Introduction

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Sets

A **set** is a collection of objects. The objects are called **elements** of the set.

A set can be described as a list, for example $D = \{5, 6, 7\}$ or with words:

$$D = \{\text{All whole numbers between 5 and 7 inclusive}\}$$

Note Repetitions in the list or changes in the order of presentation do not change the set. For example, the following lists describe the same set:

$$\{5, 5, 6, 7\} = \{5, 6, 7\} = \{6, 5, 7\}.$$

Sets

When describing the elements of a set, we should be careful that there is **no ambiguity in our description** and the set is well defined. For example to talk about "the set of the ten greatest sportsmen or sportswomen of the twentieth century" does not make sense, since the description is subject to personal opinion and different people may produce different sets from this description.

Note however that a verbal description of a well defined set is not necessarily unique. For example the set D above might be described as:
$D = \{$ All integers bigger than 4 and less than 8$\}$.

Notation

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Equal Sets We say two sets are **equal** if they consist of exactly the same elements. For example consider the following sets
$A = \{$Major league baseball players who got more than 760 home runs in their career $\}$,
$B = \{$Major league baseball players who got more than 70 home runs in a single season$\}$.

These two sets are equal and have a single element. We have $A = B = \{$Barry Bonds$\}$.

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This notation does assume we have an implicit agreement as to the formula for the remaining terms. For example $\mathbf{E}$ might have been $\{2, 4, 6, 10, 16, \dots\}$.

Set-Builder Notation

A description of the set D above may also be written using set-builder notation:

$$D = \{x |\ x \text{ is an integer between 5 and 7 inclusive}\}$$

or

$$D = \{x |\ x \text{ is an integer and } \ 5 \leq x \leq 7\}.$$

Here the symbol | is read as "such that" and the upper mathematical sentence above reads as "D is equal to the set of all x such that x is an integer between 5 and 7 inclusive".

The Empty Set

The empty set is the set with no elements, i.e. the list of its elements is a blank list. It is denoted by the symbol $\emptyset$. One can think of the empty set as an empty list: $\{ \}$. As you can imagine, this set can have many verbal descriptions: for example;
$\{$all major league baseball players who got more than 80 home runs in a single season$\} = \emptyset$.

Subsets

A **Subset** of a set A is a subcollection of elements of A. We have B is a subset of A, denoted by $B \subseteq A$, if every element of B is also an element of A. We say that B is a **proper subset** of A if $B \subseteq A$, but $B \neq A$.

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Example If $A = \{1, 2, 3, 4\}$, $B = \{2, 4, 6, 8\}$, $C = \{3, 4, 5, 6\}$ and $D = \{2, 6\}$,

Is $D \subseteq A$?

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Is $D \subseteq B$? Yes. $2 \in B$ and $6 \in B$ so every element in D is in B.

Is $D \subseteq C$? No: $2 \in D$ but $2 \notin C$.

Is D a proper subset of B? Yes: For example $8 \in B$ but $8 \notin D$.

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Two sets A and B are equal if and only if $A \subseteq B$ and $B \subseteq A$.

Universal Sets

Sometimes we wish to restrict our attention to a particular set, called a *universal set* and usually denoted by U.

Example If we wish to do a survey on the music preferences in our class, we are restricting our attention to the class. In this case our universal set is
$U = \{\text{all students in our class}\}$.

If we let $R = \{\text{students in the class who like Rap music}\}$,

$C = \{\text{students in the class who like Classical music}\}$,

and $E = \{\text{students in the class who like 80's music}\}$,

then R, C and E are all subsets of our universal set U.

Universal Sets

Note To avoid ambiguity in the definition of such sets, it is common in surveys such as this to restrict answers to the given questions to "yes" and "no"; (e.g. "Do you like Rap music? Yes No (circle one)"). In doing this the resulting sets are well defined, but of course we have ignored the tastes of those who like some rap music but not all.

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Even this is not quite right since some folks may have checked both boxes or neither box for a question. You may add additional subsets, say B for checked both and N for checked neither. Another approach is to restrict U to those students who checked precisely one box for each question.

Union

Given two sets, A and B, we define their **union**, denoted $A \cup B$, to be the set

$$A \cup B = \{x \mid x \in A \text{ or } x \in B\}$$

To be precise, when we say "or" we include both. So if $a \in A$ and $a \in B$, then $a \in A \cup B$.

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