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M20580 L.A. and D.E. Tutorial
Worksheet 2
Sections 1.3–1.4

1. Determine whether the vector w can be written as a linear combination of the vectors v_1, v_2, and v_3. If yes, find scalars a_1, a_2, a_3 such that $a_1v_1 + a_2v_2 + a_3v_3 = w$.

(a) $v_1 = \begin{bmatrix} 1 \\ -2 \\ 0 \end{bmatrix}$, $v_1 = \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix}$, $v_3 = \begin{bmatrix} 5 \\ -6 \\ 8 \end{bmatrix}$, and $w = \begin{bmatrix} 2 \\ -1 \\ 6 \end{bmatrix}$

Suppose $a_1V_1 + a_2V_2 + a_3V_3 = W$. Obtain an augmented matrix

$$\begin{bmatrix} 1 & 0 & 5 & 2 \\ -2 & 1 & -6 & -1 \\ 0 & 2 & 8 & 6 \end{bmatrix} \xrightarrow{R_2+2R_1} \begin{bmatrix} 1 & 0 & 5 & 2 \\ 0 & 1 & 4 & 3 \\ 0 & 2 & 8 & 6 \end{bmatrix} \xrightarrow{R_3-2R_2} \begin{bmatrix} 1 & 0 & 5 & 2 \\ 0 & 1 & 4 & 3 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$\Rightarrow$ yes
$$\begin{cases} a_1 + 5a_3 = 2 \\ a_2 + 4a_3 = 3 \\ a_3 \text{ free} \end{cases}$$

(b) $v_1 = \begin{bmatrix} 1 \\ 0 \\ -2 \end{bmatrix}$, $v_1 = \begin{bmatrix} -4 \\ 3 \\ 8 \end{bmatrix}$, $v_3 = \begin{bmatrix} 2 \\ 5 \\ -4 \end{bmatrix}$, and $w = \begin{bmatrix} 3 \\ -7 \\ -3 \end{bmatrix}$

Suppose $a_1V_1 + a_2V_3 + a_3V_3 = W$. Obtain an augmented matrix

$$\begin{bmatrix} 1 & -4 & 2 & 3 \\ 0 & 3 & 5 & -7 \\ -2 & 8 & -4 & -3 \end{bmatrix} \xrightarrow{R_3+2R_1} \begin{bmatrix} 1 & -4 & 2 & 3 \\ 0 & 3 & 5 & -7 \\ 0 & 0 & 0 & 3 \end{bmatrix}$$

$\Rightarrow$ $0=3$, so there are no solutions. Therefore, W is not a linear combination of V_1, V_2, V_3.

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2. Let $\mathbf{v}_1 = \begin{bmatrix} 1 \\ 0 \\ -1 \\ 2 \end{bmatrix}$, $\mathbf{v}_2 = \begin{bmatrix} 2 \\ 3 \\ 1 \\ -2 \end{bmatrix}$, $\mathbf{v}_3 = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 2 \end{bmatrix}$. Find the value of h such that $\mathbf{w} = \begin{bmatrix} 4 \\ 3 \\ h \\ 0 \end{bmatrix}$ is a linear combination of $\mathbf{v}_1$, $\mathbf{v}_2$, $\mathbf{v}_3$.

$$\begin{bmatrix} 1 & 2 & 0 & 4 \\ 0 & 3 & 0 & 3 \\ -1 & 1 & 1 & h \\ 2 & -2 & 2 & 0 \end{bmatrix} \xrightarrow[R_4-2R_1]{R_3+R_1} \begin{bmatrix} 1 & 2 & 0 & 4 \\ 0 & 3 & 0 & 3 \\ 0 & 3 & 1 & 4+h \\ 0 & -6 & 2 & -8 \end{bmatrix} \xrightarrow{(\frac{1}{3})R_2} \begin{bmatrix} 1 & 2 & 0 & 4 \\ 0 & 1 & 0 & 1 \\ 0 & 3 & 1 & 4+h \\ 0 & -6 & 2 & -8 \end{bmatrix}$$

$$\xrightarrow[R_3-3R_2]{R_1-2R_2} \begin{bmatrix} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & h+1 \\ 0 & -6 & 2 & -8 \end{bmatrix} \xrightarrow{R_4+6R_2} \begin{bmatrix} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & h+1 \\ 0 & 0 & 2 & -2 \end{bmatrix} \xrightarrow{R_4-2R_3} \begin{bmatrix} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & h+1 \\ 0 & 0 & 0 & -2h-4 \end{bmatrix}$$

Thus, w is a linear combination of v_1, v_2, v_3 iff

$$0 = -2h-4 \iff \boxed{h=-2}$$

3. Let $\mathbf{v}_1 = \begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix}$ and $\mathbf{v}_1 = \begin{bmatrix} 2 \\ 1 \\ -1 \end{bmatrix}$. Describe Span$\{\mathbf{v}_1, \mathbf{v}_2\}$.

(a) a plane
(b) $\mathbb{R}^3$
(c) a line
(d) a point
(e) three points

$$\begin{bmatrix} 1 & 2 \\ -1 & 1 \\ 2 & -1 \end{bmatrix} \xrightarrow{R_2+R_1} \begin{bmatrix} 1 & 2 \\ 0 & 3 \\ 2 & -1 \end{bmatrix} \xrightarrow{R_3-2R_1} \begin{bmatrix} 1 & 2 \\ 0 & 3 \\ 0 & -5 \end{bmatrix}$$

$$\Rightarrow \boxed{\text{plane}}$$

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4. Let $A = \begin{bmatrix} 1 & 3 & 0 & 3 \\ -1 & -1 & -1 & 1 \\ 0 & -4 & 2 & -8 \\ 2 & 0 & 3 & -1 \end{bmatrix}$.

(a) How many rows of A contain a pivot position? Does the equation $A\mathbf{x}=\mathbf{b}$ have a solution for each $\mathbf{b}$ in $\mathbb{R}^4$?

$$\begin{bmatrix} 1 & 3 & 0 & 3 \\ -1 & -1 & -1 & 1 \\ 0 & -4 & 2 & -8 \\ 2 & 0 & 3 & -1 \end{bmatrix} \xrightarrow[R_4-2R_1]{R_2+R_1} \begin{bmatrix} 1 & 3 & 0 & 3 \\ 0 & 2 & -1 & 4 \\ 0 & -4 & 2 & -8 \\ 0 & -6 & 3 & -7 \end{bmatrix} \xrightarrow[R_4+3R_2]{R_3+2R_2} \begin{bmatrix} 1 & 3 & 0 & 3 \\ 0 & 2 & -4 & 4 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 5 \end{bmatrix}$$

$$\xrightarrow{R_3 \leftrightarrow R_4} \begin{bmatrix} 1 & 3 & 0 & 3 \\ 0 & 2 & -4 & 4 \\ 0 & 0 & 0 & 5 \\ 0 & 0 & 0 & 0 \end{bmatrix} \Rightarrow \boxed{3 \text{ pivots}}$$

By Theorem 4 in Section 1.4, the answer to the second question is $\boxed{\text{No}}$

(b) Do the columns of A span $\mathbb{R}^4$?

By Theorem 4 again, the answer is $\boxed{\text{No}}$