Name: Date: 11/01/2018

M20550 Calculus III Tutorial
Worksheet 7

1. Using spherical coordinates, compute the volume, $V(R)$ of a sphere of radius R.

2. Now compute the surface area, $A(R)$, of a sphere of radius R. Hint: Recall the Fundamental Theorem of Calculus:

$$\frac{d}{dx}\left[\int_a^x f(t)dt\right] = f(x).$$

And recall the common problem from single variable calculus where you have to find the volume of a water tank of height h by integrating the cross sectional area, $A(y)$, over the height.

$$Volume(Tank) = \int_0^h A(y)dy$$

We have a similar formula for the volume of the sphere;

$$V(R) = \int_0^R A(\rho)d\rho.$$

3. (a) Let E_1 be the solid that lies under the plane $z = 1$ and above the region in the xy-plane bounded by $x = 0$, $y = 0$, and $2x + y = 2$. Write the triple integral $\iiint_{E_1} xz\,dV$ but do not evaluate it.

   (b) Let E_2 be the solid region in the first octant that lies under the paraboloid $z = 2 - x^2 - y^2$. Write the triple integral $\iiint_{E_2} xz\,dV$ in cylindrical coordinates (you don't need to evaluate it).

   (c) Let E_3 be the solid region that lies above the cone $z = \sqrt{x^2 + y^2}$ and below the plane $z = 2$. Write the triple integral $\iiint_{E_3} xz\,dV$ in spherical coordinates (you don't need to evaluate it).

4. Write the integral that computes the volume of the part of the solid cylinder $x^2 + y^2 \leq 1$ that lies between the planes $z = 0$ and $z = 2 - y$.

5. Find the mass of the solid between the spheres $x^2 + y^2 + z^2 = 1$ and $x^2 + y^2 + z^2 = 4$ whose density is $\delta(x, y, z) = x^2 + y^2 + z^2$.

6. Find the center of mass of the solid S bounded by the paraboloid $z = x^2 + y^2$ and the plane $z = 1$ if S has constant density 1 and total mass $\frac{\pi}{2}$. (Hint: $\overline{x}$ and $\overline{y}$ can be found by symmetry of the solid being considered).

7. In this problem, we are going to calculate the same integral in two different ways by changing coordinates. Compute the following integral;

$$\int_0^1 \int_0^1 x^3 y dx dy$$

first, by making the coordinate change $u = x^2, v = xy$, and then as you normally would. (Don't forget to multiply by the Jacobian!)