Section 3.3: Linear programming: A geometric approach

In addition to constraints, linear programming problems usually involve some quantity to maximize or minimize such as profits or costs. The quantity to be maximized or minimized translates to some linear combinations of the variables called an objective function. These problems involve choosing a solution from the feasible set for the constraints which gives an optimum value (maximum or a minimum) for the objective function.

Example

A juice stand sells two types of fresh juice in 12 oz cups. The Refresher and the Super Duper. The Refresher is made from 3 oranges, 2 apples and a slice of ginger. The Super Duper is made from one slice of watermelon and 3 apples and one orange. The owners of the juice stand have 50 oranges, 40 apples, 10 slices of watermelon and 15 slices of ginger. Let x denote the number of Refreshers they make and let y denote the number of Super Dupers they make.

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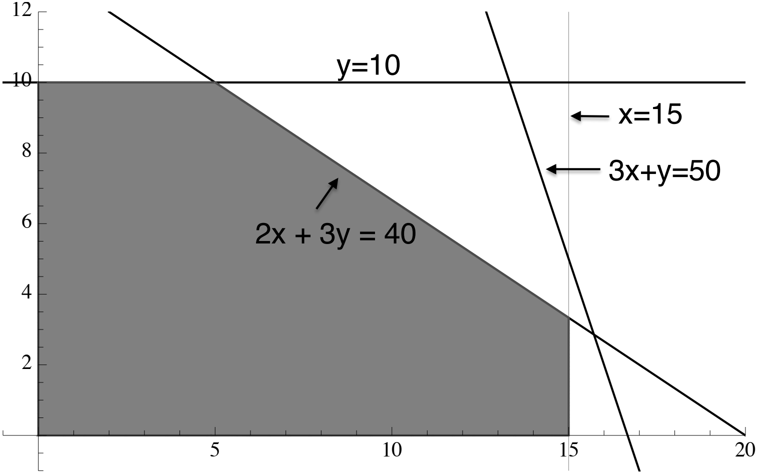
Last time, we saw that the **set of constraints** on x and y were of the form :

$$3x + y \leqslant 50 \qquad 2x + 3y \leqslant 40$$

$$x \leqslant 15 \qquad y \leqslant 10$$

$$x \geqslant 0 \qquad y \geqslant 0$$

The conditions $x \geqslant 0$ and $y \geqslant 0$ are called **non-negative conditions.** We can now draw the feasible set to see what combinations of x and y are possible given the limited supply of ingredients:



Now suppose that the Refreshers sell for \$6 each and the Super Dupers sell for \$8 each. Let's suppose also that the juice stand will sell all of the drinks they can make on this day, so their revenue for the day is $6x + 8y$. Let's assume also that one of the goals of the juice stand is to maximize revenue. Thus they want to maximize the value of $6x + 8y$ given the constraints on production listed above.

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In other words they want to find a point (x, y) in the feasible set which gives a maximum value for the **objective function** $6x + 8y$. [Note that the value of the objective function ($6x + 8y =$ revenue) varies as (x, y) varies over the points in the feasible set. For example if $(x, y) = (2, 5)$, revenue $= 6(2) + 8(5) = \$52$, whereas if $(x, y) = (5, 10)$, revenue $= 6(5) + 8(10) = \$110$.]

Terminology: Recall from last time that a linear inequality of the form

$$a_0x + a_1y \leqslant b, \qquad a_0x + a_1y < b,$$

$$a_0x + a_1y \geqslant b, \qquad a_0x + a_1y > b,$$

where a_0, a_1 and b are constants, is called a **constraint** in a linear programming problem. The corresponding **constraint line** is $a_0x + a_1y = b$. The restrictions $x \geqslant 0$, $y \geqslant 0$ are called **non-negative conditions**.

A **linear objective function** is an expression of the form $cx + dy$, where c and d are constants, for which one needs to find a maximum or minimum on the feasible set. The term **optimal value** refers to the sought after maximum or minimum as the case may be.

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Given an objective function $O(x, y) = cx + dy$ we seek for a point $(x_0, y_0) \in F$, F the feasible set, such that $O(x_0, y_0)$ is as big as possible (or as small as possible if we are doing a minimum problem).

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Here is another general theorem.
Theorem: Given a linear objective function subject to constraints in the form of linear inequalities, if the feasible set is bounded, then the objective function has both a minimum and a maximum value and each does occur at a vertex.

Here are some simple examples. Some of them are counterexamples to the Theorem on page 239 of the book.

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If the feasible set is $y \geqslant 0$ then there are no vertices. The objective function $O(x, y) = y$ has a minimum but no maximum; $O(x, y) = -y$ has a maximum but no minimum; and $O(x, y) = y - x$ has neither a maximum nor a minimum.
If the feasible set is $y \geqslant 0$, $y \leqslant 2$ then there are still no vertices but $O(x, y) = y$ has both a minimum and a maximum.

Here are some facts about lines to explain what is really going on.

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If $a_0x + a_1y = b$ is **any line** in the plane then

1. $O(x, y)$ may be constant along $a_0x + a_1y = b$. This occurs if and only if $a_0x + a_1y = b$ and $cx + dy = 0$ are parallel. Equivalently, this occurs if and only if $\delta = O(-a_1, a_0) = 0$.
2. If not case 1, then there is one direction along $a_0x + a_1y = b$ so that $O(x, y)$ increases in that direction. In the other direction, $O(x, y)$ decreases.
3. In case 2, to decide which direction on $a_0x + a_1y = b$ is the increasing direction, compute $\delta = O(-a_1, a_0)$. Given any point $P_0 = (x_0, y_0)$ on the line, let $P_1 = (x_0 - a_1, y_0 + a_0)$. The point P_1 is on the line and you don't need to do any calculations to see the relation on the line between P_0 and P_1. If $\delta > 0$ then the increasing direction is P_0 to P_1. If $\delta < 0$ then the increasing direction is P_1 to P_0.

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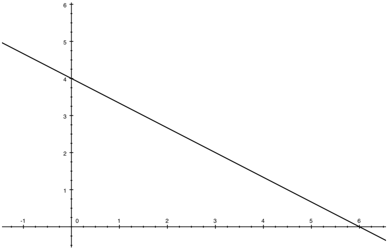

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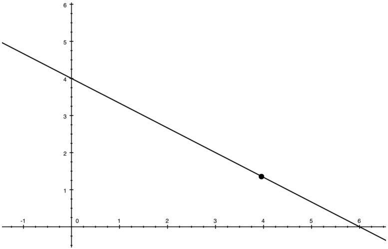

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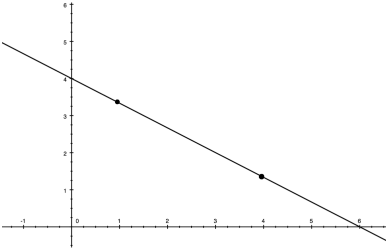

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1. Draw the line:
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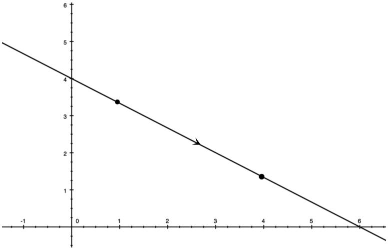
3. Move back 3 and up 2 to get P_1:
4. Since $O(-3, 2) < 0$, the increasing direction is P_1 to P_0.

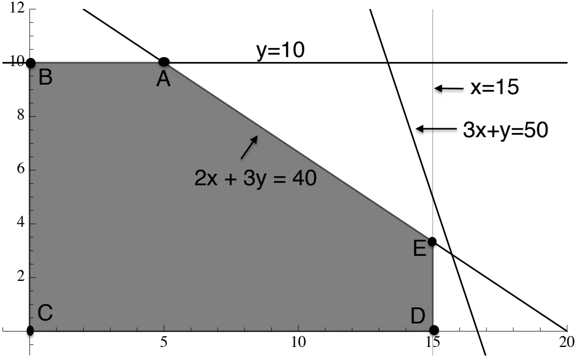


Example from above

We can summarize our problem from the previous example in the following form:
Find the maximum of the objective function $6x + 8y$ subject to the constraints

$$
\begin{array}{cc}
3x + y \leqslant 50, & 2x + 3y \leqslant 40, \\
x \leqslant 15, & y \leqslant 10, \\
x \geqslant 0, & y \geqslant 0.
\end{array}
$$

From the above theorems, we know that the maximum of $6x + 8y$ on the feasible set occurs at a corner of the feasible set since the feasible set is bounded (it may occur at more than one corner, but it occurs at at least one). We already have a picture of the feasible set and below, we have labelled the corners, A, B, C, D and E.



To find the maximum value of $6x + 8y$ and a point (x, y) in the feasible set at which it is achieved, we need only calculate the co-ordinates of the points A, B, C, D and E and compare the value of $6x + 8y$ at each.

Point	Coordinates	Value of $6x + 8y$
A		
B		
C	(0, 0)	0
D		
E		

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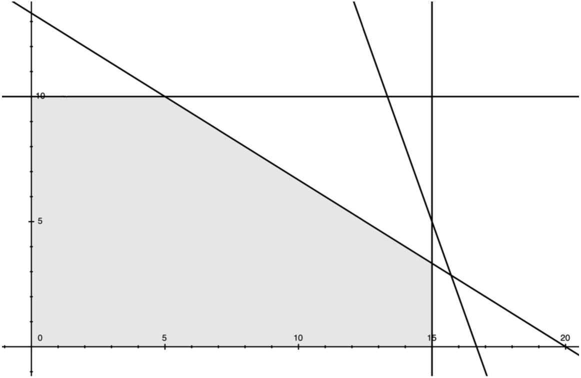
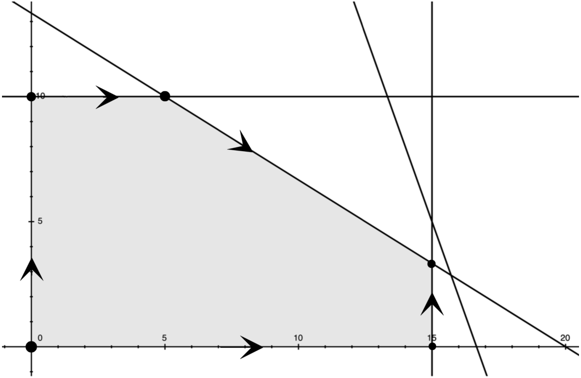
Point	Coordinates	Value of $6x + 8y$
A	(5,10)	110
B	(0,10)	80
C	(0, 0)	0
D	(15,0)	90
E	(15,10/3)	$\frac{350}{3}$

Hence E is the largest value (and C is the smallest).

If there are a lot of vertices, the method above can be tedious. Here is another approach. It has the additional advantage of working for non-bounded regions.
First draw the feasible region.



Then mark the vertices in the boundary of the feasible region and use the objective function $O(x, y) = 6x + 8y$ to draw the increasing directions on the boundary segments.

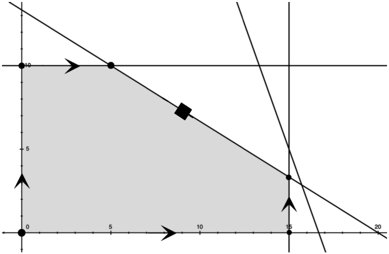


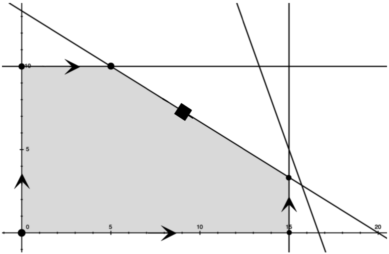
Start anywhere on the boundary you like and keep moving as long as you can go long a piece of line in the direction of the arrow. No matter where you start, you end up at vertex E where you are stuck. Hence the maximum value occurs at vertex E.

Now solve only for vertex E: $x = 15$, $2x + 3y = 40$ so
$30 + 3y = 40$ or $y = \dfrac{10}{3}$:
$O\left(15, \frac{10}{3}\right) = 6 \cdot 15 + 8 \cdot \frac{10}{3} = 90 + \frac{80}{3} = 116\frac{2}{3}$.
This is the correct solution to the formal optimization problem posed, but if we return to its roots we remember that we are selling drinks and we can not sell a partial drink. (If you are OK with selling partial drinks, rewrite the problem so the business makes TVs or cars.) Then the optimal point must have integer coordinates whereas
$\mathrm{E} = \left(15, \frac{10}{3}\right)$.
If this were my business, I'd make $(15, 3)$ drinks. If I were cautious I might note that $(14, 4)$ is in the feasible set and $O(15, 3) = 114$ and $O(14, 4) = 116$. Hence the actual maximum value is 116 and it occurs at the point $(14, 4)$.

In general, if there are integral constraints, the way to proceed is to solve the problem without worrying about this additional condition and then look at the nearby integer points to find the maximum.

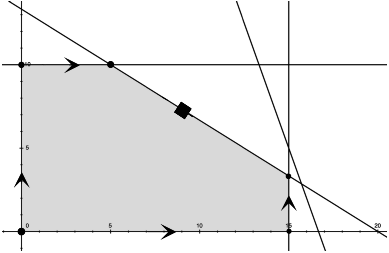
Here is an additional wrinkle which can occur. Use the same feasible set as above but use $O(x, y) = 6x + 9y$ for the objective function. This time the objective function is constant along the line $2x + 3y = 40$: $O(-3, 2) = 0$. This time we draw a solid square on the line $2x + 3y = 40$ to indicate that as we move along this line the objective function is constant.





Again we go around the boundary in the increasing direction except if we encounter a piece of boundary with a solid square label we can go along it any any direction we like. Again we end up at E but now the entire segment with the solid square consists of points where the objective function has value $O\left(15, \frac{10}{3}\right) = 6 \cdot 15 + 9 \cdot \frac{10}{3} = 120$. This time the maximum value is an integer but the point $E = \left(15, \frac{10}{3}\right)$ still does not have integer coordinates. This time $O(15, 3) = 6 \cdot 15 + 9 \cdot 3 = 117$ but $O(14, 4) = 6 \cdot 14 + 9 \cdot 4 = 120$ does have integer coordinates.

Suppose we want to minimize $O(x, y) = 6x + 9y$. We draw the same picture.



Again we go around the boundary but now in the decreasing direction except if we encounter a piece of boundary with a solid square label we can go along it any any direction we like. This time we end up at the origin.

Example Mr. Carter eats a mix of Cereal A and Cereal B for breakfast. The amount of calories, sodium and protein per ounce for each is shown in the table below. Mr. Carter's breakfast should provide at least 480 calories but less than or equal to 700 milligrams of sodium. Mr. Carter would like to maximize the amount of protein in his breakfast mix.

	Cereal A	**Cereal B**
Calories(per ounce)	100	140
Sodium(mg per ounce)	150	190
Protein(g per ounce)	9	10

Let x denote the number of ounces of Cereal A that Mr. Carter has for breakfast and let y denote the number of ounces of Cereal B that Mr. Carter has for breakfast. In the last lecture, we found that the set of constraints for x and y were

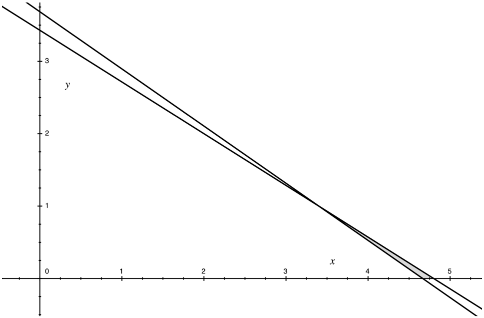
$$100x + 140y \geqslant 480, \quad 150x + 190y \leqslant 700, \quad x \geqslant 0, \quad y \geqslant 0$$

(a) What is the objective function?

$O(x, y) = 9x + 10y.$

(b) Graph the feasible set.

$$100x + 140y \geqslant 480, \quad 150x + 190y \leqslant 700, \quad x \geqslant 0, \quad y \geqslant 0$$



The feasible set is the skinny triangle just above the x-axis.

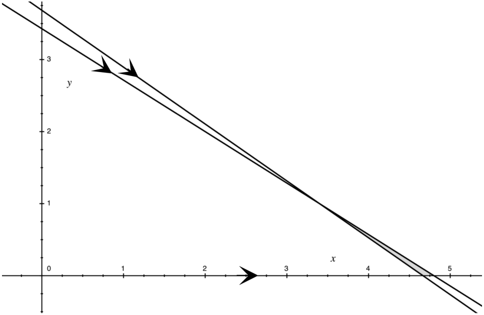
(c) Find the vertices of the feasible set and the maximum of the objective function on the feasible set.

The two vertices on the x-axis are $(4.8, 0)$ and $\left(\frac{16}{3}, 0\right)$. The intersection of $100x + 140y = 480$ and $150x + 190y = 700$ is the point $\left(\frac{17}{5}, 1\right)$. The values of the objective function are 42.2, 48 and $\frac{203}{5} = 40.6$. Hence the maximum of the objective function is 48 and it occurs at the point $\left(\frac{16}{3}, 0\right)$ on the boundary of the feasible region and nowhere else. The minimum occurs at 40.6 where the two lines intersect.

Using the other method:

$$100x + 140y \geqslant 480, \quad 150x + 190y \leqslant 700, \quad x \geqslant 0, \quad y \geqslant 0$$

$O(x, y) = 9x + 10y$. Here are the arrows.



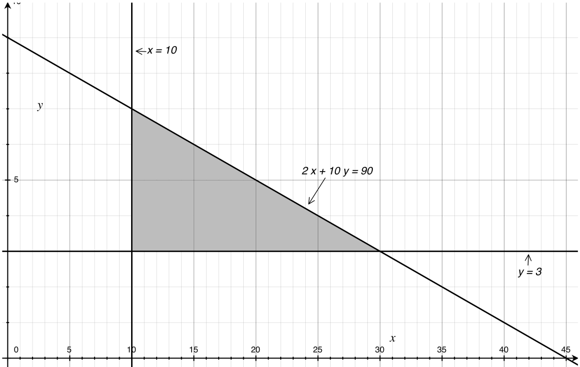
It is easy to see that the maximum value occurs at the far right vertex on the x-axis and the minimum occurs at the intersection of the two other marked lines.

This problem is a slight change in the feasible set from the previous version where Mr. Carter wanted less than 700 milligrams of sodium. In terms of the picture of the feasible set, the line $150x + 190y = 700$ was not in the feasible set in the previous lecture but it is here.
Linear programing problems where some of the lines are not in the feasible set are tricky. All you can do is first pretend that all the lines **are** in the feasible set, do the problem and if the solution is at a point where either (or both) lines are dotted, use your common sense.

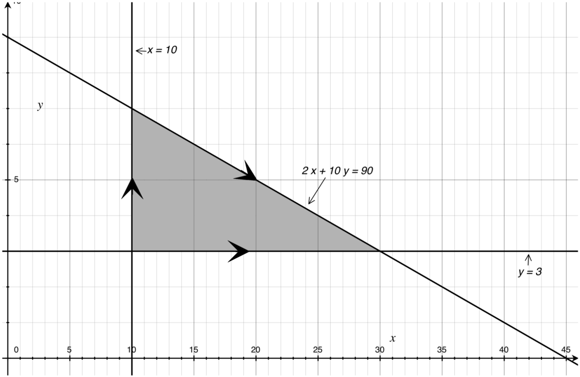
Example Michael is taking a timed exam in order to become a volunteer firefighter. The exam has 10 essay questions and 50 multiple choice questions. He has 90 minutes to take the exam and knows he cannot possibly answer every question. The essay questions are worth 20 points each and the short-answer questions are worth 5 points each. An essay question takes 10 minutes to answer and a shot-answer question takes 2 minutes. Michael must do at least 3 essay questions and at least 10 short-answer questions. Michael knows the material well enough to get full points on all questions he attempts and wants to maximize the number of points he will get. Let x denote the number of multiple choice questions that Michael will attempt and let y denote the number of essay questions that Michael will attempt. Write down the constraints and objective function in terms of x and y and find the/a combination of x and y which will allow Michael to gain the maximum number of points possible.

$2x + 10y \leqslant 90$ (time needed to answer the questions).
$x \geqslant 10$ (at least 10 short-answer questions).
$x \leqslant 50$ (at most 50 short-answer questions).
$y \geqslant 3$ (at least 3 essay questions).
$y \leqslant 10$ (at least 3 essay questions).
$5x + 20y$ is the objective function (Michael's total score).
Here is the feasible region.

The lines $x = 50$ and $y = 10$ are outside the part of the plane shown.



The three vertices are $(10, 3)$, $(10, 7)$ and $(30, 3)$. The values of the objective function are 60, 190 and 210. Hence Michael can maximize his score by answering 3 essay questions and 30 short-answer questions.
By adding the arrows to the boundary we can also see $(30, 3)$ is the point where the maximum value occurs.



Example with unbounded region A local politician is budgeting for her media campaign. She will distribute her funds between TV ads and radio ads. She has been given the following advice by her campaign advisers;

- She should run at least 120 TV ads and at least 30 radio ads.
- The number of TV ads she runs should be at least twice the number of radio ads she runs but not more than three times the number of radio ads she runs.

The cost of a TV ad is \$8000 and the cost of a radio ad is \$2000. Which combination of TV and radio ads should she choose to minimize the cost of her media campaign?

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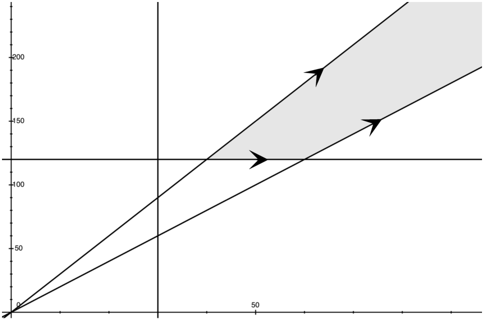
The cost of a TV ad is \$8000 and the cost of a radio ad is \$2000. Which combination of TV and radio ads should she choose to minimize the cost of her media campaign?

Let t be the number of TV ads and r the number of radio ads.

$$t \geqslant 120 \quad r \geqslant 30 \quad 2r \leqslant t \leqslant 3r$$

The objective function is $8000t + 2000r$ which she wishes to minimize.

Here is the feasible region which we see is unbounded.



Common sense: She can always increase her cost by buying more ads so there is no maximum value.
Mathematics: No matter where you start you can go along either $t = 3r$ or $t = 2r$ and the maximum value keeps getting bigger forever.

The minimum value occurs at the vertex where $t = 120$ and $t = 3r$ meet. This is the point $r = 40$, $t = 120$. Hence she should buy 40 radio ads and 120 TV ads.

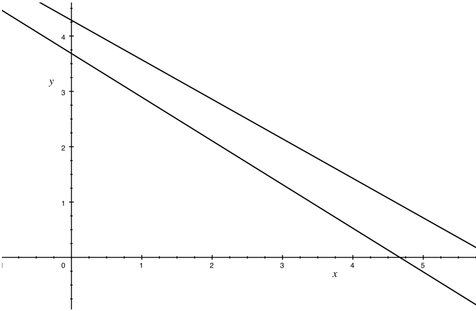
Example: No feasible region Mr. Baker eats a mix of Cereal A and Cereal B for breakfast. The amount of calories, sodium and protein per ounce for each is shown in the table below. Mr. Baker's breakfast should provide at least 600 calories but less than 700 milligrams of sodium. Mr. Baker would like to maximize the amount of protein in his breakfast mix.

	Cereal A	**Cereal B**
Calories(per ounce)	100	140
Sodium(mg per ounce)	150	190
Protein(g per ounce)	9	10

Let x denote the number of ounces of Cereal A that Mr. Baker has for breakfast and let y denote the number of ounces of Cereal B that Mr. Baker has for breakfast.

(a) Show that the feasible set is empty here, so that there is no feasible combination of the cereals for Mr. Baker's breakfast.

$100x + 140y \geqslant 600$ (calories)
$150x + 190y < 700$ (sodium)
$x \geqslant 0,\ y \geqslant 0$



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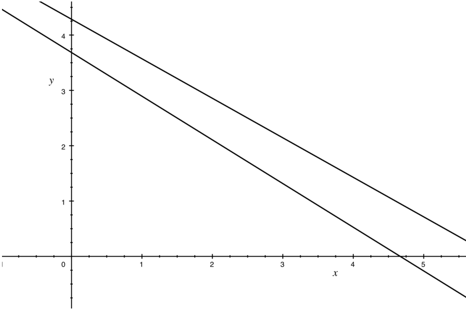

Because we have $x \geqslant 0$, $y \geqslant 0$ we need only look at regions in the first quadrant.

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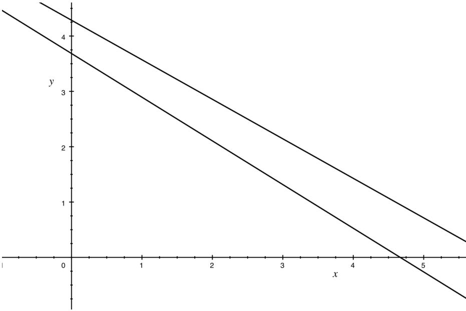

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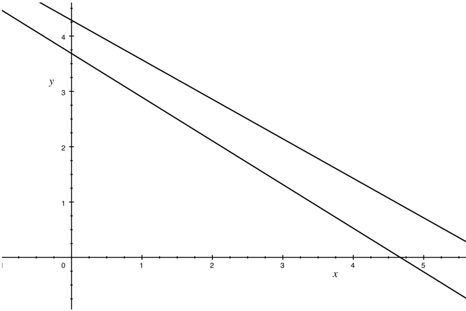

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Since $(0, 0)$ satisfies $100x + 140y < 600$, $\mathrm{P}_1 = \emptyset$.

(b) If Mr. Baker goes shopping for new cereals, what should he look for on the chart giving the nutritional value, so that he can have some feasible combination of the cereals for breakfast?

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This is an essay question with no single right answer. Basically Mr. Baker needs to choose cereals with either more calories or less sodium per ounce.