Name: **Version #1**

Instructor: _____

Math 10120, Final
December 18, 2014

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- Please turn =>off all cellphones and electronic devices.
- Calculators **are** allowed=>.
- The exam lasts for 2 hours.
- Be sure that your name and instructor's name are on the front page of your exam.
- Be sure that you have all 22 pages of the test.

PLEASE MARK YOUR ANSWERS WITH AN X, not a circle!

1.	(•)	(b)	(c)	(d)	(e)		17.	(•)	(b)	(c)	(d)	(e)
2.	(•)	(b)	(c)	(d)	(e)		18.	(•)	(b)	(c)	(d)	(e)

.... 2 .... 11 ....

| 3. | (•) | (b) | (c) | (d) | (e) | | 19. | (•) | (b) | (c) | (d) | (e) |
| 4. | (•) | (b) | (c) | (d) | (e) | | 20. | (•) | (b) | (c) | (d) | (e) |

.... 3 .... 12 ....

| 5. | (•) | (b) | (c) | (d) | (e) | | 21. | (•) | (b) | (c) | (d) | (e) |
| 6. | (•) | (b) | (c) | (d) | (e) | | 22. | (•) | (b) | (c) | (d) | (e) |

.... 4 .... 13 ....

| 7. | (•) | (b) | (c) | (d) | (e) | | 23. | (•) | (b) | (c) | (d) | (e) |
| 8. | (•) | (b) | (c) | (d) | (e) | |

.... 5 .... 14 ....

| 24. | (•) | (b) | (c) | (d) | (e) |

| 9. | (•) | (b) | (c) | (d) | (e) |
| 10. | (•) | (b) | (c) | (d) | (e) |

.... 6 .... 15 ....

| 25. | (•) | (b) | (c) | (d) | (e) |
| 26. | (•) | (b) | (c) | (d) | (e) |

| 11. | (•) | (b) | (c) | (d) | (e) |
| 12. | (•) | (b) | (c) | (d) | (e) |

.... 7 .... 16 ....

| 27. | (•) | (b) | (c) | (d) | (e) |
| 28. | (•) | (b) | (c) | (d) | (e) |

| 13. | (•) | (b) | (c) | (d) | (e) |

.... 8 .... 17 ....

| 29. | (•) | (b) | (c) | (d) | (e) |

| 14. | (•) | (b) | (c) | (d) | (e) | | 30. | (•) | (b) | (c) | (d) | (e) |

.... 9 .... 18 ....

| 15. | (•) | (b) | (c) | (d) | (e) |
| 16. | (•) | (b) | (c) | (d) | (e) |

.... 10 ....

Please do NOT write in this box.

Total _____

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| 2. | (a) | (b) | (c) | (d) | (e) |

| 3. | (a) | (b) | (c) | (d) | (e) |
| 4. | (a) | (b) | (c) | (d) | (e) |

| 5. | (a) | (b) | (c) | (d) | (e) |
| 6. | (a) | (b) | (c) | (d) | (e) |

| 7. | (a) | (b) | (c) | (d) | (e) |
| 8. | (a) | (b) | (c) | (d) | |

| 9. | (a) | (b) | (c) | (d) | (e) |
| 10. | (a) | (b) | (c) | (d) | (e) |

| 11. | (a) | (b) | (c) | (d) | (e) |
| 12. | (a) | (b) | (c) | (d) | (e) |

| 13. | (a) | (b) | (c) | (d) | (e) |

| 14. | (a) | (b) | (c) | (d) | (e) |

| 15. | (a) | (b) | (c) | (d) | (e) |
| 16. | (a) | (b) | (c) | (d) | (e) |

| 17. | (a) | (b) | (c) | (d) | (e) |
| 18. | (a) | (b) | (c) | (d) | (e) |

| 19. | (a) | (b) | (c) | (d) | (e) |
| 20. | (a) | (b) | (c) | (d) | (e) |

| 21. | (a) | (b) | (c) | (d) | (e) |
| 22. | (a) | (b) | (c) | (d) | (e) |

| 23. | (a) | (b) | (c) | (d) | (e) |

| 24. | (a) | (b) | (c) | (d) | (e) |

| 25. | (a) | (b) | (c) | (d) | (e) |
| 26. | (a) | (b) | (c) | (d) | (e) |

| 27. | (a) | (b) | (c) | (d) | (e) |
| 28. | (a) | (b) | (c) | (d) | (e) |

| 29. | (a) | (b) | (c) | (d) | (e) |
| 30. | (a) | (b) | (c) | (d) | (e) |

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Total _____

2.   Initials: _____

Note: In this exam P(*event*) denotes the probability of *event*; **Pr**(n, m) denotes permutations; and **C**(n, m) denotes combinations.

1.(5pts) What is the 1×3 entry in the product $\begin{bmatrix} 1 & 2 & 3 & 4 \end{bmatrix} \begin{bmatrix} 1 & -5 & 5 & 1 \\ 2 & 6 & -6 & 2 \\ 3 & -7 & 7 & 3 \\ 4 & -8 & -8 & 4 \end{bmatrix}$?

(a) -18 (b) 30   (c) -46 (d) 0   (e) 11

2.(5pts) Below is a payoff matrix for R. Which set of equations is the set of strategy lines for R?

	C1	C2	C3
R1	10	30	−20
R2	2	11	−4

(a) $y = 2 + 8p, \quad y = 11 + 19p, \quad y = -4 - 16p$

(b) $y = 10 - 2p, \quad y = 30 - 11p, \quad y = -20 + 4p$

(c) $y = 10 + 2p, \quad y = 30 + 11p, \quad y = -20 - 4p$

(d) $y = 2 - 8p, \quad y = 11 - 19p, \quad y = -4 + 16p$

(e) $y = -2 + 8p, \quad y = -11 + 19p, \quad y = 4 - 16p$

3. Initials: _____

3.(5pts) Find all the saddle points for the payoff matrix below.

	C1	C2	C3	C4
R1	1	2	3	4
R2	12	3	3	10
R3	3	2	3	4

(a) $(2, 2)$ and $(2, 3)$ (b) $(3, 1)$ and $(3, 2)$

(c) $(3, 1)$, $(2, 2)$, $(2, 3)$ and $(3, 3)$ (d) $(3, 1)$, $(2, 2)$

(e) There are no saddle points.

4.(5pts) If R plays mixed strategy $\begin{bmatrix} 0.1 & 0.3 & 0.6 \end{bmatrix}$ and C plays mixed strategy $\begin{bmatrix} 0.3 \\ 0.3 \\ 0.4 \end{bmatrix}$ what is

R's expected payoff if the payoff matrix is $\begin{bmatrix} 1 & 2 & 3 \\ 12 & 3 & 3 \\ 3 & 2 & 3 \end{bmatrix}$?

(a) 3.54   (b) 4.21   (c) 2.19   (d) 9.34   (e) 8.25

5.(5pts) How many distinct sequences can you make from the letters in Tennessee by rearranging them?

(a) 3, 780   (b) 362, 880   (c) 35, 231   (d) 1, 545   (e) 450

6.(5pts) The labels for multiple choice problems are traditionally selected from the universal set of all letters of the English alphabet. The set of labels for the multiple choice answers used in this test is $L = \{a, b, c, d, e\}$. If $V = \{a, e, i, o, u\}$, what is the set $L \cap V'$?

(a) $\{b, c, d\}$ (b) $\{a, e\}$ (c) $\{i, o, u\}$ (d) $\{c\}$ (e) $\emptyset$

7.(5pts) Suppose you wish to photograph 5 schoolchildren on a basketball team consisting of 9 children. You want to line the children up in a row and if Sally is in the picture, she insists on standing in the middle. How many ways can you line the children up for the photograph?

(a) 8, 400   (b) 1, 680   (c) 40, 320   (d) 18, 144   (e) 3, 024

8.(5pts) A student here at Notre Dame is doing a genetics project. She goes around to all 256 students in her dorm and records eye color: Blue, Brown, Gray, Hazel, or Green, and gets the following numbers:

Eye Color	Blue	Brown	Gray	Green	Hazel
# of Students	62	38	75	42	39

Which number below is the *relative frequency* of brown-eyed students in her dorm, rounded to 4 decimal places?

(a) 0.1484   (b) 38   (c) 0.1640   (d) 0.1743

(e) Can not be determined from the given information.

6.

9.(5pts) The test for a certain disease is 90% effective if you have the disease and 80% effective if you do not. (You will test positive 90% of the time if you have the disease and you will test negative 80% of the time if you do not.) It is estimated that 40% of the population has the disease. If a person chosen at random takes the test and it comes back negative, what is the probability that they actually do not have the disease?

(a) $\dfrac{12}{13}$ (b) $\dfrac{8}{9}$ (c) $\dfrac{4}{5}$ (d) $\dfrac{8}{11}$ (e) $\dfrac{1}{20}$

10.(5pts) Find the median of the following data set $\{\mathbf{1, 4, 7, 8, 10, 12, 13, 14, 15, 450}\}$.

(a) 11   (b) 53.5   (c) 10   (d) 12   (e) 51.2

11.(5pts) An experiment consists of flipping a fair coin 9 times and observing the sequence of heads and tails. If I perform this experiment, what is the probability that the resulting sequence will have four heads?

(a) $\dfrac{\mathbf{C}(9,4)}{2^9}$

(b) $\dfrac{\mathbf{Pr}(9,4)}{2^9}$

(c) $\dfrac{4}{2^9}$

(d) $\dfrac{\mathbf{Pr}(9,4) + \mathbf{Pr}(9,3) + \mathbf{Pr}(9,2) + \mathbf{Pr}(9,1) + \mathbf{Pr}(9,0)}{2^9}$

(e) $1 - \dfrac{\mathbf{C}(9,4) + \mathbf{C}(9,3) + \mathbf{C}(9,2) + \mathbf{C}(9,1) + \mathbf{C}(9,0)}{2^9}$

12.(5pts) A standardized exam to receive a wizard certificate has a mean of 600 with a standard deviation of 20. Ten thousand would-be wizards take this exam every year and magic insures that the scores have a normal distribution. The exam consists entirely of essay questions and so a score can be any positive number. The top 15% of applicants are awarded their certificates. What is the minimum score on the list below that is needed to achieve a certificate?

(a) 621   (b) 526   (c) 430   (d) 718   (e) 646

13. (5pts) Healthy Deli makes super-healthy soups by mixing three stocks. Stock A has 500 calories, 600 grams of protein and 90 mg of salt per cup. Stock B has 600 calories, 700 grams of protein and 50 mg of salt per cup. Stock C has 400 calories, 500 grams of protein and 70 mg of salt per cup. You need a cup of soup with at least 500 calories and at most 60 mg of salt. You want to maximize your protein. Which box below consists of constraints and the objective function for this problem if A, B and C denote the cups of each stock in your soup?

(a)
$$500A + 600B + 400C \geqslant 500$$
$$90A + 50B + 70C \leqslant 60$$
$$A \geqslant 0 \quad B \geqslant 0 \quad C \geqslant 0$$
objective function $\quad 600A + 700B + 500C$

(b)
$$500A + 600B + 400C \leqslant 500$$
$$90A + 50B + 70C \geqslant 60$$
$$A \geqslant 0 \quad B \geqslant 0 \quad C \geqslant 0$$
objective function $\quad 600A + 700B + 500C$

(c)
$$500A + 600B + 400C \leqslant 500$$
$$90A + 50B + 70C \leqslant 60$$
$$A \geqslant 0 \quad B \geqslant 0 \quad C \geqslant 0$$
objective function $\quad 600A + 700B + 500C$

(d)
$$500A + 600B + 400C \geqslant 500$$
$$90A + 50B + 70C \geqslant 60$$
$$A \geqslant 0 \quad B \geqslant 0 \quad C \geqslant 0$$
objective function $\quad 600A + 700B + 500C$

(e)
$$500A + 600B + 400C = 500$$
$$90A + 50B + 70C \geqslant 60$$
$$A \geqslant 0 \quad B \geqslant 0 \quad C \geqslant 0$$
objective function $\quad 600A + 700B + 500C$

9.

14. (5pts) Find the maximum value of $10x + 12y$ subject to the constraints
$$3x + 2y \geqslant 18, \quad x + 2y \leqslant 10, \quad 6x - 5y \geqslant 30, \quad x \geqslant 0, \quad y \geqslant 0.$$
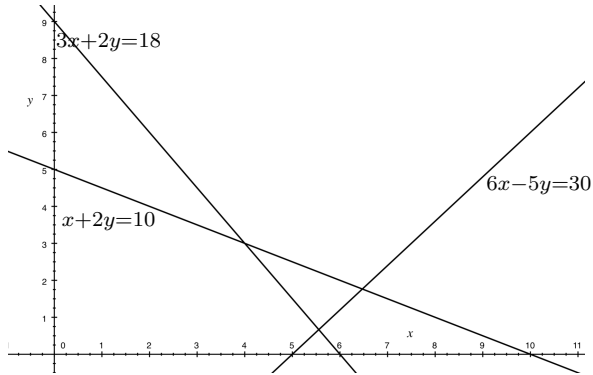
(a) 100   (b) 1140/17   (c) 200   (d) 111   (e) 570/9

15.(5pts) A random variable X has the following probability distribution:

X	P(X)
-10	1/3
0	1/3
1	1/6
2	1/6

What is the variance, $\sigma^2(X)$, of X to two decimal places?

(a) 26.14   (b) 35   (c) 16.85   (d) 3.33   (e) 19.87

16.(5pts) Ricardo and Carlo run hot dog stands on opposite sides of the same street at lunch hour. Each morning, both owners decide simultaneously and independently whether to set up their stand at intersection A, intersection B or intersection C. Both vendors are competing for the same set of customers each day.

- If both set up their stands at the same intersection, Ricardo gets three times as many customers as Carlo.
- If they set up their stands at different intersections, then
    - if one of the vendors is located at intersection A, that vendor gets 60% of the customers,
    - otherwise (if the vendors are located at intersections B and C) the customers are split equally between the vendors.

Which of the following gives the pay-off matrix for Ricardo (The Row Player) (where the pay-off for this constant sum game is the percentage of customers that go to Ricardo's stand)?

(a)

	A	B	C
A	75%	60%	60%
B	40%	75%	50%
C	40%	50%	75%

(b)

	A	B	C
A	75%	60%	50%
B	40%	75%	50%
C	50%	50%	75%

(c)

	A	B	C
A	75%	40%	40%
B	60%	75%	50%
C	60%	50%	75%

(d)

	A	B	C
A	66%	60%	60%
B	40%	66%	50%
C	40%	50%	66%

(e)

	A	B	C
A	60%	60%	60%
B	40%	60%	50%
C	40%	50%	60%

17. (5pts) Ragnar (R) and Count Odo (C) play a zero-sum game with payoff matrix for Ragnar given by

	C1	C2	C3	C4	C5
R1	2	5	−2	1	−3
R2	4	−6	2	5	−2
R3	1	−1	3	3	−2

If Ragnar always plays the pure strategy $R3$, what is the optimal counterstrategy for Count Odo?

(a) Always play C5   (b) Always play C1   (c) Always play C2

(d) Always play C3   (e) Always play C4

18. (5pts) Rubio (R) and Cruz (C) play a simultaneous move, zero sum game where the payoff matrix for Rubio is shown below.

	C1	C2
R1	2	5
R2	4	−6

What is Rubio's optimal strategy for this game?

Note: The formulas given at the end of the exam may help.

(a) $\begin{bmatrix} \frac{10}{13} & \frac{3}{13} \end{bmatrix}$ (b) $\begin{bmatrix} \frac{3}{13} & \frac{10}{13} \end{bmatrix}$ (c) $\begin{bmatrix} \frac{11}{13} & \frac{2}{13} \end{bmatrix}$ (d) $\begin{bmatrix} \frac{2}{13} & \frac{11}{13} \end{bmatrix}$ (e) $\begin{bmatrix} \frac{1}{5} & \frac{4}{5} \end{bmatrix}$

19.(5pts) In (a simplified game of) soccer, when a player takes a penalty kick, the goalie must anticipate the direction in which the ball will go in order to have a chance of stopping it, and the kicker must kick to the left or to the right. The payoff matrix for a particular goalie and penalty taker are shown below where the payoff shown is the probability of the kicker getting a goal in each situation.

		Goalie	
		Left	Right
	Left	0.4	0.7
Kicker			
	Right	0.6	0.5

Find the value of the game.

(a) 0.55   (b) 0.5   (c) 0.7   (d) 0.25   (e) 0.4

20.(5pts) The Skipping Club at Notre Dame wants to appoint a committee of three persons to arrange their monthly master classes. Three seniors and four juniors are eligible for a position on the committee. How many different committees can be formed which consist of one senior and two juniors?

(a) $\mathbf{C}(3,1)\mathbf{C}(4,2)$ (b) $\mathbf{Pr}(3,1)\mathbf{Pr}(4,2)$ (c) $\mathbf{C}(7,3) - 4$

(d) $\dfrac{\mathbf{C}(7,3)}{2!}$ (e) $\mathbf{C}(3,2)\mathbf{C}(4,1)$

21. (5pts) At Giovanni's Pizzeria you can choose from three different sizes for your pizza; small, medium or large. You can also choose from three styles of crust; thin, regular or stuffed. All pizza's have cheese and tomato sauce. You can choose any combination of toppings (including none) from the 10 different toppings available. How many different pizzas can be ordered from Giovanni's Pizzeria?

(a) 9,216   (b) 90   (c) 4,096   (d) 10,512   (e) 9

22. (5pts) An animal shelter has a group of 15 volunteers to help on Saturday. They wish to partition the volunteers into three groups; a group of 4 to bring some cats to the local retirement home, a group of 6 to clean the kennels and a group of 5 to walk the dogs. How many such unordered partitions of the volunteers are possible?

(a) $630, 630$ (b) $1, 365$ (c) $105, 105$ (d) $6, 534$ (e) $75, 675, 600$

23.(5pts) The following shows a street map of Jurassic Island. A Pterodactyl is on the loose and is sitting on top of a building at intersection P.
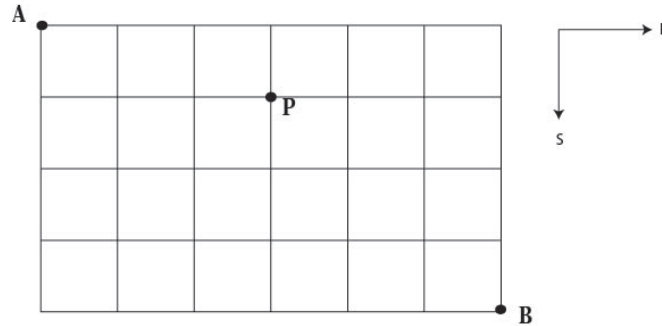
If you choose a route at random from A to B with no backtracking (always traveling South or East) what is the probability that you will go past the Pterodactyl at P?
(Round your answer to three decimal places.)

(a) 0.381   (b) 0.019   (c) 0.981   (d) 0.619   (e) 0.421

24.(5pts)   An experiment consists of rolling a pair of six-sided dice, one red and one green and observing the pair of numbers on the uppermost face. What's the probability that the numbers **do not** add up to 7?

(a) $\dfrac{5}{6}$ 　　　　　(b) $\dfrac{1}{6}$ 　　　　　(c) $\dfrac{1}{36}$ 　　　　　(d) $\dfrac{35}{36}$ 　　　　　(e) $\dfrac{29}{36}$

25. (5pts) The US Senate voted on a particular bill, for which the results are shown in the table (Abstain = did not vote). A Senator is selected at random (from the 100 Senators) and found to have voted "Yes". What's the probability that he/she is a democrat?

Affiliation	Yes	No	Abstain
Democrat	6	34	4
Republican	50	2	2
Independent	1	1	0

(a) $\dfrac{6}{57}$
(b) $\dfrac{6}{44}$
(c) $\dfrac{6}{100}$
(d) $\dfrac{44}{100}$
(e) $\dfrac{57}{100}$

26. (5pts) A basketball player has a 60% chance of making a basket each time she takes a shot from the free throw line. If she takes four independent shots from the free throw line, what's the probability that she makes a basket on at least one?

(a) 0.9744
(b) 0.0256
(c) 0.1296
(d) 0.8704
(e) 0.0384

27.(5pts) Ten percent of the new cars made by the Volksota car company will require engine repair in the first year after purchase, 20% of their new cars will require a software patch in the first year after purchase and 5% will require both in the first year after purchase. You have just bought a new Volksota car. What is the probability that your new car will require either engine repair or a software patch or both in the next year.

(a) 0.25   (b) 0.3   (c) 0.05   (d) 0.06   (e) 0.95

28.(5pts) In target practice, a pistol shooter has a 60% chance of hitting the target each time he shoots. If he takes six independent shots at practice, what is the probability that the number of times he hits the target is greater than the number of times he misses it? (Round your answer to 4 decimal places.)

(a) 0.5443   (b) 0.4557   (c) 0.2765   (d) 0.7235   (e) 0.6134

29.(5pts) Determine the expected value of the random variable X whose probability distribution is given below.

X	P(X)
0	0.2
1	0.4
2	0.3
3	0.1

(a) 1.3   (b) 2.1   (c) 1.5   (d) 1.9   (e) 3.1

30.(5pts) The number of completed passes made by a quarterback in 10 consecutive games this season is shown below.

$$10, \ 20, \ 15, \ 19, \ 20, \ 15, \ 20, \ 19, \ 13, \ 19$$

The sample average is $\bar{x} = 17$. Find the **sample** standard deviation (s) for the number of completions per game for the quarterback in question?
Round your answer to 4 decimal places.

(a) $s = 3.528$ (b) $s = 12.4444$ (c) $s = 11.2034$ (d) $s = 3.3535$ (e) $s = 4.5247$

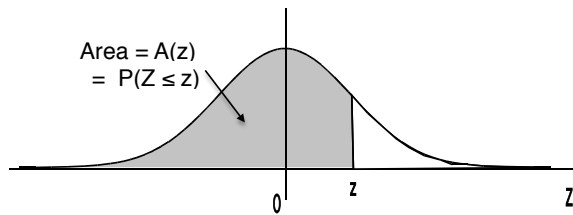For 2×2 payoff matrix

	C_1	C_2
R_1	a	b
R_2	c	d

$$p = \frac{d - c}{(a + d) - (b + c)}$$

$$q = \frac{d - b}{(a + d) - (b + c)}$$

$$\nu = \frac{ad - bc}{(a + d) - (b + c)}$$

Areas under the Standard Normal Curve

z	$A(z)$	z	$A(z)$	z	$A(z)$	z	$A(z)$	z	$A(z)$
-3.50	.0002	-2.00	.0228	$-.50$	.3085	1.00	.8413	2.50	.9938
-3.45	.0003	-1.95	.0256	$-.45$	.3264	1.05	.8531	2.55	.9946
-3.40	.0003	-1.90	.0287	$-.40$	.3446	1.10	.8643	2.60	.9953
-3.35	.0004	-1.85	.0322	$-.35$	.3632	1.15	.8749	2.65	.9960
-3.30	.0005	-1.80	.0359	$-.30$	.3821	1.20	.8849	2.70	.9965
-3.25	.0006	-1.75	.0401	$-.25$	.4013	1.25	.8944	2.75	.9970
-3.20	.0007	-1.70	.0446	$-.20$	.4207	1.30	.9032	2.80	.9974
-3.15	.0008	-1.65	.0495	$-.15$	.4404	1.35	.9115	2.85	.9978
-3.10	.0010	-1.60	.0548	$-.10$	.4602	1.40	.9192	2.90	.9981
-3.05	.0011	-1.55	.0606	$-.05$	.4801	1.45	.9265	2.95	.9984
-3.00	.0013	-1.50	.0668	.00	.5000	1.50	.9332	3.00	.9987
-2.95	.0016	-1.45	.0735	.05	.5199	1.55	.9394	3.05	.9989
-2.90	.0019	-1.40	.0808	.10	.5398	1.60	.9452	3.10	.9990
-2.85	.0022	-1.35	.0885	.15	.5596	1.65	.9505	3.15	.9992
-2.80	.0026	-1.30	.0968	.20	.5793	1.70	.9554	3.20	.9993
-2.75	.0030	-1.25	.1056	.25	.5987	1.75	.9599	3.25	.9994
-2.70	.0035	-1.20	.1151	.30	.6179	1.80	.9641	3.30	.9995
-2.65	.0040	-1.15	.1251	.35	.6368	1.85	.9678	3.35	.9996
-2.60	.0047	-1.10	.1357	.40	.6554	1.90	.9713	3.40	.9997
-2.55	.0054	-1.05	.1469	.45	.6736	1.95	.9744	3.45	.9997
-2.50	.0062	-1.00	.1587	.50	.6915	2.00	.9772	3.50	.9998
-2.45	.0071	$-.95$	.1711	.55	.7088	2.05	.9798		
-2.40	.0082	$-.90$	.1841	.60	.7257	2.10	.9821		
-2.35	.0094	$-.85$	.1977	.65	.7422	2.15	.9842		
-2.30	.0107	$-.80$	.2119	.70	.7580	2.20	.9861		
-2.25	.0122	$-.75$	.2266	.75	.7734	2.25	.9878		
-2.20	.0139	$-.70$	.2420	.80	.7881	2.30	.9893		
-2.15	.0158	$-.65$	.2578	.85	.8023	2.35	.9906		
-2.10	.0179	$-.60$	.2743	.90	.8159	2.40	.9918		
-2.05	.0202	$-.55$	.2912	.95	.8289	2.45	.9929		