Math 20580 Name:____________________
Midterm 3 Instructor:____________________
April 16, 2020 Section:____________________

Calculators are NOT allowed. You will be allowed 120 minutes to do the test.

There are 15 multiple choice questions worth 6 points each and you will receive 10 points for following the instructions. Record your answers by placing an $\times$ through one letter for each problem on this answer sheet.

Sign the pledge. "On my honor, I have neither given nor received unauthorized aid on this Exam":

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1. a b c d e
2. a b c d e
3. a b c d e
4. a b c d e
5. a b c d e
6. a b c d e
7. a b c d e
8. a b c d e
9. a b c d e
10. a b c d e
11. a b c d e
12. a b c d e
13. a b c d e
14. a b c d e
15. a b c d e

1. The differential equation

$$\frac{dy}{dt} + ty^2 = 0$$

is

(a) linear	(b) a partial differential equation	(c) separable
(d) an equation of order 2	(e) none of the above	

2. Let A be an m by n matrix. Which of the following must be true:

(a) $A\vec{x} = \vec{b}$ has a solution for any $\vec{b}$.
(b) $A\vec{x} = \vec{b}$ always has a unique least-squares solution.
(c) $A^T A\vec{x} = A^T\vec{b}$ has a solution for any $\vec{b}$.
(d) $\vec{b}$ is orthogonal to $A\vec{x}$ for any $\vec{x}$.
(e) None of the above.

3. The matrix $\begin{bmatrix} -2 & 2 \\ -15 & 9 \end{bmatrix}$ has eigenvalues 3 and 4. Then we know that $A = PDP^{-1}$ for $D = \begin{bmatrix} 3 & 0 \\ 0 & 4 \end{bmatrix}$ and P given by:

(a) $\begin{bmatrix} 2 & 1 \\ 5 & 3 \end{bmatrix}$ (b) $\begin{bmatrix} 3 & 0 \\ 0 & 4 \end{bmatrix}$ (c) $\begin{bmatrix} 7 & 5 \\ 3 & 2 \end{bmatrix}$ (d) $\begin{bmatrix} 2 & 1 \\ -1 & 3 \end{bmatrix}$ (e) $\begin{bmatrix} 9 & -2 \\ 15 & -2 \end{bmatrix}$

4. Suppose you wish to solve the differential equation $y' + \dfrac{2}{t}y = t^4$, $t > 0$, using integrating factors. After multiplying the equation by the integrating factor $\mu(t)$, the equation becomes:

(a) $ty' + 2y = t^5$ (b) $e^t ty' + 2e^t y = e^5 t^5$ (c) $e^{2t} ty' + 2e^{2t} y = e^{2t} t^5$
(d) $t^2 y' + 2ty = t^6$ (e) none of the above

5. Applying the Gram-Schmidt process to the vectors $\vec{x}_1 = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$ and $\vec{x}_2 = \begin{bmatrix} 5 \\ 5 \end{bmatrix}$ gives

(a) $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$ and $\begin{bmatrix} -1 \\ 1 \end{bmatrix}$ (b) $\begin{bmatrix} 2 \\ 1 \end{bmatrix}$ and $\begin{bmatrix} -1 \\ 2 \end{bmatrix}$ (c) $\begin{bmatrix} 2 \\ 1 \end{bmatrix}$ and $\begin{bmatrix} 1 \\ 2 \end{bmatrix}$
(d) $\begin{bmatrix} 2 \\ 1 \end{bmatrix}$ and $\begin{bmatrix} 6 \\ 3 \end{bmatrix}$ (e) None of the above

6. Find the general solution to $y' = \dfrac{y^2}{t^3}$, $t > 0$.

(a) $y = 2t^2 + C$ (b) $y = Ce^{2t} + 3$ (c) $y = -t^2 + C$ (d) $y = e^{3t} + C$
(e) None of the above

7. The vector $\vec{v} = \begin{bmatrix} -1-i \\ 1 \end{bmatrix}$ is a complex eigenvector of the matrix $A = \begin{bmatrix} 1 & -2 \\ 1 & 3 \end{bmatrix}$. What is the corresponding eigenvalue?

(a) $\lambda = 2i$ (b) $\lambda = 2+i$ (c) $\lambda = -2+3i$ (d) $\lambda = 2-i$ (e) $\lambda = -1-i$

8. Recall that $\mathbb{P}_n$ denotes the vector space of polynomials of degree at most n, and consider the linear transformation $T : \mathbb{P}_2 \to \mathbb{P}_3$ defined by

$$T(y) = y'' - y' + ty.$$

The matrix of T relative to the basis $\{1, t, t^2\}$ of $\mathbb{P}_2$ and the basis $\{1, t, t^2, t^3\}$ of $\mathbb{P}_3$ is

(a) $\begin{bmatrix} 0 & -1 & 2 \\ 1 & 0 & -2 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ (b) $\begin{bmatrix} t^2 \\ -2t \\ 1 \end{bmatrix}$ (c) $\begin{bmatrix} t^3 \\ -3t^2 \\ 6t \\ 1 \end{bmatrix}$ (d) $\begin{bmatrix} 1 & 1 & 0 & 0 \\ -1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}$

(e) it cannot be determined from the given information.

9. Consider the line L spanned by the vector $\vec{v} = \begin{bmatrix} -2 \\ 1 \end{bmatrix}$. The distance from the vector $\vec{x} = \begin{bmatrix} -7 \\ 1 \end{bmatrix}$ to the line L is

(a) $\sqrt{45}$ (b) $\sqrt{5}$ (c) $2\sqrt{3}$ (d) 5 (e) $\sqrt{50}$

10. Which of the following functions is a solution of the initial value problem

$$(y' - \sin x)^2 = 4 - 4y^2, \quad y(0) = 1$$

(a) $-\sin x$ (b) 1 (c) $\cos x$ (d) $x \cos x - \sin x$ (e) $\sin x - \cos x$

11. Consider the matrices

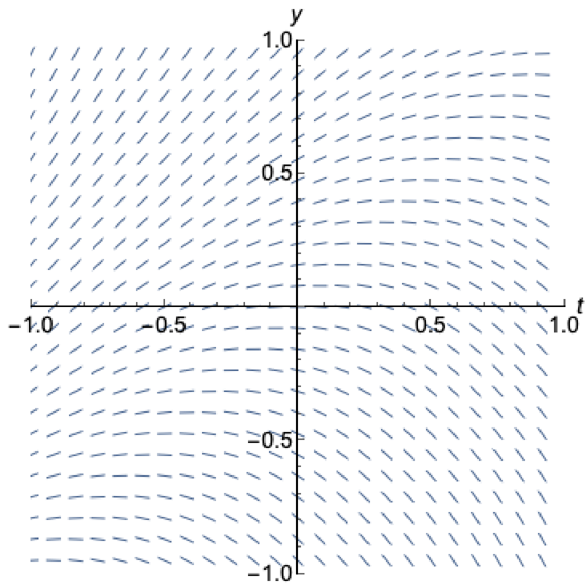
$$A = \begin{bmatrix} 2 & 4 & -2 & 1 & 11 \\ 3 & 6 & -3 & 1 & 15 \\ -1 & -2 & 1 & 2 & 2 \\ 4 & 8 & -4 & 4 & 28 \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} 1 & 2 & -1 & 0 & 4 \\ 0 & 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

where B is the reduced echelon form of A. A basis for the orthogonal complement of the row space of A is given by

(a) $\left\{ \begin{bmatrix} 2 \\ 3 \\ -1 \\ 4 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 2 \\ 4 \end{bmatrix} \right\}$ (b) $\left\{ \begin{bmatrix} -2 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} -4 \\ 0 \\ 0 \\ -3 \\ 1 \end{bmatrix} \right\}$ (c) $\left\{ \begin{bmatrix} -1 \\ -2 \\ 1 \\ 0 \\ -4 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \\ -1 \\ -3 \end{bmatrix} \right\}$

(d) $\left\{ \begin{bmatrix} -1 \\ -2 \\ 1 \\ 2 \\ 2 \end{bmatrix}, \begin{bmatrix} 4 \\ 8 \\ -4 \\ 4 \\ 28 \end{bmatrix} \right\}$ (e) $\left\{ \begin{bmatrix} 4 \\ 6 \\ -2 \\ 8 \end{bmatrix}, \begin{bmatrix} -2 \\ -3 \\ 1 \\ -4 \end{bmatrix}, \begin{bmatrix} 11 \\ 15 \\ 2 \\ 28 \end{bmatrix} \right\}$

12. Determine $f(t, y)$ if the differential equation $y' = f(t, y)$ has direction field (the value of t is measured on the horizontal axis, and the value of y on the vertical axis)



(a) $t + y$ (b) $t - y$ (c) y (d) $-t$ (e) $y - t$

13. Consider the line L spanned by the vector $\vec{u} = \begin{bmatrix} 3/5 \\ -4/5 \end{bmatrix}$, and let $\mathrm{proj}_L : \mathbb{R}^2 \to \mathbb{R}^2$ denote the linear transformation that sends a vector to its orthogonal projection onto the line L. The standard matrix of the transformation proj_L is

(a) $\begin{bmatrix} 3 & 0 \\ 0 & -4 \end{bmatrix}$ (b) $\frac{1}{5}\begin{bmatrix} 3 & 5 \\ 5 & -4 \end{bmatrix}$ (c) $\frac{1}{25}\begin{bmatrix} 9 & -12 \\ -12 & 16 \end{bmatrix}$ (d) $\frac{1}{5}\begin{bmatrix} 3 & 0 \\ 0 & -4 \end{bmatrix}$
(e) none of the above

14. Find the solution to the initial value problem

$$\frac{dy}{dt} - ty = t, \qquad y(0) = 1.$$

(a) $y = e^{2t^2}$ (b) $t = \ln(1 - y)$ (c) $y = e^t$
(d) $y = 2 - t^2$ (e) $y = 2e^{t^2/2} - 1$

15. The QR factorization of the matrix $A = \begin{bmatrix} 1 & 5 \\ 2 & 4 \\ -2 & 2 \end{bmatrix}$ has $Q = \frac{1}{3}\begin{bmatrix} 1 & 2 \\ 2 & 1 \\ -2 & 2 \end{bmatrix}$. What is R?

(a) $\frac{1}{3}\begin{bmatrix} 1 & 5 \\ 2 & 4 \end{bmatrix}$ (b) $\begin{bmatrix} 6 & 3 \\ 0 & 3 \end{bmatrix}$ (c) $\begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$ (d) $\begin{bmatrix} 3 & 3 \\ 0 & 6 \end{bmatrix}$ (e) $\begin{bmatrix} 5 & 1 \\ 0 & -3 \end{bmatrix}$.