Name: ______________________________

Instructor: __________________________

Math 10120, Final
December 18, 2014

- The Honor Code is in effect for this examination. All work is to be your own.
  Honor Pledge: As a member of the Notre Dame community,
  I will not participate in nor tolerate academic dishonesty.

  Signature:____

- Please turn off all cellphones and electronic devices.
- Calculators **are** allowed.
- The exam lasts for 2 hours.
- Be sure that your name and instructor's name are on the front page of your exam.
- Be sure that you have all 21 pages of the test.

PLEASE MARK YOUR ANSWERS WITH AN X, not a circle!

1. (a) (b) (c) (d) (e)
2. (a) (b) (c) (d) (e)

.... 2 ....

3. (a) (b) (c) (d) (e)
4. (a) (b) (c) (d) (e)

.... 3 ....

5. (a) (b) (c) (d) (e)
6. (a) (b) (c) (d) (e)

.... 4 ....

7. (a) (b) (c) (d) (e)
8. (a) (b) (c) (d) (e)

.... 5 ....

9. (a) (b) (c) (d) (e)
10. (a) (b) (c) (d) (e)

.... 6 ....

11. (a) (b) (c) (d) (e)
12. (a) (b) (c) (d) (e)

.... 7 ....

13. (a) (b) (c) (d) (e)

.... 8 ....

14. (a) (b) (c) (d) (e)

.... 9 ....

15. (a) (b) (c) (d) (e)

.... 10 ....

16. (a) (b) (c) (d) (e)

.... 11 ....

17. (a) (b) (c) (d) (e)
18. (a) (b) (c) (d) (e)

.... 12 ....

19. (a) (b) (c) (d) (e)
20. (a) (b) (c) (d) (e)

.... 13 ....

21. (a) (b) (c) (d) (e)
22. (a) (b) (c) (d) (e)

.... 14 ....

23. (a) (b) (c) (d) (e)

.... 15 ....

24. (a) (b) (c) (d) (e)

.... 16 ....

25. (a) (b) (c) (d) (e)
26. (a) (b) (c) (d) (e)

.... 17 ....

27. (a) (b) (c) (d) (e)
28. (a) (b) (c) (d) (e)

.... 18 ....

29. (a) (b) (c) (d) (e)
30. (a) (b) (c) (d) (e)

.... 19 ....

Please do NOT write in this box.

Total ____________________________

1.(5pts) What is the 1×3 entry in the product $\begin{bmatrix} 1 & 2 & 3 & 4 \end{bmatrix} \begin{bmatrix} 1 & -5 & 5 & 1 \\ 2 & 6 & -6 & 2 \\ 3 & -7 & 7 & 3 \\ 4 & -8 & -8 & 4 \end{bmatrix}$?

(a) -18 (b) 30 (c) -46 (d) 0 (e) 11

Solution.

$$\begin{bmatrix} 1 & 2 & 3 & 4 \end{bmatrix} \begin{bmatrix} 1 & -5 & 5 & 1 \\ 2 & 6 & -6 & 2 \\ 3 & -7 & 7 & 3 \\ 4 & -8 & -8 & 4 \end{bmatrix} = \begin{bmatrix} 30 & -46 & \boxed{-18} & 30 \end{bmatrix}$$

OR more quickly

$$\begin{bmatrix} 1 & 2 & 3 & 4 \end{bmatrix} \begin{bmatrix} 5 \\ -6 \\ 7 \\ -8 \end{bmatrix} = [-18]$$

2.(5pts) Below is a pay off matrix. Which set of equations is the set of strategy lines for R?

	$C1$	$C2$	$C3$
$R1$	10	30	-20
$R2$	2	11	-4

(a) $q = 2 + 8p \quad q = 11 + 19p \quad q = -4 - 16p$

(b) $q = 10 - 2p \quad q = 30 - 11p \quad q = -20 + 4p$

(c) $q = 10 + 2p \quad q = 30 + 11p \quad q = -20 - 4p$

(d) $q = 2 - 8p \quad q = 11 - 19p \quad q = -4 + 16p$

(e) $q = -2 + 8p \quad q = -11 + 19p \quad q = 4 - 16p$

Solution. $[p \quad 1-p] \begin{bmatrix} 10 & 30 & -20 \\ 2 & 11 & -4 \end{bmatrix} = [10p+2(1-p) \quad 30p+11(1-p) \quad -20p+(-4)(1-p)] =$
$[2 + 8p \quad 11 + 19p \quad -4 - 16p]$

3.(5pts) Find all the saddle points for the pay off matrix below.

	$C1$	$C2$	$C3$	$C4$
$R1$	1	2	3	4
$R2$	12	3	3	10
$R3$	3	2	3	4

(a) $(2,2)$ and $(2,3)$

(b) $(3,1)$ and $(3,2)$

(c) $(3,1)$, $(2,2)$, $(2,3)$ and $(3,3)$

(d) $(3,1)$, $(2,2)$

(e) There are no saddle points.

Solution.

	$C1$	$C2$	$C3$	$C4$	min
$R1$	1	2	3	4	1
$R2$	12	$\boxed{3}$	$\boxed{3}$	10	**3**
$R3$	3	2	3	4	2
max	12	**3**	**3**	10	

4.(5pts) If R plays mixed strategy $[0.1 \quad 0.3 \quad 0.6]$ and C plays mixed strategy $\begin{bmatrix} 0.3 \\ 0.3 \\ 0.4 \end{bmatrix}$ what is R's expected pay off if the pay off matrix is $\begin{bmatrix} 1 & 2 & 3 \\ 12 & 3 & 3 \\ 3 & 2 & 3 \end{bmatrix}$?

(a) 3.54 (b) 4.21 (c) 2.19 (d) 9.34 (e) 8.25

Solution. $[0.1 \quad 0.3 \quad 0.6] \begin{bmatrix} 1 & 2 & 3 \\ 12 & 3 & 3 \\ 3 & 2 & 3 \end{bmatrix} \begin{bmatrix} 0.3 \\ 0.3 \\ 0.4 \end{bmatrix} = [0.1 \quad 0.3 \quad 0.6] \begin{bmatrix} 2.1 \\ 5.7 \\ 2.7 \end{bmatrix} = [3.54]$

5.(5pts) How many distinct sequences can you make from the letters in Tennessee by rearranging them?

(a) $3,780$ (b) $362,880$ (c) $35,231$ (d) $1,545$ (e) 450

Solution. Here is the letter count: T $\mapsto$ 1; e $\mapsto$ 4; n $\mapsto$ 2; s $\mapsto$ 2. Next note $1+4+2+2=9$. Hence the answer is $\dfrac{9!}{1!\cdot 4!\cdot 2!\cdot 2!} = 34,650$

6.(5pts) Suppose the universal set for a problem is the set of letters of the English alphabet. The set of labels for the multiple choice answers if $L = \{a, b, c, d, e\}$. If $V = \{a, e, i, o, u\}$, what is the set Then $L \cap V' =$?

(a) $\{b, c, d\}$ (b) $\{a, e\}$ (c) $\{i, o, u\}$ (d) $\{c\}$ (e) $\emptyset$

Solution. V' consists of all the letters in the alphabet except for the vowels. Hence $L \cap V'$ consists of the non-vowels in L, namely $\{b, c, d\}$.

7.(5pts) Suppose you wish to photograph 5 schoolchildren on a basketball team consisting of 9 children. You want to line the children up in a row and if Sally is in the picture, she insists on standing in the middle. How many ways can you line the children up for the photograph?

(a) $2,016$ (b) $1,680$ (c) $40,320$ (d) $18,144$ (e) $3,024$

Solution. There are 2 distinct possibilities, Sally is in the picture or Sally not in the picture. If Sally is in the picture, there are 8 children remaining so if Sally is in the picture there are $\Pr(8,4) = 8 \cdot 7 \cdot 6 \cdot 5 = 1,680$. If Sally is not in the picture then the answer is $\Pr(8,5) = 8 \cdot 7 \cdot 6 = 336$. Hence the answer is

$$\Pr(8,4) + \Pr(8,5) = 1,680 + 336 = 2,016 \ .$$

8.(5pts) A student at here at Notre Dame is doing a genetics project. She goes around to all 256 students in her dorm and records eye color: Blue, Brown, Gray, Hazel, or Green, and gets the following numbers:

Eye Color	Blue	Brown	Gray	Green	Hazel
# of Students	62	38	75	42	39

Which number below is the *relative frequency* of brown-eyed students in her dorm, rounded to 4 decimal places?

(a) 0.1484 (b) 38 (c) 0.1640 (d) 0.1743

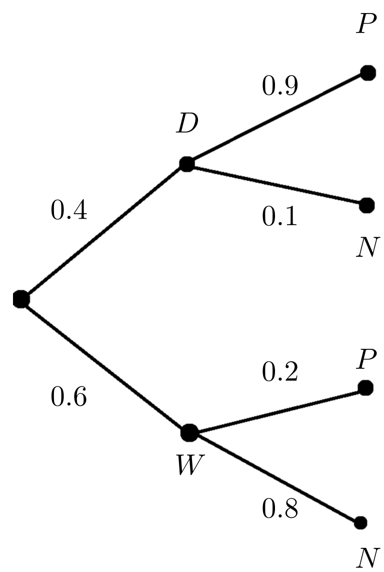
(e) Can not be determined from the given information.

Solution. The relative frequency is the number of occurrences of brown eyes divided by the total number of students: hence $\dfrac{38}{256} \approx 0.1484375$.

9.(5pts) The test for a certain disease is 90% effective if you have the disease and 80% effective if you do not. (You will test positive 90% of the time if you have the disease and you will test negative 80% of the time if you do not.) It is estimated that 40% of the population has the disease. If a person takes the test and it comes back negative, what is the probability that they are actually do not have the disease?

(a) $\frac{12}{13}$ (b) $\frac{8}{9}$ (c) $\frac{4}{5}$ (d) $\frac{8}{11}$ (e) $\frac{8}{17}$

Solution. Let D be the set of people who have the disease and let W denote the set of people who do not have the disease. Let P be the set of people who test positive and N the set of people who test negative.



$$\frac{0.6 \cdot 0.8}{0.6 \cdot 0.8 + 0.4 \cdot 0.1} = \frac{0.48}{0.48 + 0.04} = \frac{48}{52} = \frac{12}{13}.$$

10.(5pts) Find the median of the following data set $\{\mathbf{1, 4, 7, 8, 10, 12, 13, 14, 15, 450}\}$.

(a) 11 (b) 53.5 (c) 10 (d) 12 (e) 51.2

Solution. There are 10 elements so we need to count $5 = \frac{10}{2}$ from both the left, where we get 10, and the right, where we get 12, so the median is $\frac{10+12}{2} = 11$.

11.(5pts) An experiment consists of flipping a fair coin 9 times and observing the sequence of heads and tails. If I perform this experiment, what is the probability that the resulting sequence will have four heads?

(a) $\dfrac{\mathbf{C}(9,4)}{2^9}$ (b) $\dfrac{\mathrm{P}(9,4)}{2^9}$

(c) $\dfrac{4}{2^9}$ (d) $\dfrac{\mathrm{P}(9,4)+\mathrm{P}(9,3)+\mathrm{P}(9,2)+\mathrm{P}(9,1)+\mathrm{P}(9,0)}{2^9}$

(e) $1-\dfrac{\mathbf{C}(9,4)+\mathbf{C}(9,3)+\mathbf{C}(9,2)+\mathbf{C}(9,1)+\mathbf{C}(9,0)}{2^9}$

Solution. The number of sequences in the sample space is 2^9. There are $\mathbf{C}(9,4)=126$ different locations for the four heads. Hence the answer is $\dfrac{126}{2^9}=0.24609375$.

12.(5pts) A standardized exam to receive a wizard certificate has a mean of 600 with a standard deviation of 20. Ten thousand would-be wizards take this exam every year and magic insures that the scores have a normal distribution. The top 15% of applicants are awarded their certificates. What is the minimum score on the list below that is needed to achieve a certificate?

(a) 621 (b) 526 (c) 430 (d) 718 (e) 646

Solution. You need a such that $\Pr(X \leqslant a) \geqslant 0.85$. In other words, if you get a or higher, you are better than 85% of the people taking the exam.

Converting to Z-scores $\Pr(X \geqslant a) = \Pr\left(Z \geqslant \dfrac{a-600}{20}\right) \geqslant 0.85$. From the table, $\Pr(Z \leqslant 1.05) = 0.8531$. Hence if your Z-score is greater than 1.05 you are in.

But $\dfrac{a-600}{20} = 1.05$ so $a = 600 + 20\cdot 1.05 = 621$.

13.(5pts) Healthy Deli makes super-healthy soups by mixing three stocks. Stock A has 500 calories, 600 grams of protein and 90 mg of salt per cup. Stock B has 600 calories, 700 grams of protein and 50 mg of salt per cup. Stock C has 400 calories, 500 grams of protein and 70 mg of salt per cup. You need a cup of soup with at least 500 calories and at most 60 mg of salt. You want to maximize your protein. Write down the constraints and the objective function for this problem if A, B and C denote the cups of each stock in your soup.

(a)
$$\boxed{\begin{array}{l} 500A + 600B + 400C \geqslant 500 \\ \ 90A + \ 50B + \ 70C \leqslant \ 60 \\ A \geqslant 0 \qquad B \geqslant 0 \qquad C \geqslant 0 \\ \text{objective function} \quad 600A + 700B + 500C \end{array}}$$

(b)
$$\boxed{\begin{array}{l} 500A + 600B + 400C \leqslant 500 \\ \ 90A + \ 50B + \ 70C \geqslant \ 60 \\ A \geqslant 0 \qquad B \geqslant 0 \qquad C \geqslant 0 \\ \text{objective function} \quad 600A + 700B + 500C \end{array}}$$

(c)
$$\boxed{\begin{array}{l} 500A + 600B + 400C \leqslant 500 \\ \ 90A + \ 50B + \ 70C \leqslant \ 60 \\ A \geqslant 0 \qquad B \geqslant 0 \qquad C \geqslant 0 \\ \text{objective function} \quad 600A + 700B + 500C \end{array}}$$

(d)
$$\boxed{\begin{array}{l} 500A + 600B + 400C \geqslant 500 \\ \ 90A + \ 50B + \ 70C \geqslant \ 60 \\ A \geqslant 0 \qquad B \geqslant 0 \qquad C \geqslant 0 \\ \text{objective function} \quad 600A + 700B + 500C \end{array}}$$

(e)
$$\boxed{\begin{array}{l} 500A + 600B + 400C = 500 \\ \ 90A + \ 50B + \ 70C \geqslant \ 60 \\ A \geqslant 0 \qquad B \geqslant 0 \qquad C \geqslant 0 \\ \text{objective function} \quad 600A + 700B + 500C \end{array}}$$

Solution. $500A + 600B + 400C \geqslant 500$ is the constraint on the calories.

$90A + 50B + 70C \leqslant 60$ is the constraint on the salt.

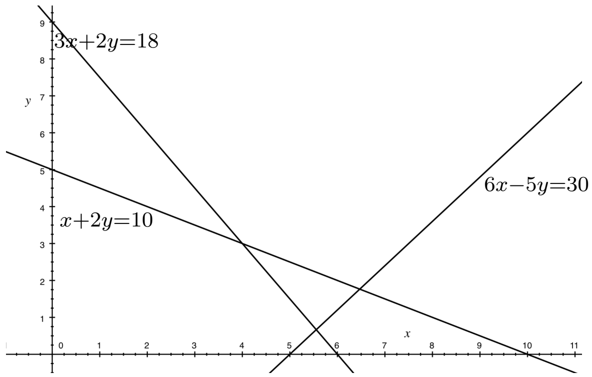
The non-negative constraints come from common sense.

The total amount of protein consumed is $600A + 700B + 500C$ and so this is the function that we wish to maximize.

Initials: ____________

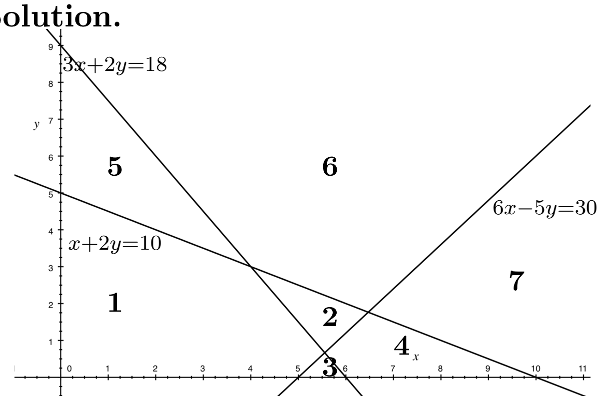
14.(5pts) Find the maximum value of $10x + 12y$ subject to the constraints

$$3x + 2y \geqslant 18, \quad x + 2y \leqslant 10, \quad 6x - 5y \geqslant 30, \quad x \geqslant 0, \quad y \geqslant 0.$$



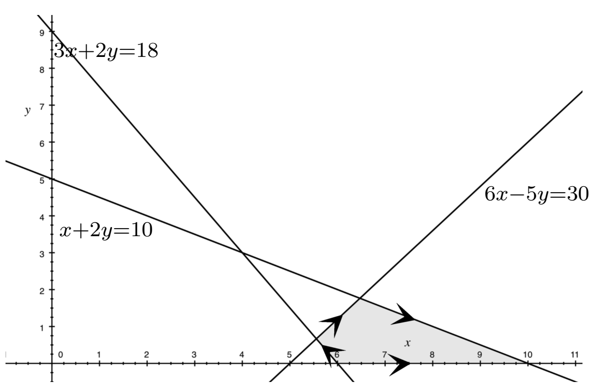
(a) 100 (b) 1140/17 (c) 200 (d) 111 (e) 570/9

Solution.



At $(x, y) = (0, 0)$, $3x+2y = 0 < 18$; so **2**, **4**, **6** & **7**.

At $(x, y) = (0, 0)$, $x + 2y = 0 < 10$; so **4**.



The gray region is the feasible set. The objective function is $O(x, y) = 10x + 12y$.
On $3x + 2y = 18$,
$O(-2, 3) = 16 > 0$ so the arrow points as shown.
On $x + 2y = 10$,
$O(-2, 1) = -8 < 0$ so the arrow points as shown.
On $6x - 5y = 30$,
$O(5, 6) = 112 > 0$ so the arrow points as shown.

Hence the maximum occurs at the vertex $(10, 0)$ where the objective function has value 100.

OR you can check the value at the vertices. One of them is $(6, 0)$ and $O(6, 0) = 60$.

$6x - 5y = 30$, $3x + 2y = 18$; $6x + 4y = 36$; $9y = 6$, $y = 2/3$; $3x + 4/3 = 18$; $(50/9, 2/3)$ so $O\,(50/9, 2/3) = 500/9 + 24/3 = 572/9 \approx 63.6 < 64$.

$6x - 5y = 30$, $x + 2y = 10$; $6x + 12y = 60$; $17x = 30$, $x = 30/17$; $30/17 + 2y = 10$; $2y = 140/17$, $y = 70/17$ and $O(30/17, 70/17) = (300 + 840)/17 = 1140/17 \approx 67.0588235294 < 68$.

15.(5pts) A random variable X has the following probability distribution:

X	Pr(X)
-10	1/3
0	1/3
1	1/6
2	1/6

What is the variance, $\sigma^2(X)$, of X to two decimal places?

(a) 26.14 (b) 35 (c) 16.85 (d) 3.33 (e) 19.87

Solution. The expected value of X is the sum of $X\Pr(X)$.

X	Pr(X)	X Pr(X)	$X(X\Pr(X))$
-10	1/3	$-10/3$	100/3
0	1/3	0	0
1	1/6	1/6	1/6
2	1/6	2/6	4/6
		$E(X) = -17/6$	$E(X^2) = 205/6$

$$\sigma^2(X) = E(X^2) - \big(E(X)\big)^2 = \frac{941}{36} \approx 26.1388888889.$$

16.(5pts) Ricardo and Carlo run hot dog stands on opposite sides of the same street at lunch hour. Each morning, both owners decide simultaneously and independently whether to set up their stand at intersection A, intersection B or intersection C. Both vendors are competing for the same set of customers each day.

- If both set up their stands at the same intersection, Ricardo gets three times as many customers as Carlo.
- If they set up their stands at different intersections, then
  - if one of the vendors is located at intersection A, that vendor gets 60% of the customers,
  - otherwise (if the vendors are located at intersections B and C) the customers are split equally between the vendors.

Which of the following gives the pay-off matrix for Ricardo (The Row Player) (where the pay-off for this constant sum game is the percentage of customers that go to Ricardo's stand)?

(a)

	A	B	C
A	75%	60%	60%
B	40%	75%	50%
C	40%	50%	75%

(b)

	A	B	C
A	75%	60%	50%
B	40%	75%	50%
C	50%	50%	75%

(c)

	A	B	C
A	75%	40%	40%
B	60%	75%	50%
C	60%	50%	75%

(d)

	A	B	C
A	66%	60%	60%
B	40%	66%	50%
C	40%	50%	66%

(e)

	A	B	C
A	60%	60%	60%
B	40%	60%	50%
C	40%	50%	60%

Solution. If both are at the same intersection and Ricardo gets 3 times and many customers as Carlo, who gets x% of the customers, then x + 3x = 100 and x = 25, 3x = 75. Therefore the diagonal entries should be 75%. If Ricardo is at A and Carlo is elsewhere thaen Ricardo's payoff is 60%. Thus the first row must be 75%, 60%, 60%. Likewise if Carlo is at A and Ricardo is elsewhere, Ricardo gets 40% of the customers. This tells us that the (2,1) and (3,1) entries are 40% and this is enough to distinguish the solution from the others:

	A	*B*	*C*
A	75%	60%	60%
B	40%	75%	50%
C	40%	50%	75%

17.(5pts) Ragnar (R) and Count Odo (C) play a zero-sum game with payoff matrix for Ragnar given by

	$C1$	$C2$	$C3$	$C4$	$C5$
$R1$	2	5	-2	1	-3
$R2$	4	-6	2	5	-2
$R3$	1	-1	3	3	-2

If Ragnar always plays the pure strategy $R3$, what is optimal counterstrategy for Count Odo?

(a) Always play C5

(b) Always play C1

(c) Always play C2

(d) Always play C3

(e) Always play C4

Solution. If Ragnar always plays the pure strategy $R3$, then Count Odo minimizes Ragnar's payoff by playing C5 on every play.

18.(5pts) Rubio (R) and Cruz (C) play a simultaneous move, zero sum game where the payoff matrix for Rubio is shown below.

	$C1$	$C2$
$R1$	2	5
$R2$	4	-6

What is Rubio's optimal strategy for this game?
Note: The formulas given at the end of the exam may help.

(a) $\begin{bmatrix} \frac{10}{13} & \frac{3}{13} \end{bmatrix}$ (b) $\begin{bmatrix} \frac{3}{13} & \frac{10}{13} \end{bmatrix}$ (c) $\begin{bmatrix} \frac{11}{13} & \frac{2}{13} \end{bmatrix}$ (d) $\begin{bmatrix} \frac{2}{13} & \frac{11}{13} \end{bmatrix}$ (e) $\begin{bmatrix} \frac{1}{5} & \frac{4}{5} \end{bmatrix}$

Solution. The optimal strategy for the row player is $\begin{bmatrix} p & 1-p \end{bmatrix}$ where

$$p = \frac{-6-4}{(2-6)-(4+5)} = \frac{10}{13}.$$

Therefore the optimal strategy is

$$\begin{bmatrix} \frac{10}{13} & \frac{3}{13} \end{bmatrix}$$

19.(5pts) In (a simplified game of)soccer, when a player takes a penalty kick, the goalie must anticipate the direction in which the ball will go in order to have a chance of stopping it, and the kicker must kick to the left or to the right. The payoff matrix for a particular goalie and penalty taker are shown below where the payoff shown is the probability of the kicker getting a goal in each situation.

		Goalie Left	Goalie Right
Kicker	Left	0.4	0.7
Kicker	Right	0.6	0.5

Find the value of the game.

(a) 0.55 (b) 0.5 (c) 0.7 (d) 0.25 (e) 0.4

Solution. The value of the game is $\mu = \dfrac{(0.4)(0.5) - (0.6)(0.7)}{-0.4} = \dfrac{-0.22}{-0.4} = 0.55.$

20.(5pts) The Skipping Club at Notre Dame wants to appoint a committee of three persons to arrange their monthly master classes. Three seniors and four juniors are eligible for a position on the committee. How many different committees can be formed which consist of one senior and two juniors?

(a) $C(3,1)C(4,2)$ (b) $P(3,1)P(4,2)$

(c) $C(7,3) - 4$ (d) $\dfrac{C(7,3)}{2!}$

(e) $C(3,2)C(4,1)$

Solution. One can break the problem of choosing such a committee into steps:
Step 1: Choose a senior ($C(3,1)$ ways)
Step 2: Choose two juniors ($C(4,2)$ ways)
Using the multiplication principle, we see that the task of forming the committee can be completed in $C(3,1)C(4,2)$ ways.

21.(5pts) At Giovanni's Pizzeria you can choose from three different sizes for your pizza; small, medium or large. You can also choose from three styles of crust; thin, regular or stuffed. All pizza's have cheese and tomato sauce. You can choose and combination of toppings (including none) from the 10 different toppings available. How many different pizzas can be ordered from Giovanni's Pizzeria?

(a) 9,216 (b) 90 (c) 4,096 (d) 10,512 (e) 9

Solution. You can break the task of ordering the pizza into steps:
Step 1: Choose a size (3 choices)
Step 2: Choose a crust (3 choices)
Step 3: Choose a subset of toppings ($2^{10} = 1,024$ choices)
Using the multiplication principle we see that the number of ways of completing the task is

$$3 \cdot 3 \cdot 1,024 = 9,216.$$

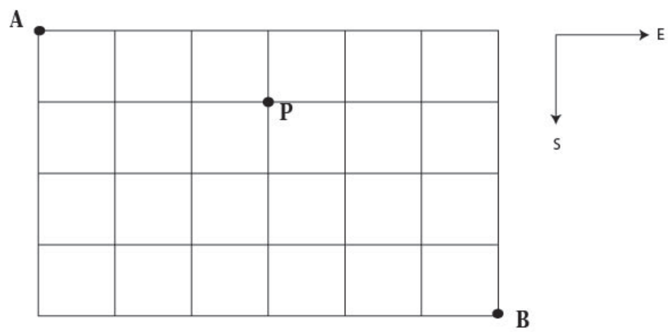
22.(5pts) An animal shelter has a group of 15 volunteers to help on Saturday. They wish to partition the volunteers into three groups; a group of 4 to bring some cats to the local retirement home, a group of 6 to clean the kennels and a group of 5 to walk the dogs. How many such ordered partitions of the volunteers are possible?

(a) $630,630$ (b) $1,365$ (c) $105,105$ (d) $6,534$ (e) $75,675,600$

Solution. The number of ways to create an unordered partition of a group 15 volunteers into groups of 4, 5, and 6 is

$$\frac{15!}{4! \cdot 5! \cdot 6!} = 630,630 = C(15,4)C(11,5)C(6,6).$$

23.(5pts) The following shows a street map of Jurassic Island. A Pterodactyl is on the loose and is sitting on top of a building at intersection P.



If you choose a route at random from A to B with no backtracking (always traveling South or East) what is the probability that you will go past the Pterodactyl at P?
(Choose the number nearest your answer)

(a) 0.381 (b) 0.019 (c) 0.981 (d) 0.619 (e) 0.421

Solution. The number of routes from A to B with no backtracking is (# such routes from A to P)× (# such routes from P to B) $= C(4,3) \cdot C(6,3)$.
The total number of such routes from A to B is $C(10,6)$.
Therefore if you choose a route at random, the probability that it will go past the point P is

$$\frac{C(4,3) \cdot C(6,3)}{C(10,6)}.$$

24.(5pts) An experiment consists of rolling a pair of dice, one red and one green and observing the pair of numbers on the uppermost face. What's the probability that the numbers **do not** add up to 7?

(a) $\frac{5}{6}$ (b) $\frac{1}{6}$ (c) $\frac{1}{36}$ (d) $\frac{35}{36}$ (e) $\frac{29}{36}$

Solution. The pairs of numbers (with the number on the red die appearing first) which add to 7 are given by $\{(6,1),(5,2),(4,3),(3,4),(2,5),(1,6)\}$. Since all pairs are equally likely and 36 such outcomes are possible, the probability that the numbers add to 7 is $6/36 = 1/6$. By the complement rule, the probability that the numbers do not add to 7 is $1 - 1/6 = 5/6$.

25.(5pts) The US Senate voted on a particular bill, for which the results are shown in the table (Abstain = did not vote). A Senator is selected at random (from the 100 Senators) and found to have voted "Yes". What's the probability that he/she is a democrat?

Affiliation	Yes	No	Abstain
Democrat	6	34	4
Republican	50	2	2
Independent	1	1	0

(a) $\frac{6}{57}$ (b) $\frac{6}{44}$ (c) $\frac{6}{100}$ (d) $\frac{44}{100}$ (e) $\frac{57}{100}$

Solution.

Affiliation	Yes	No	Abstain	Total
Democrat	6	34	4	44
Republican	50	2	2	54
Independent	1	1	0	2
Total	57	37	6	100

We wish to calculate

$$P(\text{Democrat}|\text{voted yes}) = \frac{\#\text{ Democrat and voted yes}}{\#\text{ Voted Yes}} = \frac{6}{57}$$

26.(5pts) A basketball player has a 60% chance of making a basket each time she takes a shot from the free throw line. If she takes four independent shots from the free throw line, what's the probability that she makes a basket on at least one?

(a) 0.9744 (b) 0.0256 (c) 0.1296 (d) 0.8704 (e) 0.0384

Solution. The probability that she misses all 4 shots is $(0.4)^4$ because of independence. By the complement rule, the probability that she makes at least one basket is $1-(0.4)^4 = 0.9744$.

27.(5pts) Ten percent of the new cars made by the Volksota car company will require engine repair in the first year after purchase, 20% of their new cars will require a software patch in the first year after purchase and 5% will require both in the first year after purchase. You have just bought a new Volksota car, what is the probability that your new car will require either engine repair or a software patch or both in the next year.

(a) 0.25 (b) 0.3 (c) 0.05 (d) 0.06 (e) 0.95

Solution. Let ER denote the event that your new car will require engine repair in the next year and let SP denote the event that it will require a software patch. We have $P(ER) = .1$, $P(SP) = .2$ and $P(ER \cap SP) = .05$. We want to calculate $P(ER \cup SP)$. By the inclusion-exclusion principle (or additive principle) we have

$$P(ER \cup SP) = P(ER) + P(SP) - P(ER \cap SP) = .1 + .2 - .05 = 0.25.$$

28.(5pts) In target practice, a pistol shooter has a 60% chance of hitting the target each time he shoots. If he takes six independent shots at practice, what is the probability that the number of times he hits the target is greater than the number of times he misses it?

(a) 0.5443 (b) 0.4557 (c) 0.2765 (d) 0.7235 (e) 0.6134

Solution. The shooter takes $n = 6$ independent shots with a probability of $p = 0.6$ of hitting the target on each shot. Let X denote the number of times he hits the target in the 6 shots, then X has a binomial distribution with $n = 6$ and $p = 0.6$. The probability that the number of times he hits the target is greater than the number of times he misses it is the same as

$$P(X \geq 4) = P(X = 4) + P(X = 5) + P(X = 6) = C(6,4)(0.6)^4(0.4)^2 + C(6,5)(0.6)^5(0.6)^1 + C(6,6)(0.6)^6$$

$$= 0.31104 + 0.1866 + 0.0467 = 0.5443.$$

29.(5pts) Determine the expected value of the random variable X whose probability distribution is given below.

X	Pr(X)
0	0.2
1	0.4
2	0.3
3	0.1

(a) 1.3 (b) 2.1 (c) 1.5 (d) 1.9 (e) 3.1

Solution. To find the expected value, we multiply each outcome by its probability and add to get $E(X) = 1.3$.

X	Pr(X)	XP(X)
0	.2	0
1	.4	0.4
2	.3	0.6
3	.1	0.3
	E(X)	1.3

30.(5pts) The number of completed passes made by a quarterback in 10 consecutive games this season is shown below.

$$10,\ 20,\ 15,\ 19,\ 20,\ 15,\ 20,\ 19,\ 13,\ 19$$

The sample average is $\bar{x} = 17$. Find the **sample** standard deviation (s) for the number of completions per game for the quarterback in question?
Choose the answer closest to yours.

(a) $s = 3.53$ (b) $s = 12.44$ (c) $s = 11.2$ (d) $s = 3.35$ (e) $s = 4.52$

21. Initials: ____________

Solution. The formula for the sample standard deviation is given by

$$s = \sqrt{\frac{(x_1 - \bar{x})^2 + (x_2 - \bar{x})^2 + \cdots + (x_n - \bar{x})^2}{n-1}}$$

where n is the sample size and $x_1, x_2, \ldots, x_n$ are the observations. The calculations in the frequency table below show that $s = \sqrt{s^2} = \sqrt{\frac{112}{9}} \approx 3.53$.

O_i	f_i	$(O_i - 17)$	$(O_i - 17)^2$	$(O_i - 17)^2 f_i$
10	1	-7	49	49
13	1	-4	16	16
15	2	-2	4	8
19	3	2	4	12
20	3	3	9	27
			SUM	112

Initials: _____________

For 2×2 payoff matrix

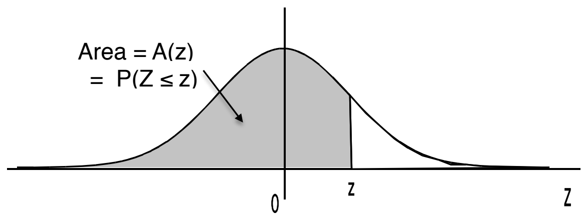
$$\begin{array}{c|cc} & C_1 & C_2 \\ \hline R_1 & a & b \\ R_2 & c & d \end{array}$$

$$p = \frac{d-c}{(a+d)-(b+c)}$$

$$q = \frac{d-b}{(a+d)-(b+c)}$$

$$\nu = \frac{ad-bc}{(a+d)-(b+c)}$$

Areas under the Standard Normal Curve



z	$A(z)$	z	$A(z)$	z	$A(z)$	z	$A(z)$	z	$A(z)$
−3.50	.0002	−2.00	.0228	−.50	.3085	1.00	.8413	2.50	.9938
−3.45	.0003	−1.95	.0256	−.45	.3264	1.05	.8531	2.55	.9946
−3.40	.0003	−1.90	.0287	−.40	.3446	1.10	.8643	2.60	.9953
−3.35	.0004	−1.85	.0322	−.35	.3632	1.15	.8749	2.65	.9960
−3.30	.0005	−1.80	.0359	−.30	.3821	1.20	.8849	2.70	.9965
−3.25	.0006	−1.75	.0401	−.25	.4013	1.25	.8944	2.75	.9970
−3.20	.0007	−1.70	.0446	−.20	.4207	1.30	.9032	2.80	.9974
−3.15	.0008	−1.65	.0495	−.15	.4404	1.35	.9115	2.85	.9978
−3.10	.0010	−1.60	.0548	−.10	.4602	1.40	.9192	2.90	.9981
−3.05	.0011	−1.55	.0606	−.05	.4801	1.45	.9265	2.95	.9984
−3.00	.0013	−1.50	.0668	.00	.5000	1.50	.9332	3.00	.9987
−2.95	.0016	−1.45	.0735	.05	.5199	1.55	.9394	3.05	.9989
−2.90	.0019	−1.40	.0808	.10	.5398	1.60	.9452	3.10	.9990
−2.85	.0022	−1.35	.0885	.15	.5596	1.65	.9505	3.15	.9992
−2.80	.0026	−1.30	.0968	.20	.5793	1.70	.9554	3.20	.9993
−2.75	.0030	−1.25	.1056	.25	.5987	1.75	.9599	3.25	.9994
−2.70	.0035	−1.20	.1151	.30	.6179	1.80	.9641	3.30	.9995
−2.65	.0040	−1.15	.1251	.35	.6368	1.85	.9678	3.35	.9996
−2.60	.0047	−1.10	.1357	.40	.6554	1.90	.9713	3.40	.9997
−2.55	.0054	−1.05	.1469	.45	.6736	1.95	.9744	3.45	.9997
−2.50	.0062	−1.00	.1587	.50	.6915	2.00	.9772	3.50	.9998
−2.45	.0071	−.95	.1711	.55	.7088	2.05	.9798		
−2.40	.0082	−.90	.1841	.60	.7257	2.10	.9821		
−2.35	.0094	−.85	.1977	.65	.7422	2.15	.9842		
−2.30	.0107	−.80	.2119	.70	.7580	2.20	.9861		
−2.25	.0122	−.75	.2266	.75	.7734	2.25	.9878		
−2.20	.0139	−.70	.2420	.80	.7881	2.30	.9893		
−2.15	.0158	−.65	.2578	.85	.8023	2.35	.9906		
−2.10	.0179	−.60	.2743	.90	.8159	2.40	.9918		
−2.05	.0202	−.55	.2912	.95	.8289	2.45	.9929		