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A line which runs through the point $(0, 0)$ has an equation of the form

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Given any two district points, we can draw the line by joining the points with a straight edge and extending.

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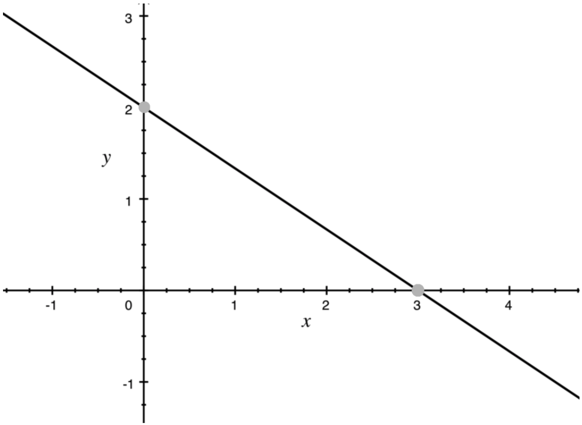

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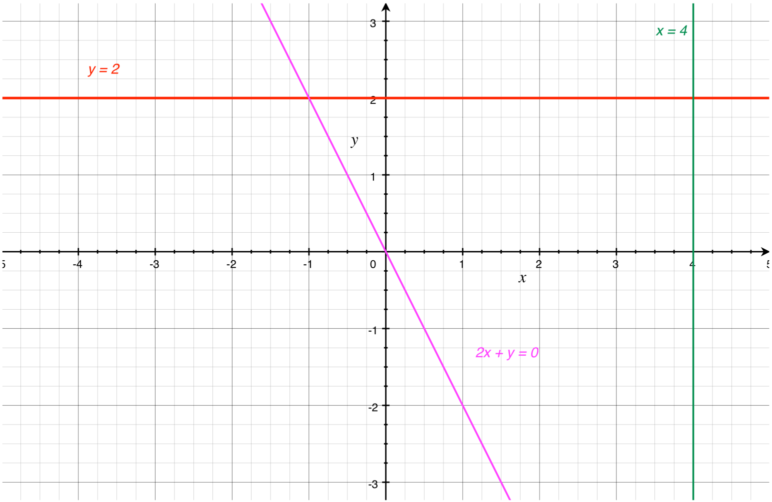

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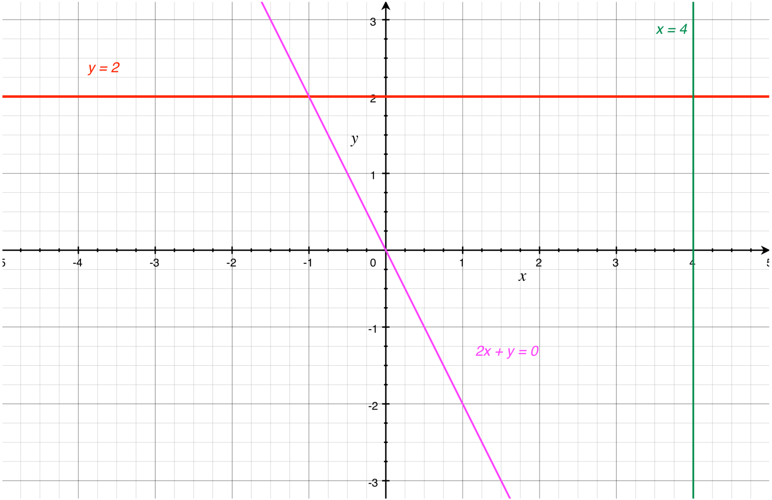


To draw $2x + y = 0$ note $(0, 0)$ is one point. Pick any non-zero value for $x$ and solve for $y$; if $x = 1$, $y = -2$.

# Linear Inequalities in Two Variables

To solve a linear programming problem, we must deal with **linear inequalities** of the form

$$ax + by \geqslant c \text{ or } ax + by \leqslant c \text{ or } ax + by > c \text{ or } ax + by < c,$$

where $a$, $b$ and $c$ are given numbers. Constraints on the values of $x$ and $y$ that we can choose to solve our problem, will be described by such inequalities.

**Example** Michael is taking a timed exam in order to become a volunteer firefighter. The exam has 10 essay questions and 50 Multiple choice questions. Michael has 90 minutes to take the exam and knows he cannot possibly answer every question. An essay question takes 10 minutes to answer and a short-answer question takes 2 minutes. Let $x$ denote the number of multiple choice questions that Michael will attempt and let $y$ denote the number of essay questions that Michael will attempt, what linear inequalities describes the constraints on Michael's time given above?

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Additionally, since he can not answer fewer than 0 questions, there are constraints $x \geqslant 0$ and $y \geqslant 0$. Furthermore, since he can not answer more questions than there are, $x \leqslant 50$ and $y \leqslant 10$.

- A **point** $(x_1, y_1)$ **is said to satisfy the inequality** $ax + by < c$ if $ax_1 + by_1 < c$.
- A **point** $(x_1, y_1)$ **is said to satisfy the inequality** $ax + by > c$ if $ax_1 + by_1 > c$.
- The **graph of a linear inequality** is the set of all points in the plane which satisfy the inequality.
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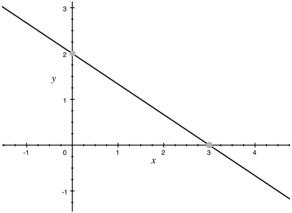

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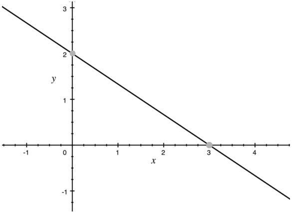


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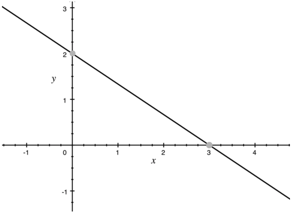
Recall the graph: 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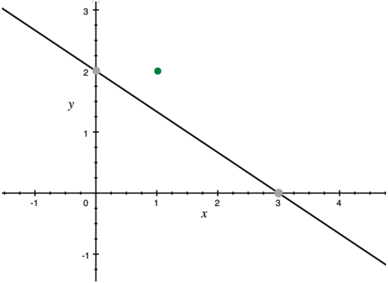

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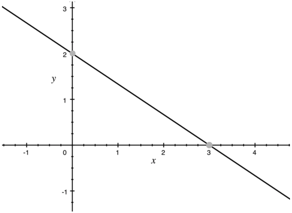


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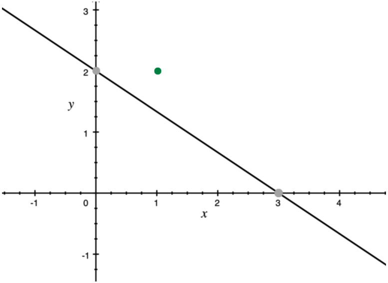
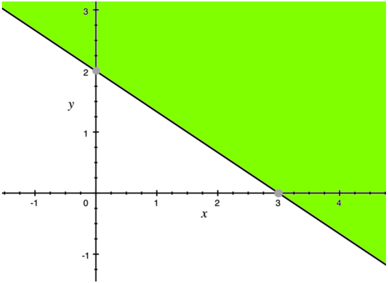
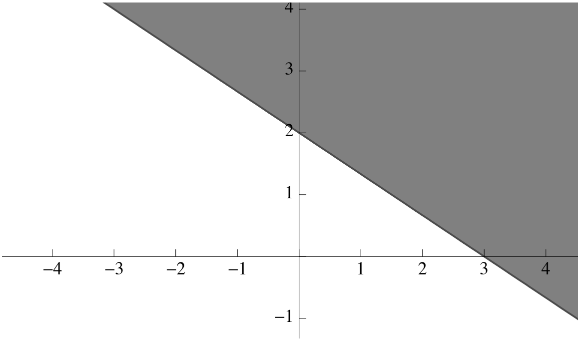
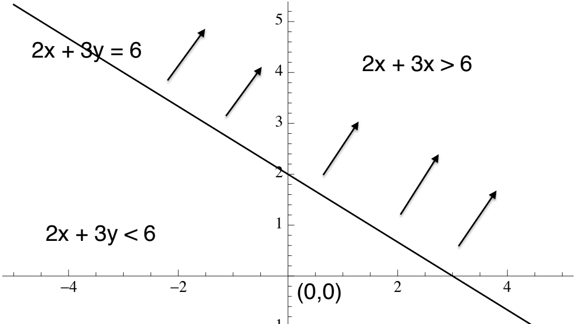


Notice that all points on the line $2x + 3y = 6$ satisfy this inequality. This line cuts the plane in half. One half contains all points $(x, y)$ with $2x + 3y > 6$ and the other half contains all points with $2x + 3y < 6$. To find which half is which, we need only check one point on one side of the line. (if the line does not cut through $(0, 0)$, we can check that point easily.) In this case we find that $2(0) + 3(0) < 6$. Therefore the solution to the inequality $2x + 3y \geqslant 6$ is the half plane not containing $(0, 0)$ shaded below. We can also represent it with arrows as in the diagram on the right.

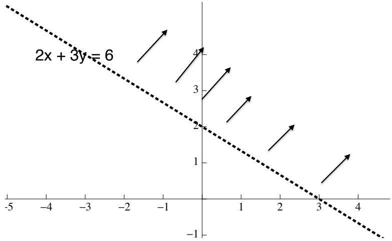
## Graph of the inequality $2x + 3y \geqslant 6$





The plot on the right will be a more useful representation when we want to plot many inequalities on the same graph. Since the **region includes the points along the line** $2x + 3y = 6$**, we draw a solid line**. We **use a dotted line when we want to indicate strict inequality** as in the solution set to $2x + 3y > 6$ shown below:

**Graph of the inequality**
$2x + 3y > 6$



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- If the line is not vertical, there is an *upper half-plane* and a *lower half-plane.*
- If the line is not horizontal, there is a *right half-plane* and a *left half-plane*.
- If the line is neither vertical or horizontal then
  - the right half-plane equals the upper half plane
  - the left half-plane equals the lower half-plane
- Using our set theory terminology, the union of the two half-planes is the complement of the line.

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- If the line is not horizontal, there is a *right half-plane* and a *left half-plane*.
- If the line is neither vertical or horizontal then
  - the right half-plane equals the upper half plane
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- Using our set theory terminology, the union of the two half-planes is the complement of the line.

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Once you draw the line, it is easy to pick out the upper/right and lower/left half-planes.

To graph an inequality of the form $ax + by \leqslant c$, first draw the line $ax + by = c$.

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(The technique described here also works if the inequality symbol is one of $<$, $>$, or $\geqslant$.)

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One of the two half-planes is the solution set for $ax + by > c$ and the other is the solution set for $ax + by < c$.

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To decide which is which, pick a point in one of the half-planes and see which inequality holds.

**Example** Graph the set of points satisfying the inequality:

$$2x - 3y \geqslant 15$$

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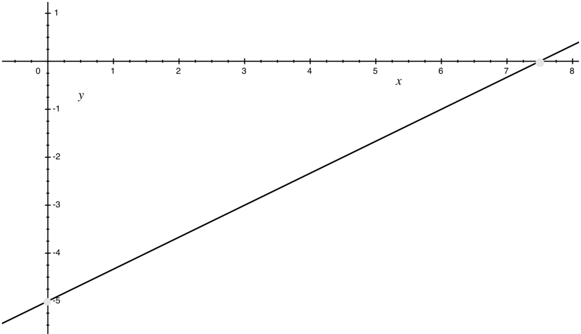
The graph:



**Example** Graph the set of points satisfying the inequality:

$$2x - 3y \geqslant 15$$

At $(0, 0)$, $2x - 3y = 0$ which is less than 15. Hence

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$$x - 3y \geqslant 0, \quad x > 2, \qquad y \leqslant 10$$

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A second way to proceed.

- Draw the lines.
- Ignore the axes *unless* they are explicitly some of the lines.
- Identify the regions into which the plane is divided. Some regions are infinite so you only see a small part of them.

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- Add the axes back.
- Pick a point in the region you think is the solution set.
- Check that your pick satisfies the inequalities.
- Shade the region containing your point.

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- Check that your pick satisfies the inequalities.
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Lets return to a previous example:
**Example** Michael is taking a timed exam in order to become a volunteer firefighter. The exam has 10 essay questions and 50 multiple choice questions. Michael has 90 minutes to take the exam and knows he cannot possibly answer every question. An essay question takes 10 minutes to answer and a short-answer question takes 2 minutes. Let $x$ denote the number of multiple choice questions that Michael will attempt and let $y$ denote the number of essay questions that Michael will attempt. We found that the linear inequality describing this time constraint was

$$2x + 10y \leqslant 90 \ .$$

Graph this inequality below.

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Note that the point $(9, 6)$ is a feasible option, i.e. Michael attempts 9 multiple choice questions and 6 partial credit questions. Note also that $(-1, 9)$ is also in the shaded region, however this is not really a feasible option for Michael. What other constraints limiting Michael's feasible choices can you write down?

$x \geqslant 0$ and $y \leqslant 0$.

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