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Title: Dualizability in Low Dimensional Higher Category Theory.

Abstract: The cobordism hypothesis establishes a powerful relationship between extended topological field theories taking values in a symmetric monoidal higher category and objects in that higher category with various kinds of duality. In this setting an extended topological field theory is a higher categorical extension of the Atiyah-Segal axioms which allows for topological bordisms to be decomposed along submanifolds of arbitrary codimension.

The principle example of this phenomenon asserts that for a given symmetric monoidal n-category C, the higher groupoid of "fully dualizable" objects in C is equivalent to the higher category of fully extended framed n-dimensional field theories in C, i.e. those where each bordism is equipped with a tangential framing. Since the bordism category has a natural action of the orthogonal group (acting by change of framing), such an equivalence induces a (homotopically coherent) O(n)-action of the groupoid of fully-dualizable objects.

Understanding this action is fundamental in applications of the cobordism hypothesis, as it provides a bridge to understanding extended topological field theories for bordisms equipped with a different tangential structure group. This includes Spin, Oriented, and Unoriented field theories as well as many others. The groupoid of such field theories, say with structure group G, are obtained by starting with the groupoid of fully-dualizable objects and passing to the G-homotopy fixed points. Unfortunately the Hopkins-Lurie proof of the cobordism hypothesis is inductive, and the origin and nature of the induced O(n)-action on the groupoid of fully-dualizable objects remains largely mysterious and elusive.

Motivated by these considerations, we will explain the O(n)-action on the n-groupoid of fully-dualizable objects in a range of "low" category numbers (n less than or equal to three). In coming to grips with how a topological group can act on a higher category, we will touch upon the beautiful connections between modern homotopy theory and higher category theory. We hope to be as explicit as possible, and describe several concrete examples and applications along the way.