Section 3.2: Feasible Sets

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The graph of the feasible set for a system of inequalities is the set of all points in intersection of the graphs of the individual inequalities.

Terminology: A linear inequality of the form

$$a_0x + a_1y \leqslant b, \qquad a_0x + a_1y < b,$$

$$a_0x + a_1y \geqslant b, \qquad a_0x + a_1y > b,$$

where a_0, a_1 and b are constants, is called a **constraint** in a linear programming problem. The corresponding **constraint line** is $a_0x + a_1y = b$. The restrictions $x \geqslant 0$, $y \geqslant 0$ are called **non-negative conditions**.

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A system of constraint lines divides the plane into a number of regions. The feasible set consists of one of these regions.

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1. For each constraint inequality, decide which side of the constraint line satisfies the inequality. Take the intersection of each of the sets.
2. Pick a point in a region and see if it satisfies the inequality. If it does, the region containing this point is the feasible set. If not, pick a point in a different region. Continue until you find the feasible set. If you check all the regions and none work then the feasible set is empty.

3. This method is a combination of the first two.

Step 1. Select a point in the interior of each region.

Step 2. Pick a constraint line and divide the set of selected points into the set which satisfies the $>$ inequality and the set which satisfies the $<$ inequality. Denote the set which satisfy the inequality you want as the "possible set", $\mathbf{P}_0$.

Step 3. Pick another constraint line and divide the "possible set" into the set which satisfies the $>$ inequality and the set which satisfies the $<$ inequality. The new "possible set" is the subset of the previous "possible set" which satisfy the second inequality, $\mathbf{P}_1$.

Step 4 $\vdots$ Repeat step 3 until you have used all the constraint lines getting $\mathbf{P}_2, \ldots, \mathbf{P}_n$ If at any time $\mathbf{P}_r$ is empty you are done and the feasible set is empty.

After you have used all the constraint lines, the "possible set" will have one point left in it and the region containing this point is the feasible set.

Example Determine if $(x, y) = (1, 2)$ is in the feasible set for the system of inequalities shown below and graph the feasible set for the system of inequalities:

$$2x + 3y \geqslant 6$$
$$2x - 3y \geqslant 15$$

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$$2 \cdot 1 - 3 \cdot 2 = -4 \not\geqslant 15$$

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$$2 \cdot 1 + 3 \cdot 2 = 8 \geqslant 6$$
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so $(1, 2)$ is not in the feasibility set. Next draw the feasible set.

The two lines divide the plane into four regions:

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First method: $2x + 3y \geqslant 6 \quad 2x - 3y \geqslant 15$

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Third method: $2x + 3y \geqslant 6 \quad 2x - 3y \geqslant 15$

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Third method: $2x + 3y \geqslant 6 \quad 2x - 3y \geqslant 15$

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A and **D** lie on one side of $2x + 3y = 6$ while **B** and **C** lie on the other.

Third method: $2x + 3y \geqslant 6 \quad 2x - 3y \geqslant 15$

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A and **D** lie on one side of $2x + 3y = 6$ while **B** and **C** lie on the other. **A** satisfies $2x + 3y < 6$ hence so does **D**; $\mathbf{P}_0 = \{\mathbf{B}, \mathbf{C}\}$.

Third method: $2x + 3y \geqslant 6 \quad 2x - 3y \geqslant 15$

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A and **D** lie on one side of $2x + 3y = 6$ while **B** and **C** lie on the other. **A** satisfies $2x + 3y < 6$ hence so does **D**; $\mathbf{P}_0 = \{\mathbf{B}, \mathbf{C}\}$.

B lies on one side of $2x - 3y = 15$ and **C** lies on the other. **B** satisfies $2x - 3y < 15$. $\mathbf{P}_1 = \{\mathbf{C}\}$.

Third method: $2x + 3y \geqslant 6 \quad 2x - 3y \geqslant 15$

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A and **D** lie on one side of $2x + 3y = 6$ while **B** and **C** lie on the other. **A** satisfies $2x + 3y < 6$ hence so does **D**; $\mathbf{P}_0 = \{\mathbf{B}, \mathbf{C}\}$.

B lies on one side of $2x - 3y = 15$ and **C** lies on the other. **B** satisfies $2x - 3y < 15$. $\mathbf{P}_1 = \{\mathbf{C}\}$.

These are all the constraint lines so **C** is a point in the feasible set.

The feasible set is shaded: $2x + 3y \geqslant 6 \quad 2x - 3y \geqslant 15$

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Here is a more efficient version of the third method: Draw the lines and label the regions. The axes are not constraint lines for this problem.

$$2x + 3y \geqslant 6 \quad 2x - 3y \geqslant 15$$

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Here is a more efficient version of the third method: Draw the lines and label the regions. The axes are not constraint lines for this problem.

$$2x + 3y \geqslant 6 \quad 2x - 3y \geqslant 15$$

1. $(5, 0)$ satisfies $2x + 3y \geqslant 6$: $\mathrm{P}_0 = \{2, 4, 5\}$.

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$$2x + 3y \geqslant 6 \quad 2x - 3y \geqslant 15$$

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1. $(5, 0)$ satisfies $2x + 3y \geqslant 6$: $\mathrm{P}_0 = \{2, 4, 5\}$.
2. $(5, 0)$ satisfies $2x - 3y < 15$: $\mathrm{P}_1 = \{5\}$.

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- A **line** in the plane is a the graph of $a_0x + a_1y = b$ with not both a_0 and a_1 equal to 0.
- A **ray** is a subset of a line consisting of a point on that line (the *initial point*) and all points on the line lying to one side of the initial point.
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The **boundary** of the feasible set consists of all subsets of the constraint lines which satisfy the inequalities $a_0x + a_1y \leqslant b$ or $a_0x + a_1y \geqslant b$.

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The **boundary** of the feasible set consists of all subsets of the constraint lines which satisfy the inequalities
$a_0 x + a_1 y \leqslant b$ or $a_0 x + a_1 y \geqslant b$.
It is a theorem that each constraint line contributes a line, a ray, a segment or the empty set to the boundary of the feasible set.

Here is the boundary of the feasible set in the last example. It consists of two rays.

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This would be the boundary of the feasible set for any of the four systems

$$2x + 3y \geqslant 6 \quad 2x - 3y \geqslant 15 \qquad 2x + 3y \geqslant 6 \quad 2x - 3y > 15$$
$$2x + 3y > 6 \quad 2x - 3y \geqslant 15 \qquad 2x + 3y > 6 \quad 2x - 3y > 15$$

With strict inequalities, you need to draw some of the boundary as dotted lines.

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$2x + 3y \geqslant 6 \quad 2x - 3y \geqslant 15$ $\qquad$ $2x + 3y \geqslant 6 \quad 2x - 3y > 15$

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$2x + 3y > 6 \quad 2x - 3y \geqslant 15$ $\qquad$ $2x + 3y > 6 \quad 2x - 3y > 15$

Example Graph the feasible set for the system of inequalities:

$$x - y \geqslant 2 \qquad y + 2x \geqslant 6 \qquad y \geqslant 2$$

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The three lines:

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Warning: The x and y axes are NOT part of the system of constraint lines!

There are 7 regions: $x - y \geqslant 2$, $2x + y \geqslant 6$, $y \geqslant 2$

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For $x - y \geqslant 2$ the "possible set" is $\mathbf{P}_0 = \{\mathrm{C}, \mathrm{E}, \mathrm{F}\}$ since $(4, 0)$ satisfies $x - y > 2$.

For $2x + y \geqslant 6$ is $\mathbf{P}_1 = \{\mathrm{C}, \mathrm{E}\}$ since $(4, 0)$ satisfies $2x + y > 2$.

Finally, if $y > 2$, $\mathbf{P}_2 = \{\mathrm{C}\}$ is all that is left and we have used all the lines.

Here are the 7 regions that the constraint lines carve out.
The feasible set is gray.

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Why aren't there 8 regions?

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$x - y \leqslant 2$	$y + 2x \leqslant 6$	$y \leqslant 2$	$x - y \leqslant 2$	$y + 2x \leqslant 6$	$y \geqslant 2$
$x - y \leqslant 2$	$y + 2x \geqslant 6$	$y \leqslant 2$	$x - y \leqslant 2$	$y + 2x \geqslant 6$	$y \geqslant 2$
$x - y \geqslant 2$	$y + 2x \leqslant 6$	$y \leqslant 2$	$x - y \geqslant 2$	$y + 2x \leqslant 6$	$y \geqslant 2$
$x - y \geqslant 2$	$y + 2x \geqslant 6$	$y \leqslant 2$	$x - y \geqslant 2$	$y + 2x \geqslant 6$	$y \geqslant 2$

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$x - y \geqslant 2$	$y + 2x \leqslant 6$	$y \leqslant 2$	$\mathbf{x - y \geqslant 2}$	$\mathbf{y + 2x \leqslant 6}$	$\mathbf{y \geqslant 2}$
$x - y \geqslant 2$	$y + 2x \geqslant 6$	$y \leqslant 2$	$x - y \geqslant 2$	$y + 2x \geqslant 6$	$y \geqslant 2$

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The color of the constraints corresponds to the color in the diagram *except*:

- The black constraints yield the white region.
- The bold constraints yield an empty region.

In general, if there are n constraint equations there will be 2^n regions. This follows of course from your general counting principles.

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However, some of these regions may be empty so you may see less than 2^n regions when you draw the picture.

Typically you are only interested in one of the regions (the feasible set for your problem) and you can ignore the others.

A feasible set is **bounded** if it can be contained in some square centered at the origin (with sides of finite length). Otherwise the feasible set is **unbounded**.

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The **corners** or **vertices** of the feasible set will be points at which constraint lines intersect. We will need to find the co-ordinates of the vertices of such a feasible set to solve the linear programming problems in the next section.

Intersection of a pair of lines

An easy way to find the intersection of a pair of lines (both non vertical), is to rearrange their equation to the (standard) form shown below and equate y values;

$$y = m_1 x + b_1 \text{ and } y = m_2 x + b_2$$

intersect where

$$m_1 x + b_1 = m_2 x + b_2.$$

Example

Find the point of intersection of the lines:

$$2x + 3y = 6$$

$$2x - 3y = 15$$

Example

Find the point of intersection of the lines:

$$2x + 3y = 6$$

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$$y = -\frac{2}{3}x + 2$$

$$y = \quad \frac{2}{3}x - 5$$

so $\frac{2}{3}x - 5 = -\frac{2}{3}x + 2$ or $\frac{4}{3}x = 2 + 5$. Then $4x = 3 \cdot (7) = 21$ so $x = \frac{21}{4}$. Then $y = \frac{2}{3}\left(\frac{21}{4}\right) - 5 = \frac{7}{2} - 5 = -\frac{3}{2}$. So $\left(\frac{21}{4}, -\frac{3}{2}\right)$ is the point of intersection.

To find the vertices/corners of the feasible set, graph the feasible set and identify which lines intersect at the corners. Use the graphs you drew above to solve the problems below.

Example Find the vertices of the feasible set corresponding to the system of inequalities:

$$2x + 3y \geqslant 6$$

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Example Find the vertices of the feasible set corresponding to the system of inequalities:

$$2x + 3y \geqslant 6$$

$$2x - 3y \geqslant 15$$

This is the same problem we just worked. The two lines are not parallel or equal so they intersect in one point, $\left(\frac{21}{4}, -\frac{3}{2}\right)$.

Example Find the vertices of the feasible set corresponding to the system of inequalities:

$$x - y \geqslant 2 \qquad y + 2x \geqslant 6 \qquad y \geqslant 2$$

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No two of these three lines are parallel or equal so there are three vertices.

$x - y = 2$ and $y + 2x = 6$ intersect as follows: $y = x - 2$, $y = -2x + 6$ so $x - 2 = -2x + 6$ or $3x = 6 + 2$ so $x = \frac{8}{3}$ and then $y = \frac{8}{3} - 2 = \frac{2}{3}$ so the intersection is $\left(\frac{8}{3}, \frac{2}{3}\right)$.

$x - y = 2$ and $y = 2$ intersect as follows: $y = x - 2$, $y = 2$ so $x - 2 = 2$ or $x = 4$ and then $y = 2$ so the intersection is $(4, 2)$.

$y = 2$ and $y + 2x = 6$ intersect as follows: $y = 2$, $y = -2x + 6$ so $2 = -2x + 6$ or $x = 2$ and then $y = 2$ so the intersection is $(2, 2)$.

There is only one vertex in the feasible set, $(4, 2)$.

Empty Feasible Sets

Sometimes there are no points in the feasible set for a system of inequalities as in the following example.

Example Graph the feasible set for the system of inequalities:

$$x - y \geqslant 2 \quad x + y \leqslant 1 \quad y \geqslant 0 \quad x \geqslant 0$$

The constraint lines divide the plane into 11 regions. This time the axes are constraint lines. (5 regions must be empty.)

$$x - y \geqslant 2 \quad x + y \leqslant 1 \quad y \geqslant 0 \quad x \geqslant 0$$

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$$x - y \geqslant 2 \quad x + y \leqslant 1 \quad y \geqslant 0 \quad x \geqslant 0$$

(0, 0) satisfies $x + y \leqslant 1$:
$\mathrm{P}_0 = \{\mathbf{1}, \mathbf{3}, \mathbf{6}, \mathbf{7}, \mathbf{8}, \mathbf{10}\}$.

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No remaining region satisfies $y \geqslant 0$: $\mathrm{P}_2 = \emptyset$.

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No remaining region satisfies $y \geqslant 0$: $\mathrm{P}_2 = \emptyset$.

Warning: Do not make the mistake of stopping at $\mathrm{P}_1 = \{\mathbf{10}\}$ and claiming that is the feasible set. Do not stop until either $\mathrm{P}_n = \emptyset$ as here or until you have examined all the inequalities.

Setting up the inequalities

Example Mr. Carter eats a mix of Cereal A and Cereal B for breakfast. The amount of calories and sodium per ounce for each is shown in the table below. Mr. Carter's breakfast should provide at least 480 calories but less than 700 milligrams of sodium.

	Cereal A	**Cereal B**
Calories(per ounce)	100	140
Sodium(mg per ounce)	150	190

Let x denote the number of ounces of Cereal A that Mr. Carter has for breakfast and let y denote the number of ounces of Cereal B that Mr. Carter has for breakfast. What are the set of constraints on the amounts of each cereal that Mr. Carter can consume for breakfast.

$$
\begin{aligned}
100x + 140y &\geqslant 480 \qquad Calories \\
150x + 190y &< 700 \qquad Sodium \\
x \geqslant 0 \quad y &\geqslant 0 \qquad non-negative\ conditions
\end{aligned}
$$

Example A juice stand sells two types of fresh juice in 12 oz cups. The Refresher and the Super Duper. The Refresher is made from 3 oranges, 2 apples and a slice of ginger. The Super Duper is made from one slice of watermelon and 3 apples and one orange. The owners of the juice stand have 50 oranges, 40 apples, 10 slices of watermelon and 15 slices of ginger. Let x denote the number of Refreshers they make and let y denote the number of Super Dupers they make. What is the set of constraints on x and y?

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$$2x + 3y \leqslant 40 \quad Apples \qquad 3x + 1y \leqslant 50 \quad Oranges$$

$$0x + 1y \leqslant 10 \quad Watermelon \qquad 1x + 0y \leqslant 15 \quad Ginger$$

$$x \geqslant 0 \quad y \geqslant 0 \qquad non-negative\ conditions$$

Note for many of the old exam questions the shading is opposite to that shown above (The unshaded region denotes the feasible set)

Extras: Old Exam Questions

1

Select the graph of the feasible set of the system of linear inequalities given by:

$$\begin{array}{rcrcl} x & & & \geqslant & 0 \\ 3x & + & y & \leqslant & 3 \end{array} \quad \begin{array}{rcrcl} & & y & \geqslant & 0 \\ 2x & + & 2y & \leqslant & 4 \end{array}$$

where the shaded area is the feasible set.

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A quick solution is to note that $(0, 0)$ satisfies all the inequalities. Hence the first graph on line 2 is the only possible answer.

Or just draw the lines and shade the feasible set.

$$3x + y \leqslant 3 \quad 2x + 2y \leqslant 4 \quad x \geqslant 0 \quad y \geqslant 0$$

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The lines and regions are

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$$3x + y \leqslant 3 \quad 2x + 2y \leqslant 4 \quad x \geqslant 0 \quad y \geqslant 0$$

$(0,0)$ satisfies $3x + y \leqslant 3$:
$\mathrm{P}_0 = \{1, 2, 4, 5, 7, 8\}$

The lines and regions are

`figures/examQ1B.{ps,eps}` not found (or no BBox)

$$3x + y \leqslant 3 \quad 2x + 2y \leqslant 4 \quad x \geqslant 0 \quad y \geqslant 0$$

$(0,0)$ satisfies $3x + y \leqslant 3$:
$\mathrm{P}_0 = \{1, 2, 4, 5, 7, 8\}$
$(0,0)$ satisfies $2x + 2y \leqslant 4$:
$\mathrm{P}_1 = \{4, 5, 7, 8\}$

The lines and regions are

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$$3x + y \leqslant 3 \quad 2x + 2y \leqslant 4 \quad x \geqslant 0 \quad y \geqslant 0$$

$(0,0)$ satisfies $3x + y \leqslant 3$:

$\mathrm{P}_0 = \{1, 2, 4, 5, 7, 8\}$

$(0,0)$ satisfies $2x + 2y \leqslant 4$:

$\mathrm{P}_1 = \{4, 5, 7, 8\}$

$(1,0)$ satisfies $x \geqslant 0$:

$\mathrm{P}_2 = \{5, 8\}$

The lines and regions are

figures/examQ1B.{ps,eps} not found (or no BBox)

$$3x + y \leqslant 3 \quad 2x + 2y \leqslant 4 \quad x \geqslant 0 \quad y \geqslant 0$$

The lines and regions are

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$(0,0)$ satisfies $2x + 2y \leqslant 4$:

$P_1 = \{4, 5, 7, 8\}$

$(1,0)$ satisfies $x \geqslant 0$:

$P_2 = \{5, 8\}$

$(0,1)$ satisfies $y \geqslant 0$:

$P_3 = \{5\}$

$$3x + y \leqslant 3 \quad 2x + 2y \leqslant 4 \quad x \geqslant 0 \quad y \geqslant 0$$

$(0,0)$ satisfies $3x + y \leqslant 3$:

$\mathrm{P}_0 = \{1, 2, 4, 5, 7, 8\}$

$(0,0)$ satisfies $2x + 2y \leqslant 4$:

$\mathrm{P}_1 = \{4, 5, 7, 8\}$

$(1,0)$ satisfies $x \geqslant 0$:

$\mathrm{P}_2 = \{5, 8\}$

$(0,1)$ satisfies $y \geqslant 0$:

$\mathrm{P}_3 = \{5\}$

The lines and regions are

`figures/examQ1B.{ps,eps}` not found (or no BBox)

Hence the feasible set is

`figures/examQ1A.{ps,eps}` not found (or no BBox)

$$3x + y \leqslant 3 \quad 2x + 2y \leqslant 4 \quad x \geqslant 0 \quad y \geqslant 0$$

The lines and regions are

figures/examQ1B.{ps,eps} not found (or no BBox)

$(0,0)$ satisfies $3x + y \leqslant 3$:

$P_0 = \{1, 2, 4, 5, 7, 8\}$

$(0,0)$ satisfies $2x + 2y \leqslant 4$:

$P_1 = \{4, 5, 7, 8\}$

$(1,0)$ satisfies $x \geqslant 0$:

$P_2 = \{5, 8\}$

$(0,1)$ satisfies $y \geqslant 0$:

$P_3 = \{5\}$

Hence the feasible set is

figures/examQ1A.{ps,eps} not found (or no BBox)

Note that the feasible set is bounded and that the boundary consists entirely of segments.

2 A student spending spring break in Ireland wants to visit Galway and Cork. The student has at most 7 days available and at most 500 euros to spend. Each day spent in Galway will cost 50 euros and each day spent in Cork will cost 60 euros. Let x be the number of days the student will spend in Galway and y, the number of days the student will spend in Cork. Which of the following sets of constraints describe the constraints on the student's time and money for the visits?

2 The student has at most 7 days available and at most 500 euros to spend. Each day spent in Galway will cost 50 euros and each day spent in Cork will cost 60 euros. Let x be the number of days the student will spend in Galway and y, the number of days the student will spend in Cork.

$$(a)\quad \begin{aligned} x + y &\leqslant 7 \\ 50x + 60y &\leqslant 500 \\ x \geqslant 0, &\quad y \geqslant 0 \end{aligned}$$

$$(b)\quad \begin{aligned} x + 7y &\leqslant 500 \\ 50x + 60y &\leqslant 1000 \\ x \geqslant 0, &\quad y \geqslant 0 \end{aligned}$$

$$(c)\quad \begin{aligned} x + y &\leqslant 7 \\ 60x + 50y &\leqslant 500 \\ x \geqslant 0, &\quad y \geqslant 0 \end{aligned}$$

$$(d)\quad \begin{aligned} x + y &\geqslant 7 \\ 50x + 60y &\geqslant 500 \\ x \geqslant 0, &\quad y \geqslant 0 \end{aligned}$$

$$(e)\quad \begin{aligned} x + y &\geqslant 7 \\ 60x + 50y &\geqslant 500 \\ x \geqslant 0, &\quad y \geqslant 0 \end{aligned}$$

$$\begin{array}{rcrcrl} x & + & y & \leqslant & 7 & \text{Days} \\ 50x & + & 60y & \leqslant & 500 & \text{euros} \\ x \geqslant 0 & & y \geqslant 0 & & & \end{array}$$