Name: Date: 12/06/2018

M20550 Calculus III Tutorial
Worksheet 11

1. Compute the surface integral $\iint_S (x+y+z)\, dS$, where S is a surface given by $\mathbf{r}(u,v) = \langle u+v, u-v, 1+2u+v \rangle$ and $0 \leq u \leq 2,\ 0 \leq v \leq 1$.

2. Let S be the portion of the graph $z = 4 - 2x^2 - 3y^2$ that lies over the region in the xy-plane bounded by $x = 0,\ y = 0$, and $x + y = 1$. Write the integral that computes $\iint_S \left(x^2 + y^2 + z\right)\, dS$.

3. Compute $\iint_S \mathbf{F} \cdot d\mathbf{S}$, where $\mathbf{F} = y\mathbf{i} - x\mathbf{j} + z\mathbf{k}$ and S is a surface given by

$$x = 2u, \quad y = 2v, \quad z = 5 - u^2 - v^2,$$

   where $u^2 + v^2 \leq 1$. S has downward orientation.

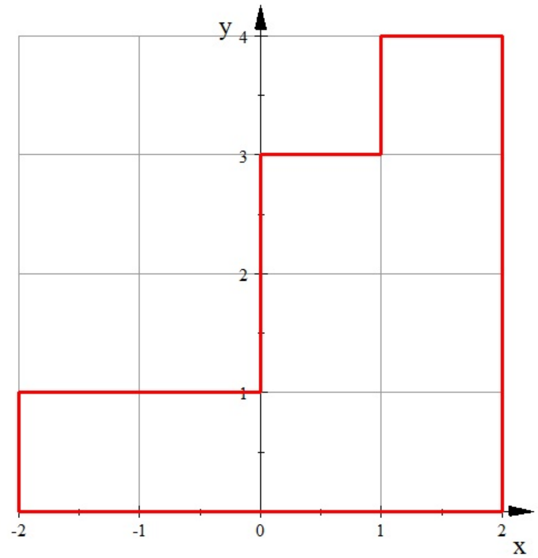
4. Compute the flux of the vector field $\mathbf{F} = x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$ over the part of the cylinder $x^2 + y^2 = 4$ that lies between the planes $z = 0$ and $z = 2$ with normal pointing away from the origin.

5. Find the flux of the vector field $\mathbf{F}(x,y,z) = \langle 0, z, 1 \rangle$ across the hemi-sphere $x^2 + y^2 + z^2 = 4$, $z \geq 0$ with orientation away from the origin.

Each of the problem below can be solved using one of these theorems: Green's Theorem, Stokes' Theorem, or Divergence Theorem

6. Let S be the surface defined as $z = 4 - 4x^2 - y^2$ with $z \geq 0$ and oriented upward. Let $\mathbf{F} = \langle x - y, x + y, ze^{xy} \rangle$. Compute $\iint_S \operatorname{curl} \mathbf{F} \cdot d\mathbf{S}$. (*Hint*: use one of the theorems you learned in class.)

7. Evaluate $\int_C (x^4y^5 - 2y)dx + (3x + x^5y^4)dy$ where C is the curve below and C is oriented in clockwise direction.



8. Let S be the boundary surface of the region bounded by $z = \sqrt{36 - x^2 - y^2}$ and $z = 0$, with outward orientation. Find $\iint_S \mathbf{F} \cdot d\mathbf{S}$, where $\mathbf{F} = x\mathbf{i} + y^2\mathbf{j} - 2yz\mathbf{k}$.

9. Let C be the boundary curve of the part of the plane $x + y + 2z = 2$ in the first octant. C has counterclockwise orientation when viewing from above. Compute $\int_C \mathbf{F} \cdot d\mathbf{r}$, where $\mathbf{F} = \langle e^{\sin x^2}, z, 3y \rangle$.

10. *(A Challenging Problem)* Evaluate

$$\int_C (y^3 + \cos x)dx + (\sin y + z^2)dy + x\,dz$$

where C is the closed curve parametrized by $\mathbf{r}(t) = \langle \cos t, \sin t, \sin 2t \rangle$ with counter-clockwise direction when viewed from above. (*Hint*: the curve C lies on the surface $z = 2xy$.)