Name: Date: 29/11/2018

M20550 Calculus III Tutorial
Worksheet 9

1. Determine whether or not the following vector fields are conservative:
   (a) $\mathbf{F} = (3 + 2xy)\,\mathbf{i} + (x^2 - 3y^2)\,\mathbf{j}$
   (b) $\mathbf{F} = \mathbf{i} + \sin z\,\mathbf{j} + y\cos z\,\mathbf{k}$

2. Evaluate $\displaystyle\int_C \mathbf{F}\cdot d\mathbf{r}$, where $\mathbf{F}(x,y,z) = -2xy\,\mathbf{i} + 4y\,\mathbf{j} + \mathbf{k}$ and $\mathbf{r}(t) = t\,\mathbf{i} + t^2\,\mathbf{j} + \mathbf{k}$, $0 \le t \le 2$.

3. Evaluate $\displaystyle\int_C \mathbf{F}\cdot d\mathbf{r}$, where $\mathbf{F} = (y^2\cos(xy^2) + 3x^2)\,\mathbf{i} + (2xy\cos(xy^2) + 2y)\,\mathbf{j}$ is a conservative vector field and C is any curve from the point $(-1,0)$ to $(1,0)$.

4. Use Green's Theorem to evaluate
$$\int_C \left(-\frac{y^3}{3} + \sin x\right)\,dx + \left(\frac{x^3}{3} + y\right)\,dy,$$
where C is the circle of radius 1 centered at $(0,0)$ oriented counterclockwise when viewed from above.

5. A particle starts at the origin $(0,0)$, moves along the x-axis to $(2,0)$, then along the curve $y = \sqrt{4 - x^2}$ to the point $(0,2)$, and then along the y-axis back to the origin. Find the work done on this particle by the force field $\mathbf{F}(x,y) = y^2\,\mathbf{i} + 2x(y+1)\,\mathbf{j}$.

6. (a) Compute div $\mathbf{F}$, where $\mathbf{F} = \langle e^y, zy, xy^2\rangle$.
   (b) Is there a vector field $\mathbf{G}$ on $\mathbb{R}^3$ such that curl $\mathbf{G} = \langle xyz, -y^2z, yz^2\rangle$? Why?

7. Parametrize the following surfaces:
   (a) Part of the cylinder $x^2 + y^2 = 9$ between $z = -1$ and $z = 2$.
   (b) Par of the sphere $x^2 + y^2 + z^2 = 4$ in the first octant.
   (c) Part of the paraboloid $z = x^2 + y^2$ which lies inside the cylinder $x^2 + y^2 = 1$

8. Write an equation of the tangent plane to the parametric surface
$$x = u^2 + 1, \quad y = v^3 + 1, \quad z = u + v,$$
at the point $(5,2,3)$.

9. Write the integral that computes the surface area of the surface S parametrized by $\mathbf{r}(u,v) = \langle u^2\cos v, u^2\sin v, v\rangle$, where $0 \le u \le 1$ and $0 \le v \le \pi$.

10. Find the area of the part of the paraboloid $z = x^2 + y^2$ which lies inside the cylinder $x^2 + y^2 = 1$.