Name: SOLUTIONS Date: 10/11/2018

M20550 Calculus III Tutorial
Worksheet 6

1. Evaluate the double integral $\iint_R (1-x)dA$, for $R = [0,1] \times [0,1]$, by identifying it as the volume of a solid.

2. Evaluate the iterated integral.

   (a) $\int_0^2 \int_0^\pi r \sin^2\theta \; d\theta dr$

   (b) $\iint_R ye^{-xy} dA$ on $R = [0,2] \times [0,3]$

3. Use polar coordinates to show that

$$\int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} e^{-(x^2+y^2)} dA = \pi$$

   and deduce that $\int_{-\infty}^{+\infty} e^{-x^2} dx = \sqrt{\pi}$.

4. Evaluate the given integral.

$$\iint_R \arctan\left(\frac{y}{x}\right) \, dA$$

   where $R = \{(x,y) : 1 \leq x^2 + y^2 \leq 4, 0 \leq y \leq x\}$.

5. Find the volume of the solid enclosed by the paraboloid $z = x^2 + y^2$ and the plane $z = 1$.

6. Set up, but do not evaluate, the integral that gives the volume of the solid region bounded by the paraboloid $z = x^2 + y^2$ and the cone $z = 1 - \sqrt{x^2 + y^2}$.

7. (*1+1=2*) Prove the integration by parts formula

$$\int_0^a f(x)g(x)dx = f(a)\int_0^a g(y)dy - \int_{x=0}^a \frac{df}{dx} \int_{y=0}^x g(y)dydx$$

   by changing the order of integration and using the fundamental theorem of calculus.

8. (Optional) Find the maximum value of the function $f(x,y,z) = x + y$ on the curve of intersection of the plane $x + y + z = 1$ and the cylinder $y^2 + z^2 = 1$.

9. (Optional) The plane $x + y + 2z = 2$ intersects the paraboloid $z = x^2 + y^2$ in an ellipse. Find the points on the ellipse that are nearest and farthest from the origin.