Passivity and Stability of Switched Systems Under Quantization

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ABSTRACT

Passivity theory is a well-established tool for analysis and synthesis of dynamical systems. Recently, this work has been extended to switched and hybrid systems where passivity and stability results of single systems as well as interconnected systems are derived. However, the results may no longer hold when quantization is present as is the case with digital controllers or communication channels. The contribution in this paper is to introduce a control framework under which passivity for switched and non-switched systems can be maintained. This framework centers on the use of an input-output coordinate transformation to recover the passivity property. In order to present these results, background material is provided on passive quantization and output strict passivity for switched and non-switched systems. The proposed framework is first presented for non-switched systems and then generalized to switched systems.

Categories and Subject Descriptors

J.2 [**Physical Sciences and Engineering**]: Engineering

General Terms

Theory

Keywords

Passivity, Switched Systems, Quantization

1. INTRODUCTION

The notion of passivity, which originated in electrical network theory, is a characterization of system input/output behavior based on a generalized notion of energy. Along with Lyapunov function techniques, passivity theory is widely used in analysis and control of nonlinear systems [24, 10, 11, 13]. It is well known that passive systems are stable. Additionally, the parallel interconnection and the negative feedback interconnection of two passive systems is still a passive system. These results provide open-loop conditions to guarantee closed-loop stability. These well known results are summarized along with some recent results in [7]. These results have been extended to switched systems in [14, 15, 16, 17, 22, 29].

Although traditional passivity theory has been applied successfully in various classical nonlinear systems, this property is vulnerable to discretization, quantization and other factors introduced by digital controllers or communication channels in modern control systems. In digital control system design, a continuous-time system is first discretized into a sampled-data system. However, it is pointed out in [21, 3, 23, 19, 5] that passivity is not preserved under discretization, which means the discretized system may not be passive even if the original continuous-time system is passive. Exactly how much passivity is lost under standard discretization has been quantified in [21]. The passivity degradation under the standard discretization can be characterized in terms of passivity indices and sampling time. In [19, 23], a novel average passivity for discrete-time systems was proposed in order to preserve the passivity property losslessly under any sampling time. Besides preserving passivity in discrete-time, stability and stabilization of discrete-time passive systems were also considered in recent work [1, 20]. The problem of finding the maximum sampling time preserving passivity for linear discrete-time systems was considered in [1]. It was shown that the feedback system is exponentially stable if the time-varying asynchronous sampling times embedded in feedback connection are bounded by the maximum sampling time. Two passivity-based control strategies for the problem of stabilizing sampled-data systems were presented in [20].

In addition to discretization, the effect of quantization also needs to be considered when digital controllers interact with the environment by means of analog-to-digital converters or digital-to-analog converters that have a finite resolution. Moreover, quantization is necessary when the information between plants and controllers is transmitted through communication networks. In fact, the problem of control using quantized feedback has been an active research area for a long time. Most of the work [8, 12, 4, 6, 18] concentrates on understanding and mitigating the effects of quantization for feedback stability and stabilization. The existing results on passivity and quantization effects mainly focus on certain specific problems, depending on what kind of systems are considered. In signal processing systems [25], passivity analysis and passification of LTI systems with quantization was treated as an uncertainty described by integral quadratic constraints. In networked control systems, conditions were derived [9] under which the closed-loop networked control system is passive in the presence of sensor quantization and network induced delay. The problem of closed-loop stability for input-affine passive systems with quantized output feedback was investigated in [2]. Recent results [26, 27] used passivity to achieve $\mathcal{L}_2$ stability in the presence of communication delays and signal quantization for networked control systems. To the authors' best knowledge, there is no published results on either preserving passivity under quantization in general or stability conditions for switched systems under quantization.

In this paper, the main contributions are the derivation of conditions under which the passive structure of an output strictly passive (OSP) system can be preserved under quantization and its application in stability for passive switched systems with passive quantizers. The passivity preservation relies on an input/output transformation on the quantized input and output. The result shows that one can find such transformation so that the same passivity index of the original OSP system, with respect to the transformed input and output, will be recovered. The result is relatively general since we only require the system to be OSP and the quantizers to be passive, which characterize many practical quantizers. Although the passivity preserving condition is initially derived for non-switched systems, it can be extended to passive switched systems where the input/output transformation can switch between different transformations according to the current active subsystem. Therefore, passivity of passive switched systems under quantization can be guaranteed and the stability conditions in [15, 16] can be applied.

The rest of the paper is as follows. In Section 2, background material on discrete-time passive systems and passive switched systems is covered. The notion of passive quantizers is introduced. The conditions on preserving passivity under quantization for OSP systems are given in Section 3. Section 4 extends the passivity-preserving conditions for non-switched systems to passive switched systems and then the stability conditions on passive switched systems are obtained. An example is provided in Section 5 to demonstrate the methods used in this paper. Some conclusions are provided in Section 6.

2. BACKGROUND MATERIAL

2.1 Passivity for Discrete-Time Systems

The work in this paper is based on passivity for discrete-time non-switched systems with time index $k \in \mathbb{Z}^+$. A system has input $u(k) \in \mathbb{R}^m$, output $y(k) \in \mathbb{R}^m$, and internal state $x(k) \in \mathbb{R}^n$ and can be modeled as

$$\begin{aligned} x(k+1) &= f(x(k), u(k)) \\ y(k) &= h(x(k), u(k)). \end{aligned} \tag{1}$$

A discrete-time system is passive if it stores and dissipates energy supplied to the system without generating its own energy. The passivity property is typically demonstrated by finding a positive energy storage function and showing that the energy stored in the system at any time step is bounded by the energy supplied to the system.

Definition 1. *A discrete-time system (1) is passive if there exists a positive energy storage function $V(x)$ ($V(x) > 0, \forall x \neq 0$) such that the following inequality holds for all $k \geq k_0$*

$$\Delta V(x(k)) := V(x(k+1)) - V(x(k)) \leq u^T(k)y(k) - \rho y^T(k)y(k) \tag{2}$$

for $\rho \geq 0$. When $\rho > 0$ this system is called output strictly passive.

2.2 Passive Quantizers

Consider a quantizer $q(\cdot)$ with an input v and an output u, where $v \in \mathbb{R}$ and $u \in \mathcal{U}$. $\mathcal{U} \subset \mathbb{R}$ is a quantized set whose elements are distinct quantized levels.

Definition 2. *[26] A quantizer is called a passive quantizer if its input v and output u satisfy*

$$av^2 \leq uv \leq bv^2 \tag{3}$$

where $u = q(v)$ and $0 \leq a \leq b < \infty$.

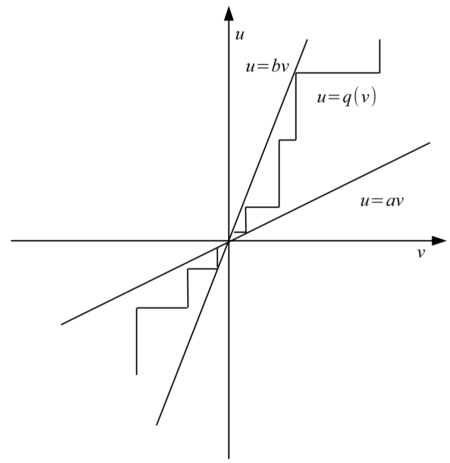

Figure 1: A general quantizer bounded by a cone

The notion of a passive quantizer [26] is based on conic systems theory [28]. A passive quantizer is a special case of a memoryless conic system. This can be seen in Fig. 1, where a quantizer satisfying (3) has its input and output mapping bounded in a cone characterized by two lines with slope a and b. The quantizer is called "passive" since the condition $uv \geq 0$ holds for all inputs v. This is the general condition for a memoryless nonlinearity to be passive [13]. The notion of passivity for quantizers can capture many quantizers

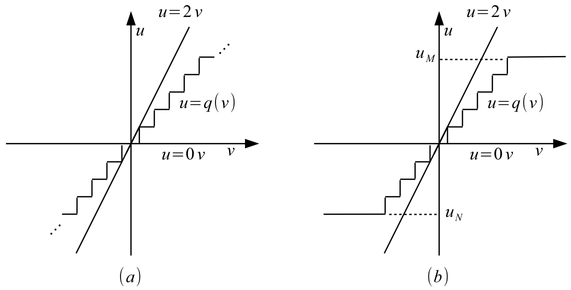

Figure 2: (a) A uniform quantizer with infinite quantization levels (b) A uniform quantizer with finite quantization levels

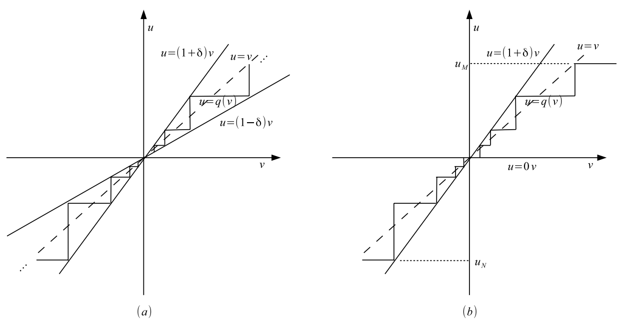

Figure 3: (a) A logarithmic quantizer with infinite quantization levels (b) A logarithmic quantizer with finite quantization levels

used in practice, such as the uniform mid-tread quantizer (Fig. 2), the logarithmic quantizer (Fig. 3) and many non-standard quantizers (Fig. 1).

We can find the values of a and b from a quantizer's input and output mapping. For example, we can show that $a = 0, b = 2$ for a uniform mid-tread quantizer with infinite/finite quantization levels; $a = 0, b = 1 + \delta$ for a logarithmic quantizer with finite quantization levels; and $a = 1 - \delta, b = 1 + \delta$ for a logarithmic quantizer with infinite quantization levels, where $1 > \delta > 0$ is a constant quantization gain.

It is worth pointing out that a quantized system in Fig. 4 is not necessarily a passive system even if the quantizers Q_c and Q_p are passive quantizers. This leads us to resort input-output transformations introduced in Section III to preserve passivity.

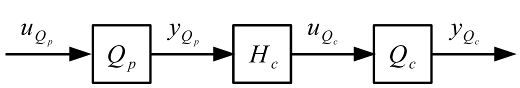

Figure 4: A general system with input and output quantization

2.3 Passivity for Switched Systems

A nonlinear switched system consists of a finite set of subsystems with nonlinear dynamics. The finite number of subsystems can be enumerated, $\{1, 2, ..., P\}$. At any point in time, a single subsystem i is active and the dynamics are nonlinear and time-invariant. The time-varying nature of these systems comes from the switching behavior. The switching signal $\sigma(k)$ is a function that maps the time to the index of the active subsystem, $\sigma : \mathbb{Z}^+ \to \{1, ..., P\}$. This function is piecewise constant and only changes at switching instants. The model with the switching signal is given by

$$\begin{aligned} x(k+1) &= f_{\sigma(k)}(x(k), u(k)) \\ y(k) &= h_{\sigma(k)}(x(k), u(k)). \end{aligned} \tag{4}$$

The switching instants can be listed in order k_1, k_2, etc. Alternatively, the notation k_{i_p} will be used to denote the p^{th} time that subsystem i becomes active. For example, the first subsystem ($i = 1$) becomes active for the first time ($p = 1$) at time k_0 ($k_0 = k_{1_1}$). The second subsystem $i = 2$ becomes active at time k_1 ($k_1 = k_{2_1}$) and so forth. By using these two notations in conjunction, it is possible to list completely the times that a system becomes active as well as the times it becomes inactive. Subsystem i becomes active the p^{th} time at time k_{i_p} and then inactive at time $k_{(i_p+1)}$. That same subsystem becomes active again at time $k_{i_{(p+1)}}$.

An indicator set will be defined to signify regions where a particular subsystem is active. Consider subsystem i that is active from k_{i_1} to $k_{(i_1+1)}$, k_{i_2} to $k_{(i_2+1)}$, etc. The set of times I_i can be defined to indicate those time intervals where subsystem i is active,

$$I_i = \bigcup_{p=1}^{K_i} \{k_{i_p}, ..., k_{(i_p+1)}\}. \tag{5}$$

This notation will be used to draw a distinction between the active and inactive time intervals of a system.

The notion of passivity for switched systems used in this paper is based on previous work on decomposable dissipativity for switched systems. This approach has been used in continuous-time [29, 22] and in discrete-time [14]. The concept of decomposable dissipativity is based on the fact that systems typically store energy differently when they are active compared to when they are inactive. The solution is to decompose the supply rate into an active portion and an inactive portion. When a subsystem is inactive, it may have a different supply rate depending on which other subsystem is active. The definition given here is a special case of [14]. While that work presented a very general definition, the authors didn't consider stability of interconnected systems. Traditionally, stability of feedback interconnections is one of the main benefits of dissipativity theory.

In decomposable dissipativity, the multiple energy storage function approach is taken. This allows for each subsystem i to have a unique notion of energy captured by the storage function $V_i(x)$. This notion of energy is positive, i.e. for all i, $V_i(x) > 0$ for all $x \neq 0$. The notion of supplied energy for a subsystem i while it is inactive may be unique for each active subsystem $j \neq i$. This results in several inactive energy supply rates for each i and j. These rates may be a function of input, output, state, and time and will be denoted as $\omega_i^j(u, y, x, k)$. When each subsystem is inactive, the following inequality holds for each active subsystem j at an appropriate time $t \in I_j$ $(\forall i)$

Passivity for discrete-time switched systems is given in the following definition. Recall that a function $\alpha : \mathbb{R}^+ \to \mathbb{R}^+$ is class K_∞ if $\alpha(0) = 0$, α is non-decreasing, and α is radially unbounded.

Definition 3. *Consider a discrete-time switched system (4). This system is passive if there exists a positive storage* *function $V_i(x)$, for each subsystem i, with the property that for some K_∞ functions $\underline{\alpha_i}$ and $\overline{\alpha_i}$,*

$$\underline{\alpha_i}(||x||) \leq V_i(x) \leq \overline{\alpha_i}(||x||),$$

such that the following conditions hold for all i.

1. *During the active time period $k \in I_i$ of each subsystem i, the system is passive ($\rho_i \geq 0$)*
$$V_i(x(k+1)) - V_i(x(k)) \leq u^T y - \rho_i y^T y. \quad (6)$$
2. *When each subsystem i is inactive, it is dissipative with respect to a cross supply rate that may be specific to the active subsystem j. For $k \in I_j$*
$$V_i(x(k+1)) - V_i(x(k)) \leq \omega_i^j(u, y, x, k). \quad (7)$$
3. *The cross supply rates are absolutely summable for all switching sequences $\forall i$ and $\forall j \neq i$,*
$$\sum_{k=k_0}^{\infty} |\omega_i^j(u, y, x, k)| < L, \quad (8)$$
where L is an arbitrarily large finite constant.

When $\rho_i > 0$ for all i, the switched system is called output strictly passive.

This definition is a natural extension of passivity for non-switched systems. Consider the case when there exists a common storage function for the switched system such that equation (6) holds for all i. In this case, passivity for switched systems reduces to the traditional notion of passivity for non-switched systems.

3. PRESERVING PASSIVITY UNDER QUANTIZATION

3.1 Proposed Passification Scheme

The main problem addressed in this paper is the problem of preserving passivity with signal quantization at the system input, the system output, or both (Fig. 4). As mentioned previously, the quantizers of interest Q_c and Q_p are passive and memoryless with

$$\begin{aligned} Q_c:\ & a_c u_{Q_c}^2 \leq u_{Q_c} y_{Q_c} \leq b_c y_{Q_c}^2, \quad \text{with } 0 \leq a_c < b_c < \infty; \\ Q_p:\ & a_p u_{Q_p}^2 \leq u_{Q_p} y_{Q_p} \leq b_p y_{Q_p}^2, \quad \text{with } 0 \leq a_p < b_p < \infty; \end{aligned} \quad (9)$$

where u_{Q_c} represents the input of the quantizer Q_c and y_{Q_c} is the output of Q_c. The same holds for Q_p. If the input to the quantizer is vector, the quantization function acts component-wise on the input vector. One can verify

$$\|y_{Q_c}\|_2^2 \leq b_c^2 \|u_{Q_c}\|_2^2 \ \text{ and } \ \|y_{Q_p}\|_2^2 \leq b_p^2 \|u_{Q_p}\|_2^2. \quad (10)$$

The passification scheme proposed in this paper is shown in Fig. 5. As mentioned, H_c is a discrete-time output strictly passive system such that

$$\Delta V_c(k) = V_c(k+1) - V_c(k) \leq u_c^T(k) y_c(k) - \rho_c y_c^T(k) y_c(k), \quad (11)$$

where $u_c, y_c \in \mathbb{R}^m$, $0 < \rho_c < \infty$, $V_c \in \mathbb{R}^+$ is the storage function of H_c.

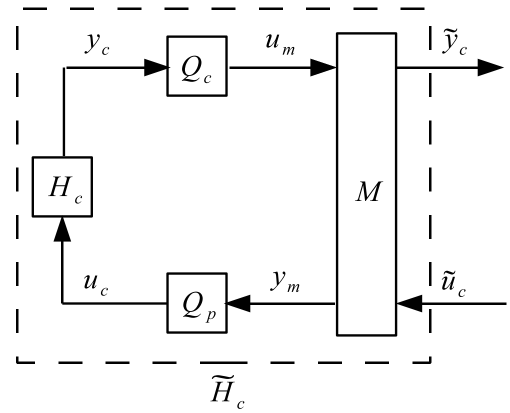

Figure 5: Proposed scheme to preserve passivity under quantization

The block M shown in Fig. 5 is an input/output coordinate transformation such that

$$\begin{bmatrix} u_m \\ y_m \end{bmatrix} = M \begin{bmatrix} \tilde{y}_c \\ \tilde{u}_c \end{bmatrix} = \begin{bmatrix} m_{11} I_m & m_{12} I_m \\ m_{21} I_m & m_{22} I_m \end{bmatrix} \begin{bmatrix} \tilde{y}_c \\ \tilde{u}_c \end{bmatrix}, \quad (12)$$

where $m_{ij} \in \mathbb{R}$, $u_m, y_m \in \mathbb{R}^m$ and $\tilde{u}_c, \tilde{y}_c \in \mathbb{R}^m$. An appropriate transformation will be found in order to maintain the passivity property of H_C.

3.2 Main Results on Preserving Passivity

In this section, we apply the proposed set-up in Fig. 5 to show how passivity of the system H_c is preserved under quantization. Similar set-up to recover passivity of the original system over communication networks under network induced delays and signal quantization have been reported in [26, 27]. The result is stated in Theorem 1.

THEOREM 1. *Consider an OSP system H_C in the proposed scheme shown in Fig. 5 with passive quantizers Q_c and Q_p. If a transformation M is chosen such that*

$$\begin{aligned} & m_{21} = 0, \quad m_{11}^2 = 2b_c^2 \\ & m_{11} m_{12} = \frac{-b_c^2}{\rho_c}\ , \quad m_{12}^2 = \frac{b_c^2 b_p^2}{\rho_c^2} m_{22}^2, \end{aligned} \quad (13)$$

then the subsystem $\widetilde{H}_c : \tilde{u}_c \to \tilde{y}_c$ is output strictly passive such that

$$\Delta V_c(k) = V_c(k+1) - V_c(k) \leq \tilde{u}_c^T(k) \tilde{y}_c(k) - \rho_c \tilde{y}_c^T(k) \tilde{y}_c(k).$$

PROOF. The system H_c being output strictly passive implies the following

$$\begin{aligned} \Delta V_c(k) &= V_c(k+1) - V_c(k) \leq u_c^T(k) y_c(k) - \rho_c y_c^T(k) y_c(k) \\ &= -\frac{1}{2\rho_c} \big[u_c(k) - \rho_c y_c(k)\big]^T \big[u_c(k) - \rho_c y_c(k)\big] \\ &+ \frac{1}{2\rho_c} u_c^T(k) u_c(k) - \frac{\rho_c}{2} y_c^T(k) y_c^T(k) \\ &\leq \frac{1}{2\rho_c} \|u_c(k)\|_2^2 - \frac{\rho_c}{2} \|y_c(k)\|_2^2. \end{aligned} \quad (14)$$

Since the quantizers function component-wise on the input vectors, in view of (9), one can verify that

$$\|u_m\|_2^2 = \sum_{i=1}^m u_{mi}^2 \leq \sum_{i=1}^m b_c^2 y_{ci}^2 = b_c^2 \|y_c\|_2^2 \quad (15)$$

where u_{m_i} is the component of vector u_m and y_{c_i} is the component of vector y_c. We can rewrite this as

$$-\|y_c\|_2 \leq -\frac{1}{b_c^2}\|u_m\|_2^2. \tag{16}$$

Similarly, we can find

$$\|u_c\|_2^2 \leq b_p^2\|y_m\|_2^2. \tag{17}$$

Substituting (16) and (17) into (14), gives

$$\Delta V_c(k) \leq \frac{b_p^2}{2\rho_c}\|y_m(k)\|_2^2 - \frac{\rho_c}{2b_c^2}\|u_m(k)\|_2^2. \tag{18}$$

Considering the transformation M,

$$\begin{cases} u_m(k) = m_{11}\tilde{y}_c(k) + m_{12}\tilde{u}_c(k) \\ y_m(k) = m_{21}\tilde{y}_c(k) + m_{22}\tilde{u}_c(k), \end{cases} \tag{19}$$

equation (18) can be written as

$$\begin{aligned} \Delta V_c(k) \leq \frac{b_p^2}{2\rho_c}&\|m_{21}\tilde{y}_c(k) + m_{22}\tilde{u}_c(k)\|_2^2 \\ - \frac{\rho_c}{2b_c^2}&\|m_{11}\tilde{y}_c(k) + m_{12}\tilde{u}_c(k)\|_2^2 \end{aligned} \tag{20}$$

thus

$$\begin{aligned} \Delta V_c(k) \leq &\Big(\frac{b_p^2}{\rho_c}m_{21}m_{22} - \frac{\rho_c}{b_c^2}m_{11}m_{12}\Big)\tilde{u}_c^T(k)\tilde{y}_c(k) \\ &- \big(\frac{\rho_c}{2b_c^2}m_{11}^2 - \frac{b_c^2}{2\rho_c}m_{21}^2\big)\|\tilde{y}_c(k)\|_2^2 \\ &- \big(\frac{\rho_c}{2b_c^2}m_{12}^2 - \frac{b_c^2}{2\rho_c}m_{22}^2\big)\|\tilde{u}_c(k)\|_2^2. \end{aligned} \tag{21}$$

With the parameters of M as chosen in (13), one can verify that

$$\Delta V_c(k) \leq \tilde{u}_c^T(k)\tilde{y}_c(k) - \rho_c\tilde{y}_c^T(k)\tilde{y}_c(k), \tag{22}$$

which shows that $\tilde{H}_C$ is OSP. □

The implementation of the transformation M chosen in Theorem 1 is illustrated in Fig. 6. The transformation chosen is a specific one that preserves passivity. In fact, the choice of transformation M is not unique. One can find a different transformation from (13), which gives designers freedom to choose from various transformation candidates according to different specifications. In general, any M is allowable as long as it is invertible and satisfies the result (22).

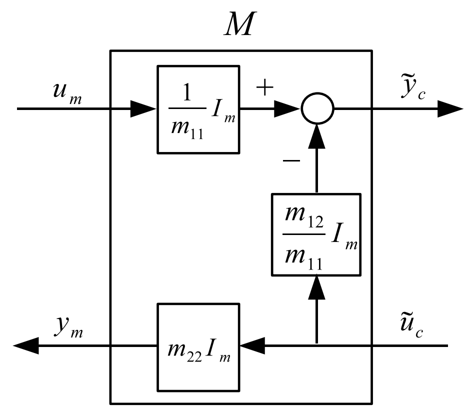

Figure 6: Implementation of M in Theorem 1

Remark 1. *Although Theorem 1 is derived based on discrete-time OSP systems, the result remains valid for continuous-time OSP systems and the same transformation can be applied to preserve passivity.*

Remark 2. *For the case where only one of the quantizers is needed, one can choose $b_c = 1$ when only input quantizer Q_p is present or $b_p = 1$ when only output quantizer Q_c is present.*

Remark 3. *Since $\tilde{H}_c$ is an OSP system, the negative feedback interconnection of $\tilde{H}_c$ with another OSP system H_p, as shown in Fig. 7, is also passive and thus the stability condition can be derived from traditional passive systems theory. The same idea is extended to switched systems in Section IV.*

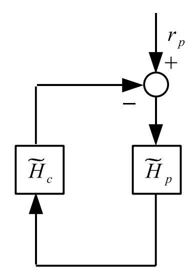

Figure 7: Negative feedback interconnection of two OSP systems

4. STABILITY OF PASSIVE SWITCHED SYSTEMS WITH QUANTIZATION

4.1 Stability of Passive Switched Systems

Passive systems form an important class of dynamical systems. For one, these systems are common in practice. Additionally, passivity can be used to simplify analysis. Passivity is a property that implies stability and the property is preserved when systems are combined in feedback. Combining these two results gives open-loop conditions for closed-loop stability. Additionally, large scale systems can be shown to be stable if each component is passive and the components are sequentially combined in feedback or in parallel. The following results are discrete-time extensions of the work presented in [15]. They will appear in [17]. The first result concerns stability of a single passive switched system.

THEOREM 2. *A passive discrete-time switched system is stable for zero input ($u(k) = 0, \forall k$).*

The passivity property can be used when considering interconnections of systems. The following result shows stability of the feedback interconnection of two passive systems.

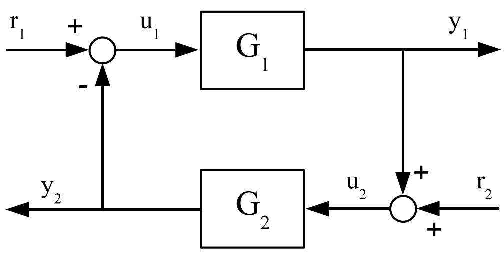

Figure 8: The negative feedback interconnection of two systems.

THEOREM 3. *The feedback interconnection (Fig. 8) of two passive switched systems G_1 and G_2 forms a passive switched system.*

As in the non-switched case, these results can be used to verify closed loop stability by showing that the two systems in feedback are passive. This result can also be used from a design perspective. When controlling a passive switched system, any passive controller is stabilizing without additional conditions. This allows for a large class of controllers to be applied directly including traditional PI controllers.

4.2 Passification of Quantized Switched Systems

The work presented in Section 3 can be extended to switched systems. The structure of the passification scheme remains the same (Fig. 5) with the system H_C being modeled as a switched system according to the dynamics (4). Now that the system dynamics are time-varying, the transformation M must also be time-varying

$$M(k) = \begin{bmatrix} m_{11}(k)I_m & m_{12}(k)I_m \\ m_{21}(k)I_m & m_{22}(k)I_m \end{bmatrix}. \tag{23}$$

The matrix $M(k)$ will be piecewise constant, belonging to a finite set of constant matrices. There will be at most one constant matrix for each subsystem of the given switched system.

The transformation M can switch as H_C switches. In order for this to be allowable, the switching signal of H_C must be known or measurable in real time. From the perspective of this paper, the system H_C is a designed controller so it should be possible to measure the switching signal. Additionally, the set of ρ_i that define the OSP switched system should be known. A function $\rho(k)$ can be defined such that

$$\rho(k) = \rho_i \quad \text{for active subsystem } i. \tag{24}$$

This function is piecewise constant and changes as the switching signal changes. This function is used to demonstrate passivity in the following theorem.

THEOREM 4. *Consider an output strictly passive discrete-time switched system H_C (4). This system is placed in the structure (Fig. 5) with passive quantizers defined by the constants a_c, b_c, a_p, and b_p. This control structure preserves the output strict passivity property of system H_C if the transformation $M(k)$ is chosen according to the following time-varying equations*

$$m_{21}(k) = 0, \quad m_{11}^2(k) = 2b_c^2 \tag{25}$$

$$m_{11}m_{12}(k) = \frac{-b_c^2}{\rho(k)}\ , \quad m_{12}^2(k)(t) = \frac{b_c^2 b_p^2}{\rho^2(k)} m_{22}^2(k), \tag{26}$$

PROOF. Since H_C is OSP, for each subsystem i there exists a V_i to satisfy the passive inequality with $\rho_i > 0$ for $i \in \{1, ..., P\}$,

$$V_i(x(k+1)) \le V_i(x(k)) + u_c^T(k) y_c(k) - \rho_i y_c(k)^T y_c(k). \tag{27}$$

The quantizers satisfy the following inequalities,

$$||u_m||_2 \le b_c\, ||y_c||_2 \text{ and } ||u_c||_2 \le b_p\, ||y_m||_2\,.$$

Applying Theorem 1, the OSP structure of each active subsystem is preserved at each time step by the transformation $M(k)$. The storage functions V_i are also preserved with the structure.

Now the inactive behavior can be analyzed. For each inactive subsystem i and for all active subsystems $j \neq i$, there exists a cross supply rate ω_i^j. For each one, a modified supply rate can be introduced such that

$$\tilde{\omega}_i^j(\tilde{u_c}, \tilde{y_c}, x, k) = \omega_i^j(u_c, y_c, x, k), \forall i, j. \tag{28}$$

These new cross supply rates imply

$$V_i(x(k+1)) \le V_i(x(k)) + \tilde{\omega}_i^j(\tilde{u_c}, \tilde{y_c}, x, k) \tag{29}$$

and

$$\sum_{k=k_0}^{\infty} |\tilde{\omega}_i^j(\tilde{u_c}, \tilde{y_c}, x, k)| < L, \tag{30}$$

where L is an arbitrarily large finite constant given by (8). Since these hold for all i and j, the inactive behavior is dissipative and the supply rates are still absolutely summable. All the conditions for the switched system to be passive are satisfied. The proposed scheme maintains passivity of the switched systems. □

As mentioned earlier, this choice of transformation $M(k)$ is not unique. The conditions listed in the theorem are sufficient to preserve passivity after the quantization effect but there is an entire class of transformations that will also preserve passivity.

This result can be used to preserve passivity of a single system. This can be used with previous results to show stability of feedback interconnections (Fig. 8). When this system is combined in negative feedback with another passive switched system, the overall interconnection is a passive switched system so is stable using Theorem 2 and 3. An example is provided in the following section to demonstrate how this result can be used.

5. EXAMPLE

The work presented in this paper is a method of maintaining passivity for discrete-time switched systems with quantization. The following example illustrates how this method can be applied to a practical system. A linear example was chosen, however, the results are valid for nonlinear switched systems. The switched system H_C chosen is a switched system with two subsystems.

The first subsystem of H_C is modeled by the following dynamics

$$x(k+1) = \begin{bmatrix} -0.060 & 0.173 \\ 0.125 & 0 \end{bmatrix} x(k) + \begin{bmatrix} 1 \\ 0 \end{bmatrix} u(k) \tag{31}$$

$$y(k) = \begin{bmatrix} -0.74 & 0.346 \end{bmatrix} x(k) + 2u(k). \tag{32}$$

The second subsystem of H_C is

$$x(k+1) = \begin{bmatrix} -0.179 & 0.169 \\ 0.125 & 0 \end{bmatrix} x(k) + \begin{bmatrix} 1 \\ 0 \end{bmatrix} u(k) \tag{33}$$

$$y(k) = \begin{bmatrix} -0.667 & 0.158 \end{bmatrix} x(k) + 0.94u(k). \tag{34}$$

This system can be shown to be a passive switched system using the definition given in this paper. The storage functions to show passivity (6) are

$$V_1(x) = x^T(k) \begin{bmatrix} 0.761 & -0.016 \\ -0.016 & 0.96 \end{bmatrix} x(k) \tag{35}$$

$$V_2(x) = x^T(k) \begin{bmatrix} 0.671 & -0.019 \\ -0.019 & 0.989 \end{bmatrix} x(k) \tag{36}$$

with cross supply rates

$$\omega_2^1(u, y, x, k) = u^T(k)y(k) + \frac{1}{10}(x_1^2 + x_2^2) \tag{37}$$

$$\omega_1^2(u, y, x, k) = u^T(k)y(k) + \frac{2}{5}x_1^2. \tag{38}$$

These rates satisfy (7-8). The system is OSP with $\rho_1 = 0.202$ and $\rho_2 = 0.295$.

Both input and output quantization are applied to the controller. The quantizers are uniform with quantization interval 0.1. It can be shown that these are passive quantizers with $a = 0$ and $b = 2$.

The transformation $M(k)$ can take on values in the set $\{M_1, M_2\}$ where

$$M_1 = \begin{bmatrix} 2.83 & -7.00 \\ 0 & 0.354 \end{bmatrix} \tag{39}$$

$$M_2 = \begin{bmatrix} 2.83 & -4.79 \\ 0 & 0.354 \end{bmatrix}, \tag{40}$$

given by (25-26). Transformation $M(k) = M_1$ when subsystem $i = 1$ is active and $M(k) = M_2$ when subsystem $i = 2$ is active.

The switched controller with quantization and transformation $M(k)$ was simulated in feedback with a passive plant. The plant has the following dynamics

$$x(k+1) = \begin{bmatrix} -0.020 & 0.865 \\ 1 & 0 \end{bmatrix} x(k) + \begin{bmatrix} 2 \\ 0 \end{bmatrix} u(k) \tag{41}$$

$$y(k) = \begin{bmatrix} -0.330 & 0.865 \end{bmatrix} x(k) + 2u(k). \tag{42}$$

The feedback interconnection of these two systems forms a passive switched system. When simulated, both the state of the plant and the controller converge to a set near the origin for arbitrary switching. The convergence of the plant state and output are as shown in Fig. 9 with switching signal Fig. 10.

This example demonstrates the methods introduced in this paper. The example chosen was straightforward, being a linear switched system with two subsystems. However,

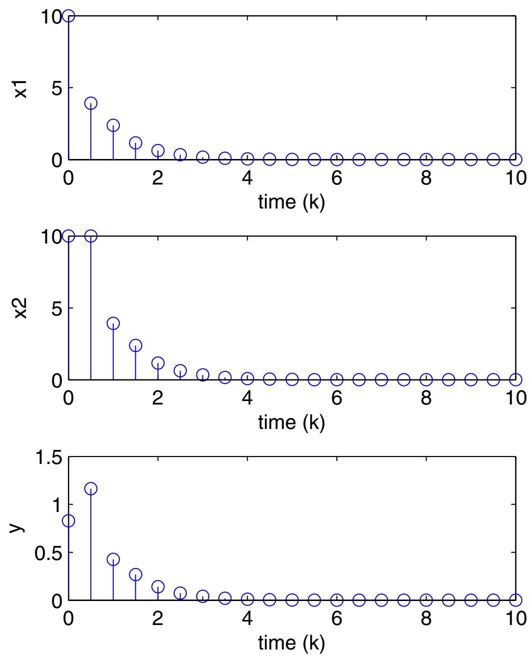

Figure 9: The first two panels show stability of the plant state x_1 and x_2. The third panel shows the system output y.

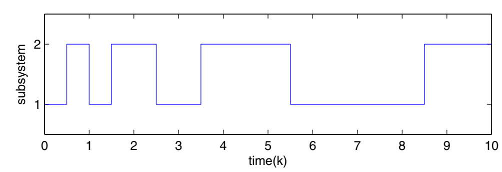

Figure 10: The switching signal of controller H_c that switches between subsystems 1 and 2 is shown.

these methods apply to nonlinear switched systems with any arbitrary finite number of subsystems.

6. CONCLUSION

In this paper, we introduced a scheme to preserve the output strict passivity property of a system with passive input and output quantization by using an input-output coordinate transformation. Then we showed that the same scheme can be applied to switched systems and thus the stability of interconnected passive switched systems can be guaranteed from the results. The example demonstrated how these methods can be applied to a practical quantized switched system.

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