ represent a set indicating time in which $\mathcal{T} = \mathbb{R}^+$ for continuous time signals and $\mathcal{T} = \mathbb{Z}^+$ for discrete time signals. Let $\mathcal{V}$ be a linear space $\mathbb{R}^m$ and denote the space of all functions $u : \mathcal{T} \rightarrow \mathcal{V}$ by the symbol $\mathcal{H}$ which satisfy the following:

$$\|u\|_2^2 = \int_0^\infty u^\mathsf{T}(t)u(t)dt < \infty, \tag{34}$$

for continuous time systems (L_2^m), and

$$\|u\|_2^2 = \sum_0^\infty u^\mathsf{T}(i)u(i) < \infty, \tag{35}$$

for discrete time systems (l_2^m). Similarly we will denote the extended space of functions $u : \mathcal{T} \rightarrow \mathcal{V}$ in $\mathcal{H}_e$ which satisfy the following:

$$\|u_T\|_2^2 = \langle u, u\rangle_T = \int_0^T u^\mathsf{T}(t)u(t)dt < \infty;\ \forall T \in \mathcal{T} \tag{36}$$

for continuous time systems (L_{2e}^m), and

$$\|u_T\|_2^2 = \langle u, u\rangle_T = \sum_0^{T-1} u^\mathsf{T}(i)u(i) < \infty;\ \forall T \in \mathcal{T} \tag{37}$$

for discrete time systems (l_{2e}^m).

Definition 2: A dynamic system $H : \mathcal{H}_e \rightarrow \mathcal{H}_e$ is L_2^m stable if

$$u \in L_2^m \implies Hu \in L_2^m. \tag{38}$$

Definition 3: A dynamic system $H : \mathcal{H}_e \rightarrow \mathcal{H}_e$ is l_2^m stable if

$$x \in l_2^m \implies Hx \in l_2^m. \tag{39}$$

Definition 4: Let $H : \mathcal{H}_e \rightarrow \mathcal{H}_e$. We say that H is

i) *passive* if $\exists\beta$ s.t.

$$\langle Hu, u\rangle_T \geq -\beta,\ \forall u \in \mathcal{H}_e,\ \forall T \in \mathcal{T} \tag{40}$$

ii) *strictly-input passive* if $\exists\delta > 0$ and $\exists\beta$ s.t.

$$\langle Hu, u\rangle_T \geq \delta\|u_T\|_2^2 - \beta,\ \forall u \in \mathcal{H}_e,\ \forall T \in \mathcal{T} \tag{41}$$

iii) *strictly-output passive* if $\exists\epsilon > 0$ and $\exists\beta$ s.t.

$$\langle Hu, u\rangle_T \geq \epsilon\|Hu_T\|_2^2 - \beta,\ \forall u \in \mathcal{H}_e,\ \forall T \in \mathcal{T} \tag{42}$$

iv) *non-expansive* if $\exists\hat{\gamma} > 0$ and $\exists\hat{\beta}$ s.t.

$$\|Hu_T\|_2^2 \leq \hat{\beta} + \hat{\gamma}^2\|u_T\|_2^2,\ \forall u \in \mathcal{H}_e,\ \forall T \in \mathcal{T} \tag{43}$$

Remark 5: A *non-expansive* system H is equivalent to any system which has finite L_2^m (l_2^m) gain in which there exists constants γ and β s.t. $0 < \gamma < \hat{\gamma}$ and satisfy

$$\|Hu_T\|_2 \leq \gamma\|u_T\|_2 + \beta,\ \forall u \in \mathcal{H}_e,\ \forall T \in \mathcal{T}. \tag{44}$$

Furthermore a *non-expansive* system implies L_2^m (l_2^m) stability [15, p.4] ([4, Remark 1]).

Theorem 3: Given a single-input single-output *LTI strictly-output passive* system with transfer function $H(s)$, real impulse response $h(t)$, and corresponding frequency response:

$$H(j\omega) = \mathtt{Re}\{H(j\omega)\} + j\mathtt{Im}\{H(j\omega)\} \tag{45}$$

in which $\mathtt{Re}\{H(j\omega)\} = \mathtt{Re}\{H(-j\omega)\}$ for the real part of the frequency response and $\mathtt{Im}\{H(j\omega)\} = -\mathtt{Im}\{H(-j\omega)\}$ for the imaginary part of the frequency response. The constant ϵ for (42) satisfies:

$$0 < \epsilon \leq \inf_{\omega\in[0,\infty)} \frac{\mathtt{Re}\{H(j\omega)\}}{\mathtt{Re}\{H(j\omega)\}^2 + \mathtt{Im}\{H(j\omega)\}^2}. \tag{46}$$

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