GRADUATE STUDENT TOPOLOGY AND GEOMETRY CONFERENCE 2013

Schedule

Saturday, April 6.

8:30 – 9:30: Breakfast and registration (Jordan Hall Galleria)
9:30 – 10:30: Plenary talk

- Dan Freed, *Wigner's* gruppenpest *and K-theory* [10]
  101 Jordan Hall

10:30 – 11:00: Coffee break (Jordan Hall Galleria)
11:00 – 11:30: Graduate student talks

- Ved Datar, *Connectedness of the space of singular Kähler-Einstein metrics on toric varieties* [8]
  105 Jordan Hall
- Mona Merling, *Equivaraint algebraic K-theory* [21]
  107 Pasquerilla Center
- Jonathan Thompson, *Cobordism and Formal Group Laws* [30]
  101 Jordan Hall
- Ruobing Zhang, *Volume entropy for collapsed Riemannian manifolds* [32]
  109 Pasquerilla Center

11:40 – 12:10: Graduate student talks

- Prasit Bhattacharya, *Higher Associativity and Moore Spectra* [3]
  101 Jordan Hall
- Fabian Hebestreit, *Positive scalar curvature on hardly spin manifolds* [15]
  105 Jordan Hall
- Kyle Kinneberg, *Entropy rigidity in coarse geometry* [17]
  109 Pasquerilla Center
- Peter Lambert-Cole, *A braid approach to combinatorial knot Floer homology* [18]
  107 Pasquerilla Center

12:10 – 13:30: Lunch break
13:30 – 14:30: Young faculty talks

- Fernando Galaz-García, *Singular Riemannian foliations by Bieberbach manifolds and applications* [11]
  105 Jordan Hall
- Ana Rita Pires, *Toric b-symplectic and origami manifolds* [24]
  101 Jordan Hall

14:40 – 15:10: Graduate student talks

- Panagiotis Gianniotis, *The Ricci flow on manifolds with boundary* [12]
  105 Jordan Hall
- Akhil Mathew, *The homology of tmf* [20]
  101 Jordan Hall
- Nathan Perlmutter, *Cobordism Categories of Manifolds With Singularities* [23]
  107 Pasquerilla Center
- Laura Starkston, *Symplectic Fillings of Seifert Fibered Spaces* [28]
  109 Pasquerilla Center

15:10 – 15:40: Coffee break (Jordan Hall Galleria)

15:40 – 16:10: Graduate student talks

- Otis Chodosh, *Uniqueness of Ricci Solitons* [6]
  105 Jordan Hall
- Robert Hank, *Massey products in A_∞ algebras* [14]
  107 Pasquerilla Center
- Donghoon Jang, *Symplectic circle actions with isolated fixed points* [16]
  109 Pasquerilla Center
- Arthur Parzygnat, *2-bundles over 2-spaces* [22]
  101 Jordan Hall

16:20 – 17:20: Young faculty talks

- Clark Barwick, *Algebraic K-theory of higher categories* [2]
  101 Jordan Hall
- Gábor Székelyhidi, *Filtrations and test-configurations* [29]
  105 Jordan Hall

17:30 – 18:00: Graduate student talks

- Alessandro Carlotto, *A new generation of uniformization theorems* [5]
  105 Jordan Hall
- Saul Glasman, *A quasicategorical theory of E-infinity bimonoids* [13]
  107 Pasquerilla Center
- Chaitanya Senapathi, *Theorems of Barth-Lefschetz type and Morse theory on the space of paths in homogeneous spaces* [26]
  109 Pasquerilla Center
- Gangotryi Sorcar, *An exotic torus* [27]
  101 Jordan Hall

18:00 – 21:00: Conference dinner (Jordan Hall Galleria)

Sunday, April 7.

8:30 – 9:30: Breakfast (Jordan Hall Galleria)

9:30 – 10:30: Plenary talk

- Burkhard Wilking, *A generalisation of Gromov almost flat manifold theorem* [31]
  101 Jordan Hall

10:30 – 11:00: Coffee break (Jordan Hall Galleria)

11:00 – 11:30: Graduate student talks

- Jess Boling, *Homogeneous Solutions of Pluriclosed Flow on Complex Surfaces* [4]
  105 Jordan Hall
- Gokhan Civan, *Partition Functions and Ray-Singer Invariants* [7]
  109 Pasquerilla Center
- Ash Lightfoot, *Obstructions to embedding 2-spheres in dimension 4* [19]
  101 Jordan Hall
- Aaron Royer, *Generalized String Topology is Derived Koszul Duality* [25]
  107 Pasquerilla Center

11:40 – 12:40: Young faculty talks

- David Ayala, *Generalizing Poincaré duality via labeled configurations* [1]
  101 Jordan Hall
- Alessio Figalli, *Optimal transport and Riemannian geometry* [9]
  105 Jordan Hall

Abstracts

[1] David Ayala (USC), *Generalizing Poincaré duality via labeled configurations.*
We will investigate configurations of dots in a manifold, each dot labeled by an element of an abelian group (akin to Dold-Thom). Using this construction we will see both the homology and the cohomology of a manifold with coefficients in the abelian group; and we will witness Poincaré duality. Examining this construction will lead to observations of an inherent connection between higher algebraic/categorical structures and differential topology, which ultimately yields the fabrication of topological field theories.

[2] Clark Barwick (MIT), *Algebraic K-theory of higher categories.*

[3] Prasit Bhattacharya (Indiana), *Higher Associativity and Moore Spectra.*
We all know that The sphere spectrum S is an E_∞ ring spectrum. But the cofiber of the map $S \xrightarrow{n} S$ for $n \in \mathbb{N}$ are not even A_∞. Results are known for n prime but otherwise not much results are known. Some constructions are made with the hope of getting some obstruction theory for higher associativity. This talk will be an expository talk on the current work along this line.

[4] Jess Boling (UC Irvine), *Homogeneous Solutions of Pluriclosed Flow on Complex Surfaces.*
This talk is expository, presents minimal original research, and should be accessible to anyone with general knowledge of Ricci flow and complex geometry.

Plurclosed flow is a generalization of Kähler-Ricci flow to the case when the metric is $\partial\overline{\partial}$ closed. All compact complex surfaces admit a pluriclosed metric, so one expects that the long time and singularity behavior of the flow will give information about such manifolds. In the talk, I will present results analogous to Jackson et al. for homogeneous Ricci flow. Namely, I present the long time behavior of homogeneous metrics under pluriclosed flow on Hopf, Inoue, Kodiara, and other complex surfaces admitting a homogeneous complex structure. I show that a homogeneous metric on the Hopf surface converges to a canonical form, while time rescaled homogeneous metrics on an Inoue surface exhibit Gromov-Hausdorff convergence to a circle. I finish the talk with a discussion of what we would like to do next with the flow.

[5] Alessandro Carlotto (Stanford), *A new generation of uniformization theorems.*
It has been known for several decades that, unlike the classical smooth setting, singular surfaces do not in general admit uniformising metrics. In this talk, I will briefly describe a variational approach to this problem (based on a Morse analytic study of the Euler-Lagrange functional associated to the curvature prescription equation) and its applications in terms of new results concerning existence, non-existence and multiplicity. This is joint work with Andrea Malchiodi.

[6] Otis Chodosh (Stanford), *Uniqueness of Ricci Solitons.*
We'll discuss uniqueness results for Ricci solitons. In particular, we will give an overview of the "approximate Killing vector method" recently used by Brendle to solve a conjecture of Perelman concerning rotational symmetry of kappa-noncollapsed steady solitons. We will also discuss our recent results with Brendle concerning expanding soliton uniqueness. The talk will not require any Ricci flow expertise, only basic notions from Riemannian geometry and PDE.

[7] Gokhan Civan (Maryland), *Partition Functions and Ray-Singer Invariants.*
This talk is expository and accessible to students experienced in geometry and analysis. We look at partition functions introduced by Albert Schwarz and relate them to Ray-Singer invariants. The partition function is an invariant attached to elliptic functionals on Hermitian vector bundles. Its definition depends on the notion of zeta function regularized determinants. We motivate the definition of the partition function using the Weyl integral formula. We then show connections with Ray-Singer invariants.

[8] Ved Datar (Rutgers), *Connectedness of the space of singular Kähler-Einstein metrics on toric varieties.*
Let X be a toric manifold and L a line bundle on it. Then given any $\alpha > 0$, we can show that there exists a unique $\omega \in c_1(L)$ satisfying

$$Ric(\omega) = \alpha\omega + D$$

for some divisor D. Moreover, if $\pi : X \to Y$ is a blow up map along some subvariety, then one can choose conical K-E metrics, ω_t and ω on X and Y respectively such that (X, ω_t) converges to (Y, ω) in the Gromov-Hausdorff topology. In the first ten minutes of the talk I will provide a quick introduction to toric varieties and state the two theorems. I will then discuss the short proof of the convergence. It relies on a beautiful trick of Gromov's. We believe that this idea will be very useful in the future to tackle similar problems of G-H convergence.This is joint work with Bin Guo, Jian Song and Xiowei Wang.

[9] Alessio Figalli (UT Austin), *Optimal transport and Riemannian geometry.*
The optimal transportation problem consists in finding the most effective way of moving mass distributions from one place to another, minimizing the transportation cost. Such a concept has been found very useful in other areas of mathematics. In this talk I'll introduce the optimal transport problem and describe some of the main results in the theory, and then I'll address the question of continuity of optimal maps on Riemannian manifolds. As we will see this will lead us to formulate a condition, called "Ma-Trudinger-Wang condition", which can then be used as a tool to obtain geometric informations on the cut locus of the underlying manifold.

[10] Dan Freed (UT Austin), *Wigner's* gruppenpest *and K-theory.*
Eugene Wigner applied the mathematics of groups in many ways to problems in quantum physics. I'll begin with his fundamental theorem about quantum symmetries, which turns out to lie in (infinite dimensional) Riemannian geometry. Then I'll discuss joint work with Gregory Moore in which we classify topological phases of certain quantum mechanical systems in condensed matter physics.

[11] Fernando Galaz-García (Münster), *Singular Riemannian foliations by Bieberbach manifolds and applications.*
Isometric actions of compact Lie groups on Riemannian manifolds are examples of singular Riemannian foliations. This talk will focus on singular Riemannian foliations whose leaves are homeomorphic to Bieberbach manifolds, generalizing isometric torus actions. I will discuss topological and geometric aspects of these foliations, including their structure, their classification in codimension 1 and 2 and some applications to nonnegatively and positively curved Riemannian manifolds. This is joint work with Marco Radeschi.

[12] Panagiotis Gianniotis (Stony Brook), *The Ricci flow on manifolds with boundary.* Despite the extensive study of Ricci flow in the past years, there is still not a well developed theory for it on manifolds with boundary. In this talk, I will discuss past work on the topic and present some new results on the short-time existence, uniqueness and continuation of the flow on arbitrary compact Riemannian manifolds with boundary.

[13] Saul Glasman (MIT), *A quasicategorical theory of E-infinity bimonoids.*
May's theory of 'pairs of operads' affords us entry to the well-fortified land of spaces which have two operad actions, one of which distributes over the other. From this springs the definition of an E_∞ ring space, the natural topological analog of a commutative semiring. We summon various combinatorial entities which allow us to reconstruct this coherent bicommutativity in the language of quasicategories. By way of application, we give a new proof that the K-theory of a commutative ring is an E_∞ ring spectrum, as well as a definition of symmetric bimonoidal quasicategory.

[14] Robert Hank (Minnesota), *Massey products in A_∞ algebras.*
This talk will be mostly expository with some research presented near the end. The content is primarily algebraic and should be understandable for most students.

Massey products arise as obstruction classes to higher order operations. In practice, we frequently encounter objects in a category equipped with a multiplication. We would like to know whether or not the multiplication is associative. The obstruction to associativity can be given in terms of a Massey product. In practice, one frequently encounters objects equipped with an "almost but not quite" associative multiplication, or more precisely a homotopy associative multiplication. Such objects are known as A_∞ objects and arise in topology as loop spaces. During the talk, I will develop the relation between Massey products and multiplicative structure in such a way that the generalization to A_∞ structure arises naturally.

[15] Fabian Hebestreit (Münster), *Positive scalar curvature on hardly spin manifolds.* Given a closed manifold M, does it admit a metric of positive scalar curvature?

In 1980 M. Gromov and B. Lawson showed that the answer to this question depends only on the class of M in a bordism ring, provided $dim(M) \geq 5$; which bordism ring to use, however, depends on some low dimensional, topological data of M. Their program has lead to a complete answer in the case of simply connected M and strong results for spinnable M, using classical oriented and spin bordism. The remaining cases involve other cobordism rings and the last one to be adressed is a twisted version of spin bordism.

In my talk I will explain some of the above in more detail and then present a generalisation of the Anderson-Brown-Peterson splitting for spin bordism to this twisted version using M. Atiyah and G. Segal's twisted K-Theory. Thus the first part of my talk is expository and will (hopefully) be understandable for everyone, while the second part will require some knowledge about stable homotopy theory.

[16] Donghoon Jang (Illinois), *Symplectic circle actions with isolated fixed points.*
This talk is introductory; we begin with the definitions of a symplectic manifold and a symplectic circle action, with basic examples. The study of fixed points of group actions is a classical and important topic in geometry. During this talk, we

focus on symplectic circle actions on compact symplectic manifolds with isolated fixed points. First, it is easy to check that one fixed point is impossible. If there are exactly two fixed points, C. Kosniowski proved that the manifold is either the 2-sphere or $\dim M = 6$. We will show that in the three fixed point case, the manifold is equivariantly symplectomorphic to $\mathbb{CP}^2$ (arxiv:1210.0458). This provides further evidence for C. Kosniowski's conjecture that symplectic circle actions with only a few fixed points are "small."

This work is closely related to the question of under what conditions symplectic circle actions are Hamiltonian, because Hamiltonian actions have at least $\frac{1}{2}\dim M + 1$ fixed points. For Kahler manifolds, a symplectic circle action is Hamiltonian if and only if it has a fixed point. On the other hand, McDuff constructed an example of a non-Hamiltonian symplectic circle action on a 6-dimensional symplectic manifold with fixed tori. For isolated fixed points, the question of whether symplectic circle actions are Hamiltonian is still open.

During the talk, we will discuss techniques to prove the theorem, including ABBV Localization formula.

[17] Kyle Kinneberg (UCLA), *Entropy rigidity in coarse geometry.*
A common theme in Riemannian geometry is the analysis of metrics that are extremal for a given functional. In the 1990s, several important results in this direction were established for negatively curved manifolds. Among them was the entropy-rigidity theorem of Hamenstädt: if a closed manifold of curvature bounded above by -1 admits a locally symmetric metric, then the volume-growth entropy is minimized precisely by the locally symmetric metrics. This result was subsequently adapted to the setting of metric geometry by the work of Bourdon and Bonk–Kleiner, who studied rigidity of geometric group actions on CAT(-1) spaces. In this talk, I will discuss analogous results in the coarse setting, namely, rigidity for geometric group actions on Gromov hyperbolic metric spaces with a suitable upper curvature bound.

[18] Peter Lambert-Cole (LSU), *A braid approach to combinatorial knot Floer homology.*
This talk is based upon original research and should be accessible to all students in geometry and topology. Knot Floer homology is an invariant of links in 3-manifolds introduced by Ozsvath and Szabo. The main approach to combinatorial computations of this invariant in S^3, due to Manolescu, Ozsvath and Sarkar, is via a grid diagram of the knot. However, computation time grows factorially in the arc index of the knot. We compute the knot Floer homoloy of a link L in terms of a braid B whose closure is L. This approach is significantly faster for certain classes of knots. Joint with Sam Connolly, Michaela Stone, Shea Vela-Vick and Ellen Weld.

[19] Ash Lightfoot (Indiana), *Obstructions to embedding* 2*-spheres in dimension* 4.
The success of the Whitney trick in dimension $k > 4$ allowed Wall to define the complete obstruction μ to turning a k-sphere immersed in a $2k$-dimensional manifold into an embedding. Its failure in dimension 4 was known then, and there has been interest since in defining "higher-order" invariants in this dimension. We begin by describing an elementary example (due to Kirk) of an immersed 2-sphere in a 4-manifold for which μ fails. We then define a secondary invariant τ (due

to Teichner and Schneiderman) and show that it detects Kirk's example. Our motivation is to have shown that τ is practicable, and nontrivial beyond the realm of examples cooked up specifically for its application. In doing so we'll introduce many of the techniques a geometric topologist employs to study 4-dimensional manifolds.

[20] Akhil Mathew (Harvard), *The homology of tmf.*
Let ko be the connective real K-theory spectrum. Then ko has torsion in its homotopy, but there is a 2-cell complex $C\eta$ such that $ko \wedge C\eta$ is the simpler specrum ku of complex K-theory. This is a theorem of Reg Wood, and has an analog in tmf. Let tmf be the (2-local) connective spectrum of topological modular forms. Then there exists an 8-cell complex C such that tmf $\wedge\, C$ is in fact a complex-orientable E_∞-ring spectrum ("topological modular forms of level 3") whose homotopy groups are torsion-free (and are a polynomial ring), also known as $BP < 2 >$. This is a folk theorem of Hopkins-Mahowald which provides an approach to certain basic computations with tmf, for instance the calculation of its mod 2 homology or complex bordism. In this talk, I'll explain an elementary proof of this result based on stacks, and some of its implications.

[21] Mona Merling (UChicago), *Equivaraint algebraic K-theory.*
In the early 1980's, Dress and Kuku and Fiedorowicz, Hauschild and May introduced space level equivariant versions of the plus and Q constructions in algebraic K-theory. However, back then, the group action on the ring in question was taken to be trivial. We generalize these definitions to the case in which a finite group G acts nontrivially on a ring, an exact category, or a Waldhausen category, and we show how to construct a genuine equivariant K-theory spectrum from a G-ring. The main example of interest is that of a Galois extension.

The equivariant constructions rely on finding categorical models for classifying spaces of equivariant bundles (a joint project with Guillou and May) and the use of equivariant infinite loop space machines such as the one developed by Guillou and May, or the equivariant version of Segal's machine. The comparison of these machines, which will allow their interchangeable use in algebraic K-theory constructions, is a joint project with May and Osorno. New ideas are needed since, among other things, the comparison theorem of May and Thomason fails equivariantly.

[22] Arthur Parzygnat (CUNY), 2-*bundles over* 2-*spaces.*
The notions of topology can be generalized to two spaces that are put together in such a way as to resemble a category. Such a category is called a 2-space. A degenerate example is an ordinary space. The notions of principle group bundles over spaces can also be extended to those of 2-bundles over 2-spaces. We will compare the theory of 2-bundles over an ordinary space to nonabelian gerbes (a definition will be given) on that space and show how the theory of 2-bundles over 2-spaces generalizes the theory of gerbes. Time permitting, we will describe the case when one adds the data of a connection. This talk is rather expository and should be accessible to students who know what topological spaces, groups, categories, functors, and natural transformations are. Knowing what bundles and 2-categories are would also help, but are not necessary.

[23] Nathan Perlmutter (Oregon), *Cobordism Categories of Manifolds With Singularities.*
This talk is based on original research and should be accessible to anyone familiar with the concept of Cobordism of Manifolds and a smattering of homotopy theory.

In addition to the standard non-singular Cobordism theories (Oriented, Complex, Spin, etc...) one can study certain "singular" Corbordism theories corresponding to manifolds with singularities. For a given set of singularity types (I'll explain exactly what this means in the talk) one can define Cobordism groups consisting of manifolds with singularities from that given set of singularity types. These singular Cobordism theories (there is one for each choice of singularity set) can be compared to each other and the non-singular theories via an exact couple known as the Bockstein-Sullivan exact couple.

Following recent work in Cobordism Categories, for a given a set of singularity types I show how to construct a Category whose objects are closed Manifolds with singularities from the given set of singularity types and whose morphisms are the singular Cobordisms connecting them. I then identify the homotopy type of the classifying space of such a singular Cobordism Category with that of the infinite loop-space of a certain spectrum, constructed out of Thom-spectra. My result is a direct generalization of the work of S. Galatius, I. Madsen, U. Tillmann, and M. Weiss in "The Homotopy Type of a Cobordism Category" and uses many similar techniques.

[24] Ana Rita Pires (Cornell), *Toric b-symplectic and origami manifolds.*
Delzant's theorem tells us that toric symplectic manifolds are classified by the image of their moment map, and therefore are in one-to-one correspondence with Delzant polytopes. In this talk we will examine what happens for two types of manifolds which fail to be symplectic on a hypersurface, and do so in two very different ways. A folded symplectic form is a closed 2-form with the mildest possible degeneracy along a hypersurface; origami manifolds are a special case of these. On the other extreme, b-symplectic manifolds are a class of Poisson manifolds for which the Poisson bivector field vanishes on a hypersurface, and therefore the dual 2-form can be regarded as a sympletic form which explodes to infinity at Z. If we endow them with a toric action, what moment images can we get in these two cases, and what classification results are possible?

[25] Aaron Royer (UT Austin), *Generalized String Topology is Derived Koszul Duality.*
Generalized string topology attaches to any bundle of algebraic objects (monoids, E_n-algebras, etc) over a closed oriented manifold a collection of intersection-type operations on the homology of the total space. We show that this rigidifies and extends to a lax monoidal functor connecting the unstable homotopy theory parametrized over a closed manifold M to the highly-structured module theory over its Atiyah dual M^{-TM}. Using the theory of $(\infty, 1)$-categories, we give a characterization of this functor as a form of derived Koszul duality. We will assume familiarity with the basics of modern stable homotopy theory, but will spend some time discussing the relevant ideas from higher category theory.

[26] Chaitanya Senapathi (Michigan State), *Theorems of Barth-Lefschetz type and Morse theory on the space of paths in homogeneous spaces.*
We look at the space of paths joining two complex submanifolds in complex G^c/P, where G^c is complex semi-simple Lie group and P is a parabolic subgroup. We study the Morse theory of this space using the energy of the path with respect to the normal metric. The Barth-Lefschetz type theorems for a wide class of manifolds of the form G^c/P are re-proven in an elegant manner. Barth-Lefschetz type theorems are homotopy vanishing theorems for intersections of complex submanifolds of generalized flag manifolds.

[27] Gangotryi Sorcar (Binghamton), *An exotic torus.*
This talk is mostly expository with some subsequent research result towards the end. We shall see how Browder constructed a manifold that is homeomorphic but not diffeomorphic to the 7-dimensional torus, and then we will see how it can be extended to $M \times S^1$, where M is a 6-dimensional manifold, easily, using Novikov's Theorem on rational Pontryagin numbers.

[28] Laura Starkston (UT Austin), *Symplectic Fillings of Seifert Fibered Spaces.*
This talk is based on original research and uses tools from 4-manifold topology and symplectic geometry, but we will not assume any background, and will have lots of pictures. A symplectic filling is a symplectic manifold with boundary, where the symplectic structure satisfies natural conditions so that it induces a contact structure on the boundary. Given a contact manifold, one can try to classify all of its symplectic fillings. Such classifications have been completed for S^3 and certain contact lens spaces. We will discuss how to determine all diffeomorphism types of symplectic 4-manifolds which convexly fill certain contact Seifert fibered spaces. The technique produces a finite list of handlebody diagrams of 4-manifolds. In certain cases, these can be shown to be symplectic fillings by understanding these manifolds as rational blow-downs or Lefschetz fibrations.

[29] Gábor Székelyhidi (Notre Dame), *Filtrations and test-configurations.*
Test-configurations are certain degenerations of projective manifolds, used in the definition of K-stability, which in turn is related to the existence of special metrics. I will explain how filtrations of the homogeneous coordinate ring of a projective manifold can be thought of as sequences of test-configurations, and that they encode the limiting behavior of these sequences. Such filtrations arise naturally when studying the Calabi flow, or when trying to minimize the Calabi functional. I will also discuss how filtrations can be used to give a strengthening of the notion of K-stability, and why this is desirable.

[30] Jonathan Thompson (Kentucky), *Cobordism and Formal Group Laws.*
This talk will be expository and broadly accessible, though perhaps of more interest to topologists. I will introduce the notions of formal group law and the complex cobordism ring, and show how the cobordism ring leads to a universal formal group law.

[31] Burkhard Wilking (Münster), *A generalisation of Gromov almost flat manifold theorem.*
By Gromov a manifold with Diameter 1 and sectional curvature suffiently small in absolute value is finitely covered by a nilmanifold. We generalize the Theorem by relaxing the curvature condition in the theorem.

[32] Ruobing Zhang (Princeton), *Volume entropy for collapsed Riemannian manifolds.*
We will give some estimates for collapsed Riemannian manifolds under various curvature conditions. Then we will talk about some metrics compactness theorems for volume entropy almost maximal.