The Cosmos of Science

Essays of Exploration

EDITED BY John Earman and
John D. Norton

University of Pittsburgh Press / Universitätsverlag Konstanz

4

A Peek Behind the Veil of Maya

Einstein, Schopenhauer, and the Historical Background of the Conception of Space as a Ground for the Individuation of Physical Systems

Don Howard
Department of Philosophy, University of Notre Dame

> We find among their many efforts to bring to light the analogy between all the phenomena of nature, many attempts, although unfortunate ones, to derive laws of nature from the mere laws of space and time. However, we cannot know how far the mind of a genius will one day realize both endeavors.
>
> —Arthur Schopenhauer, *Die Welt als Wille und Vorstellung*

Einstein's Berlin Portrait Gallery

According to Einstein's son-in-law and biographer, Rudolf Kayser, portraits of three figures hung on the wall of Einstein's Berlin study in the late 1920s: Michael Faraday, James Clerk Maxwell, and Arthur Schopenhauer (Reiser-Kayser 1930, 194).[1] One can guess why Faraday and Maxwell, the inventors of field theory, were there. But I have long been puzzled about what Schopenhauer was doing in this august company.

Something else puzzled me as well. From what source did Einstein draw the idea of "spatiotemporal separability,"—that a non-null spatiotemporal separation is a sufficient condition for the individuation of physical systems and their states, an idea fundamental both to his conception of field theory and to his famous reservations about the quantum theory? This idea makes its first appearance in Einstein's 1905 paper on the photon hypothesis and gradually finds ever-clearer expression. By the late 1940s, Einstein has disentangled it from other, related conceptions of independence, such as the concept of "locality," or "local action," and has come to regard it as the most essential aspect of his understanding of the very concept of physical reality. But, search as one will, this idea is not to be found anywhere in the scientific and philosophical literature normally regarded as having influenced Einstein's world view in his early years. It will not be found in Mach, Maxwell, Lorentz, Boltzmann, Hume, Poincaré, or Hertz.[2] It will not be found in period textbooks, either elementary texts, such as Violle (1892–1893), or more advanced ones, such as Föppl (1894). It will not be found in treatises on phoronomy, as in Aurel Voss's article in the *Encyklopädie der mathematischen Wissenschaften* (1901). And it will not be found in the popular science literature of the day, such as the works by Aaron Bernstein (1853–1857) or Ludwig Büchner (1855) that Einstein read as a youth. The idea may, of course, have been Einstein's own. Still, one must ask whether there was a source for it elsewhere in Einstein's readings.

I suggest that the solutions to the two puzzles may be related. Surprising as it may seem, Schopenhauer may well have been the source for the idea of spatiotemporal separability. Given how fundamental that idea was to Einstein's conception of a field theory, this may explain Schopenhauer's rather exalted place next to Faraday and Maxwell in Einstein's little Berlin gallery.

The argument for this conjecture will be circumstantial at many crucial places; I have found no smoking gun. All the more reason to be skeptical, as I was at first (and still am, to some small extent) about the possibility that a thinker like Schopenhauer, famous as "the philosopher of pessimism," could have had an important influence on Einstein's thinking about fundamental questions of the ontology of spacetime and field theories. Schopenhauer was certainly widely read in the late nineteenth and early twentieth centuries. His aesthetics is said to have been a crucial influence on Wagner's notion of the *Gesammtkunstwerk*. He was an important source for Nietzsche's concept of the will. And he influenced several generations of literary figures, Thomas Mann being only the most famous of many.[3] But such influences are different in kind from those I suggest he had on the young physicist, Einstein. Could Schopenhauer *really* have had an influence on Einstein's thinking about fundamental questions of the ontology of spacetime?

Patience and forbearance are required in order to permit this argument to get off the ground. One must put on hold a strong disinclination to believe in the influence suggested here. For part of that initial

skepticism results from the fact that we have all grown to philosophical maturity in an antimetaphysical age (at least the philosophers of science among us), an experience that leaves us ill-prepared, if not positively ill-disposed, to believe that Schopenhauer could have had this kind of influence on Einstein. Indeed, I suspect that one reason why better documentation for this influence is lacking is that, whereas many ardent young positivists were eager to ask Einstein about his reading of Mach and Hume, almost no one bothered to ask him about Schopenhauer.

Fortunately, a few people did ask, and Einstein himself on at least a few occasions wrote about what he learned from Schopenhauer. So we can say some things with reasonable certainty about his reading of Schopenhauer, his estimation of Schopenhauer as a writer, and the way in which Schopenhauer influenced his world view.

What I will argue, more specifically, is that several crucial features of Einstein's world view, in addition to the idea of spatiotemporal separability, could easily have been derived from his reading of Schopenhauer. At the very least, they would have found important confirmation in Schopenhauer. In particular, we will find in Schopenhauer a unique view, a critical reaction to Kant, about the equally fundamental importance of, on the one hand, space and time as the "*principium individuationis*," the "ground of being," and, on the other hand, causality, the "ground of becoming," for the constitution of representations of empirical objects in the understanding, both causality and space and time being seen by Schopenhauer as forms of the principle of sufficient reason.

There is a context for Schopenhauer's development of these themes. It goes back at least to Newton, if not still earlier, to ancient and medieval discussions of the problem of individuation. The specific issue of space and time as principles of individuation comes to the fore in the disputes between Leibniz and the Newtonians over absolute versus relational conceptions of space and time, as in the Leibniz-Clarke correspondence. An important chapter in the story concerns Kant's turn away from Leibniz and back to Newton, with his invention of the "incongruent counterparts" argument at the time of his *Inaugural Dissertation*. But it is Schopenhauer, more than anyone else—more even than the Marburg neo-Kantians—who brings the theme of space and time as principles of individuation into nineteenth-century discussions of space and time.

That Schopenhauer's philosophy could have been read in this way by philosophers of science and philosophically sophisticated physicists of Einstein's generation will be shown by looking at what thinkers as diverse as Mach, Schlick, Schrödinger, Weyl, Pauli, and Cassirer did in fact say about Schopenhauer. As one begins to appreciate *who* it was who read Schopenhauer in this way, and exactly *how* they read him, a picture begins to emerge in which Schopenhauer's distinctive views on the importance of space and time as the *principium individuationis* arguably form the background and provide the vocabulary for the early twentieth-century discussion of the way in which the spatiotemporal manner of individuating physical objects is thrown into doubt by the development of the quantum theory. Thus, Einstein's ardent defense of spatiotemporal separability as something fundamental to general relativity or any field theory, and the equally forceful critiques of separability in the work of Schrödinger and Pauli (and perhaps also Bohr) as something explicitly denied in the quantum-mechanical theory of interactions, must all be seen against this background informed by Schopenhauer.

Finally, when Einstein's defense of spatiotemporal separability is seen against this Schopenhauerian background, its place in his understanding of the ontology of general relativity assumes a new significance, inasmuch as it helps us to locate Einstein squarely within a tradition regarding the nature of space or spacetime going all the way back to Newton. What defines this tradition is not one's position in the dispute over absolute versus relative conceptions of space and time, or one's position in that more recent debate between substantival and relational conceptions of spacetime—both of which debates miss something essential in the Newtonian conception of space. What defines this tradition is, rather, a commitment to the idea that spatiotemporal separation is an objective feature of spacetime sufficient to serve as a ground for the individuation of systems and their states.

It was this characteristically Newtonian idea to which Leibniz really objected, arguing that systems are individuated not by extrinsic spatiotemporal determinations, but by intrinsic, qualitative determinations. It was this Newtonian idea that Kant reaffirmed with his incongruent counterparts argument. It was this idea that Schopenhauer bequeathed to the nineteenth century. And it was this idea that was preserved, in its most elementary form, stripping away all of the baggage of absolutist ideas, in Einstein's making the infinitesimal metric interval the fundamental invariant of the general theory of relativity.

Einstein's Reading of Schopenhauer

Schopenhauer was born in 1788 and died in 1860. His major published works include *Ueber die vierfache Wurzel des Satzes vom zureichenden Grunde* (On the fourfold root of the principle of sufficient reason, 1813); *Die Welt als Wille und Vorstellung* (The world as will and representation, 1st ed. 1819, 2nd ed., including first publication of vol. 2, 1844); *Ueber den Willen in der Natur* (On the will in nature, 1836); *Die beiden Grundprobleme der Ethik* (The two fundamental problems of ethics, 1841b); and *Parerga und Paralipomena* (1851). Schopenhauer began to win an audience for his writings only near the end of his life, but in the latter half of the nineteenth century and the first decades of the twentieth he had become perhaps the most widely read philosopher in German-speaking Europe, both among academic philosophers and among the broader public; there was also a large audience for his works in translation (see Laban 1880 and Hübscher 1981 for details on the editions and translations of Schopenhauer's works). Here is how a recent biographer explains Schopenhauer's posthumous popularity:

> In Schopenhauer, readers found an encomium of a sober sense of reality, of materialistic explanation, and a justification, based on Kant, of why our empirical curiosity must follow this road. They found a confirmation of what they were, materialistically, doing. Simultaneously, however, they found in Schopenhauer the empirical proof that this approach to reality was not the only one. Even the materially visualized world still remains an idea. Schopenhauer inaugurated a new renaissance of Kant and opened up the possibility of a "materialism as if." One could endorse strictly empirical science, one could surrender to the materialistic spirit, but one need not be totally captured by it. With Schopenhauer's "Beyond" of the self-experienced will one could now withstand the pull of a materially interpreted immanence.
>
> Even more effective than this "as if" materialism was the "as if" ethics which Schopenhauer sketched out in his "philosophy for the world." After 1850 his "Aphorisms on Practical Wisdom" [*Aphorismen zur Lebensweisheit*, part of *Parerga und Paralipomena* (1851)] rapidly became the Bible of the educated bourgeoisie. (Safranski 1990, 334)

Einstein, born in 1879 to a father just starting out in the new electrotechnology business, was a child of this educated bourgeoisie. (See Einstein 1987, xlviii–lxvi, on Einstein's early life and family circumstances.)

Rudolf Kayser, the son-in-law and biographer who gave us the story of the portraits in Einstein's Berlin study, describes Einstein's reading of Schopenhauer during his student years at the ETH in Zurich:

> Despite all the progress in understanding and knowledge achieved by the youthful physicist and mathematician, almost a dislike for science and its intellectual technique remained with him after he had finished his course of study. He overcame it only a long time afterwards. He approached the broader aspects of thought through philosophical studies, chiefly through his readings in Kant and Schopenhauer, and later through his study of Hume, with whom he felt a special kinship. (Reiser-Kayser 1930, 55)

This being the only biography of himself that Einstein read before publication, it deserves to be trusted in such matters.[4] But we have Einstein's own words to the same effect. Responding to a question about his early readings from another, later biographer, Carl Seelig, Einstein wrote on 20 April 1952:

> As a young man (and even later) I concerned myself little with poetical literature and novels. . . . I preferred books whose content concerned a whole world view [*Bücher weltanschaulichen Inhalts*] and, in particular, philosophical ones. Schopenhauer, David Hume, Mach, to some extent Kant, Plato, and Aristotle. (EA 39-019)[5]

Moreover, there is every reason to believe that this interest in Schopenhauer continued in Einstein's later years. Kayser says about Einstein in the late 1920s, "He has read the most important works of classical philosophy, and in his hours of leisure he returns with especial pleasure to Plato, Hume and Schopenhauer" (Reiser-Kayser 1930, 197). And Konrad Wachsmann, the architect of Einstein's summer house in Caputh, recalls of that same time, the late 1920s and early 1930s, "The philosophers occupied him more than *belles lettres*. Above all, he read a lot of Schopenhauer. On Haberlandstraße and also in Caputh, he often sat with one of the already well-worn Schopenhauer volumes, and as he sat there reading, he seemed so pleased, as if he were engaged with a serene and cheerful work" (Grüning 1989, 243). Wachsmann adds, "He always insisted that the engagement with Schopenhauer, Kant, the Greeks, or even Locke and Hume, gave him far and away greater pleasure than, for example, Goethe's — as it is so nicely known — great epic of human kind [*großes Menschheitsgedicht*]" (Grüning 1989, 247).

Remarks such as these suggest that Einstein read widely in Schopenhauer's works. A complete 1894–1896 edition of Schopenhauer's works, answering Wachsmann's description of the "already well-worn Schopenhauer volumes," survives in Einstein's private library.[6] But what, exactly, did Einstein read? Here, unfortunately, the documentation is thin. The only explicit reference I have found is in a letter from Einstein to his friend, ETH classmate, and future collaborator, Marcel Grossmann, on 6 September 1901: "What kinds of things do you do with your free time these days? Have you too already looked at Schopenhauer's *Aphorismen zur Lebensweisheit*? It's part of *Parerga & Paralipomena* and I liked it very much" (Einstein 1987, Doc. 122, 316). The reading of the *Aphorismen* evidently affected Einstein deeply enough that an analogy with Schopenhauer's professional isolation readily came to mind when he wrote to his future first wife, Mileva Marić, on 17 December 1901, describing the peculiar situation in which he found himself as a private tutor living and working in the home of Carl Baumer, a teacher in Schaffhausen:

> It is really a very funny life I live here, precisely in Schopenhauer's sense. That is to say that, aside from my pupil, I speak with no one the whole day long. Even Herr Baumer's company seems to me boring and insipid. I always find that I am in the best company when I am alone, except when I am together with you. (Ibid., Doc. 128, 325)

When he wrote this, one day before applying for the position at the Swiss Federal Patent Office that he occupied from 1902 to 1909 (ibid., Doc. 129), Einstein was nearing the end of a long period of despair over his failure to find a regular professional position. He could, no doubt, empathize with Schopenhauer's failure ever to win a regular academic appointment, after his failed attempt as a lecturer in Berlin in 1820 (see Safranski 1990, 250–63).

Indeed, Schopenhauer's acid cynicism about professional philosophy and the academy more generally would have found a sympathetic audience in the young Einstein. Einstein was a headstrong young man, sure enough of his own abilities that, according to legend, he did not bother with many of the required lectures during his years at the ETH (1896–1900), borrowing Grossmann's notes if necessary and otherwise reading on his own the works of Mach, Helmholtz, Maxwell, Hertz, Boltzmann, and Lorentz (see Einstein 1946, 15; 1955, 10–11; Kollros 1955, 22). Einstein did not like the rigid structure of university instruction. His recollections of this time echo Kayser's remark about the young Einstein's "distaste for science and its intellectual technique" at the time when he started reading Schopenhauer:

For people like me, with a brooding interest, university study is not an unqualified blessing. If forced to eat so many good things, one can do lasting damage to one's appetite and stomach. The light of holy curiosity can be forever extinguished. Happily, in my case, this intellectual depression endured for only one year after the successful completion of my studies. (Einstein 1955, 12)

At the end of that year, Einstein was recommending Schopenhauer to Grossmann.

Einstein's cynicism about the university and professional academic life was reinforced by the fact that his hopes for a position as *Assistent* were thwarted at every turn (see Einstein 1987, esp. letters to Marić in the spring of 1901) forcing him to take unrewarding positions as a temporary teacher at the *Technikum* in Winterthur and as a private tutor in Schaffhausen, before finally going to work at the Patent Office. What Schopenhauer says in the *Aphorismen* about finding happiness in "what one is," as opposed to "what one has" or "what one represents" (one's fame, reputation, or standing) would surely have appealed to an Einstein who lacked a steady income and whose talents were not being recognized:

A tranquil and serene temperament, the result of perfect health and successful organization, a clear, lively, penetrating, and sure intellect, a measured, gentle will, and, thus, a clear conscience, these are advantages that no fame or riches can replace. For what a person is for himself, what abides with him in his loneliness and isolation, and what no one can give or take away from him, this is obviously more essential for him than everything that he possesses or what he may be in the eyes of others. A gifted man, in complete isolation, enjoys splendid converse with his own thoughts and fancies, whereas for an obtuse stump of a man even a continuing variety of companions, plays, excursions, and amusements cannot protect against torturous boredom. . . . Thus, for one's happiness in this life, that which one *is*, one's personality, is absolutely the first and most essential thing. (Schopenhauer 1851, vol. 1, 348–49)

But was this all that Einstein took from Schopenhauer? Was providing a voice and a validation for Einstein's late-adolescent alienation enough to earn Schopenhauer a place of honor in the portrait gallery? Was this why the mature and world-wise Einstein of 1930 was still reading Schopenhauer avidly? I think not. Einstein, himself, tells us in several places what else he learned from Schopenhauer.

Consider, first, Einstein's solution to the problem of free will. Einstein's commitment to determinism is well known. Equally important to his larger world view, however, was the extension of this meta-

physics to the consideration of human freedom. One of his first public discussions of this issue was in an essay entitled, "The World as I See It":

I do not at all believe in human freedom in the philosophical sense. Everybody acts not only under external compulsion but also in accordance with inner necessity. Schopenhauer's saying, "A man can do what he wants, but not want what he wants," has been a very real inspiration to me since my youth; it has been a continual consolation in the face of life's hardships, my own and others', and an unfailing well-spring of tolerance. This realization mercifully mitigates the easily paralyzing sense of responsibility and prevents us from taking ourselves and other people all too seriously; it is conducive to a view of life which, in particular, gives humor its due. (Einstein 1931, 8–9)

As Friedrich Herneck has pointed out—in the only essay I know of that takes seriously the question of Schopenhauer's influence on Einstein—that exact quotation, "Ein Mensch kann zwar tun, was er will, aber nicht wollen, was er will," is not to be found in Schopenhauer's writings (Herneck 1969, 204), although many similar formulations are scattered throughout his works. Most relevant is his essay, "Ueber die Freiheit des menschlichen Willens" (1841). Schopenhauer begins by suggesting that we recast the question of free will not as a question about *physical freedom*—whether our doing what we will may be blocked by physical impediments ("Frei bin ich, wenn ich *thun* kann, *was ich will*")—but as a question about *moral freedom*, which he understands as a question about our capacity to want that which we will—"Kanst du auch *wollen*, was du willst?" (ibid., 364). Schopenhauer's answer, which rests on his doctrine of the will as the thing-in-itself, is that we are obviously free in the physical sense—that one does what one wills being almost an analytic truth—but that we are not free in the moral sense. Hence Einstein's paraphrase, and his remark that we act "in accordance with inner necessity." Our actions are controlled outwardly by physical causality—Einstein and Schopenhauer are both strict determinists as regards the course of physical events, such physical determinism being the first of Schopenhauer's four forms of the principle of sufficient reason. But we are also constrained inwardly—our psychological motives are no more under voluntary control than our outward actions, psychological determinism being the fourth form of the principle of sufficient reason. The will itself, as thing-in-itself, is not under the control of the principle of sufficient reason in any of its forms. It is only the objectifications of the will in phenomenal objects and events, both outer and inner, to which the principle of sufficient reason pertains.

How much of the metaphysical foundation to Schopenhauer's position on the freedom of the will Einstein accepted is hard to say. But another hint that he accepted enough to discomfit even the most tolerant logical empiricist is contained in a second essay from about this same time, a brief piece entitled "Religion and Science," first published in the *New York Times Sunday Magazine* for 9 November 1930. Einstein's main thesis is that while science and religion, viewed historically, may appear to be "irreconcilable antagonists," at a deeper level the "cosmic religious feeling," which distinguishes the "religious geniuses of all ages," is in fact "the strongest and noblest motive for scientific research," this same feeling being what gave Kepler and Newton "the strength to remain true to their purpose in spite of countless failures." And what is this "cosmic religious feeling"? It is a "third stage of religious experience" common to all historical religions, after the religion of fear and moral religion. Einstein explains:

> It is very difficult to elucidate this feeling to anyone who is entirely without it, especially as there is no anthropomorphic conception of God corresponding to it.
>
> The individual feels the futility of human desires and aims and the sublimity and marvelous order which reveal themselves both in nature and in the world of thought. Individual existence impresses him as a sort of prison and he wants to experience the universe as a single significant whole. The beginnings of cosmic religious feeling already appear at an early stage of development, e.g., in many of the Psalms of David and in some of the Prophets. Buddhism, as we have learned especially from the wonderful writings of Schopenhauer, contains a much stronger element of this. (Einstein 1930, 38)

One will not find this precise conception of "cosmic religion" anywhere in Schopenhauer, but the quoted lines echo a number of characteristically "Schopenhauerian" themes. Thus, for example, Schopenhauer writes that the historical religions and science are "natural enemies," adding that "to want to speak of peace and reconciliation between the two is most laughable; it is a *bellum ad internecionem*" (1851, vol. 2, 431).

More importantly, however, Schopenhauer distinguishes philosophy and religion as two kinds of metaphysics, the first having "its verification and credentials *in itself*, the other *outside itself*," this external verification being revelation (1859, vol. 2, 164). They are further

distinguished by philosophy's being obliged to be true *sensu stricto et proprio*, whereas religion is obliged only to be true *sensu allegorico*, neglect of religion's allegorical nature being the source of error and confusion, especially among those who seek to rationalize religion and to reconcile it with science (ibid., 166–68). Schopenhauer adds about Buddhism:

> The value of a religion will depend on the greater or lesser content of truth which it has in itself under the veil of allegory; next on the greater or lesser distinctness with which this content of truth is visible through the veil, and hence on the veil's transparency. It almost seems that, as the oldest languages are the most perfect, so too are the oldest religions. If I wished to take the results of my philosophy as the standard of truth, I should have to concede to Buddhism pre-eminence over all the others. In any case, it must be a pleasure to me to see my doctrine in such close agreement with a religion that the majority of men on earth hold as their own, for this numbers far more followers than any other. (Ibid., 169)

In this light, Einstein's conception of "cosmic religion," a "much stronger element" of which is contained in Buddhism, has a distinctly Schopenhauerian cast. At the heart of Schopenhauer's philosophy is the idea that human individuality is a kind of illusion, the necessary form in which the will objectifies itself in space and time, but not an aspect of the will in itself. The momentary overcoming of one's individuality, the recognition of "the futility of human desires and aims," a glimpse into "the sublimity and marvelous order which reveal themselves both in nature and in the world of thought," the experience of "the universe as a single significant whole"—all of these are, for Schopenhauer, marks of the way genius apprehends the world:

> Genius is the ability to leave entirely out of sight our own interest, our willing, and our aims, and consequently to discard entirely our own personality for a time, in order to remain *pure knowing subject*, the clear eye of the world; and this not merely for moments, but with the necessary continuity and conscious thought to enable us to repeat by deliberate art what has been apprehended, and "what in wavering apparition gleams fix in its place with thoughts that stand forever." (1859, vol. 1, 185–86; the quotation is from Goethe's *Faust*)

It is essential to the expression of genus, for Schopenhauer, that one apprehends the "Idea" standing behind the plurality of its phenomenal manifestations. This "Idea" is not subject to the forms of space and time, the *principium individuationis*, and is thus a unity, a whole, different in kind from individual objects in space and time.

From an early age, Einstein was captivated by the prospect of a mode of comprehension that would carry one out of oneself, and he linked this yearning with his scientific ambitions. In his "Autobiographical Notes," he writes thus about his turn away from organized religion:

It is quite clear to me that the religious paradise of youth, which was thus lost, was a first attempt to free myself from the chains of the "merely personal," and from an existence dominated by wishes, hopes, and primitive feelings. Out yonder there was this huge world, which exists independently of us human beings and which stands before us like a great, eternal riddle, at least partially accessible to our inspection and thinking. The contemplation of this world beckoned as a liberation, and I soon noticed that many a man whom I had learned to esteem and to admire had found inner freedom and security in its pursuit. The mental grasp of this extra-personal world within the frame of our capabilities presented itself to my mind, half consciously, half unconsciously, as a supreme goal. Similarly motivated men of the present and of the past, as well as the insights they had achieved, were the friends who could not be lost. The road to this paradise was not as comfortable and alluring as the road to the religious paradise; but it has shown itself reliable, and I have never regretted having chosen it. (1946, 5)

Such remarks breathe the spirit of Schopenhauer, a connection made explicitly by Einstein himself in his 1918 address in honor of Max Planck's sixtieth birthday:

In the temple of science are many mansions, and various indeed are they that dwell therein and the motives that have led them thither. Many take to science out of a joyful sense of superior intellectual power; science is their own special sport to which they look for vivid experience and the satisfaction of ambition; many others are to be found in the temple who have offered the products of their brains on this altar for purely utilitarian purposes. Were an angel of the Lord to come and drive all of the people belonging to these two categories out of the temple, the assemblage would be seriously depleted, but there would still be some men, of both present and past times, left inside. Our Planck is one of them, and that is why we love him. . . . Now let us have another look at those who have found favor with the angel. Most of them are somewhat odd, uncommunicative, solitary fellows, really less like each other, in spite of these common characteristics, than the hosts of the rejected. What has brought them to the temple? . . . To begin with, I believe with Schopenhauer that one of the strongest motives that leads men to art and science is escape from everyday life with its painful crudity and hopeless dreariness, from the fetters of one's own ever shifting desires. A finely tempered nature longs to escape from personal life into the world of objective perception and thought; this desire may be compared with the townsman's irresistible longing to escape from his noisy, cramped surroundings into the silence of high mountains, where the eye ranges freely through the still, pure air and fondly traces out the restful contours apparently built for eternity. (1918, 224–25)

Talk of escape from "personal life" into the world of "objective perception" is vintage Schopenhauer; the expression, "objective perception," is Schopenhauer's term for the kind of knowledge achieved when the genius frees himself from the will and from his own individuality enough to apprehend the Ideas (in a quasi-Platonic sense) standing behind the will's objectification in phenomenal, empirical objects and events. Ordinary scientific knowledge is termed, by contrast, "subjective" (see Schopenhauer 1859, vol. 1, book 3). Even the image of the mountaintop is a Schopenhauer hallmark (see Schopenhauer 1985, 14, for one example of many).

It has been said that Einstein appreciated Schopenhauer only as a writer, not as a thinker. Thus, an early interviewer, Alexander Moszkowski, writes: "To Schopenhauer and Nietzsche he assigns a high position as writers, as masters of language and moulders of impressive thoughts. He values them for their literary excellence, but denies them philosophic depth" (Moszkowski 1921, 237). And Philipp Frank observes:

Einstein read philosophical works from two points of view, which were sometimes mutually exclusive. He read some authors because he was actually able to learn from them something about the nature of general scientific statements, particularly about their logical connection with the laws through which we express direct observations. These philosophers were chiefly David Hume, Ernst Mach, Henri Poincaré, and, to a certain degree, Immanuel Kant. Kant, however, brings us to the second point of view. Einstein liked to read some philosophers because they made more or less superficial and obscure statements in beautiful language about all sorts of things, statements that often aroused an emotion like beautiful music and gave rise to reveries and meditations on the world. Schopenhauer was pre-eminently a writer of this kind, and Einstein liked to read him without in any way taking his views seriously. In the same category he also included philosophers like Nietzsche. Einstein read these men, as he sometimes put it, for "edification," just as other people listen to sermons. (1947, 51)

It is difficult to reconcile these characterizations with the evidence of Einstein's own words. I think that Moszkowski and Frank are simply wrong. It is worth recalling that the Moszkowski volume is notoriously unreliable, Einstein having been persuaded by close friends to try to block its publication (see Born 1969, 62–70), and that Philipp Frank was an ardent logical empiricist whose characterization of Einstein's way of reading philosophers deserves to be dismissed as nothing more than a piece of positivist propaganda, part of the positivist-revisionist reading of Einstein that, unfortunately, gained wide acceptance in the middle decades of this century in spite of Einstein's careful, repeated attempts to distance himself from positivism.[7]

If we put aside positivist prejudices, and let Einstein speak for himself, it appears that he did see in Schopenhauer a certain "philosophic depth." After all, Einstein, if not Frank, put Schopenhauer first in the list of his early philosophical readings, *before* Hume, Mach, and Kant. It is, again, only an antimetaphysical, positivist prejudice that would lead one to say that Einstein could have taken his solution to the problem of free will and key ingredients of his concept of cosmic religion from Schopenhauer "without in any way taking his views seriously."

Do such metaphysical doctrines have a place in Einstein's world view? Recall what Einstein wrote to Schlick in a letter that sealed Einstein's turn away from Schlick's brand of positivism:

> Generally speaking, your presentation does not correspond to my way of viewing things, inasmuch as I find your whole conception, so to speak, too positivistic. Indeed, physics *supplies* relations between sense experiences, but only indirectly. For me *its essence* is by no means exhaustively characterized by this assertion. I put it to you bluntly: Physics is an attempt to construct conceptually a model of the *real world* as well as of its law-governed structure. To be sure, it must represent exactly the empirical relations between those sense experiences accessible to us; but *only* thus is it chained to the latter. . . . You will be surprised by Einstein the "metaphysician." But in this sense every four- and two-legged animal is, de facto, a metaphysician. (EA 21-603)[8]

This was written on 28 November 1930, roughly the time when various sources, such as Kayser and Wachsmann, report Einstein's intense preoccupation with Schopenhauer and immediately before the period when we find Einstein citing Schopenhauer in discussions of free will and cosmic religion. Had he perhaps just read or reread, in one of those "well-worn Schopenhauer volumes," chapter 17 of volume 2 of *Die Welt als Wille und Vorstellung*, the chapter entitled "Über das metaphysische Bedürfniß des Menschen" (On man's need for metaphysics), where Schopenhauer wrote:

> [Man] is an *animal metaphysicum*. At the beginning of his consciousness, he naturally takes himself also as something that is a matter of course. This, however, does not last long, but very early, and simultaneously with the first reflection, appears that wonder which is some day to become the mother of metaphysics. (1859, vol. 2, 160)[9]

We begin to appreciate why Schopenhauer's portrait was in Einstein's study. But still one must ask, why in the company of Faraday and Maxwell?

Einstein on Spatiotemporal Separability

Important though they may be to Einstein's larger world view, cosmic religion and the problem of free will are not among our main concerns here. Our concern is rather with the question of whether or not Schopenhauer's doctrine of space and time as the *principium individuationis*, the ground of individuation for empirical objects, was the source for Einstein's doctrine of separability, according to which non-null spatiotemporal separation is a sufficient condition for the individuation of physical systems. To begin to try to answer this question, let us review what Einstein had to say about separability (see Howard 1990, 1985).

Einstein's concern with the problem of separability first emerges in his 1905 paper on the light quantum hypothesis. Einstein explains there that what he means by the assumption that light quanta behave like spatially localized, "independent" particles is that, in a two-particle system, the joint probability for the two particles occupying given cells in phase space is simply the product of the separate occupation probabilities. This factorizability is a necessary and sufficient condition for Boltzmann's principle, $S = k \cdot \log W$, to hold (1905, 140). But he adds that this way of conceiving light quanta leads not to Planck's law for black-body radiation, but to Wien's law, which is valid only in the limit of large ν/T, implying that this independence and factorizability must be *denied* in order to get the universally valid Planck formula (ibid., 143).

The fact that photons behave like independent particles in the Wien regime, but *not* like independent particles more generally was a source of great puzzlement to Einstein in subsequent years. This puzzlement led first to his speculations about wave-particle duality in 1909, when he found that expressions for the mean-square fluctuations of the energy in a radiation-filled cavity and the mean-square fluctuations in radiation pressure can each be written as a sum of two terms. Einstein interpreted one as corresponding to behavior typical of "independent," localized, particlelike light quanta appearing in the Wien regime, and the other as corresponding to nonindependent, wavelike behavior appearing in the limit of small ν/T (Einstein 1909a, 1909b).

The next step came in late 1924 and early 1925, as Einstein pondered Satyendra Nath Bose's (1924) new idea for deriving the Planck formula from a non-Boltzmannian statistics, based on a different set of assumptions about what count as distinct configurations in the particle's phase space. The key idea here is that, in a two-particle phase space, for example, mere exchange of spatial location of the two particles does not yield a different configuration, implying that the different spatial locations of the two particles do not suffice to endow them with discernible identities. In a series of three papers, Einstein (1924, 1925a, 1925b) applied Bose's idea to a quantum gas of material particles. In the second paper, prompted by a question from Paul Ehrenfest, Einstein for the first time drew attention to the way in which the application of the new statistics to material particles throws into question our classical assumptions about particle independence: "Thus, the formula [for the entropy] indirectly expresses a certain hypothesis about a mutual influence of the molecules—for the time being of a quite mysterious kind—which determines precisely the equal statistical probability of the cases here defined as 'complexions' " (1925a, 6).

Erwin Schrödinger's reading of these three papers was an important stimulus to his development of wave mechanics. As we now understand, one of his main aims was to provide an explanation for the failure of quantum systems to behave like independent classical particles. The issue of independence is at the heart of an extensive correspondence between Einstein and Schrödinger during 1925 and 1926, beginning with Einstein's explaining to Schrödinger in a letter of 28 February 1925, "In the Bose statistics employed by me, the quanta or molecules are not treated as being *independent of one another.*" Einstein added in a postscript that, in relatively dense gases, where the difference between Boltzmann and Bose statistics should be especially noticeable: "There the interaction between the molecules makes itself felt,—the interaction which, for the present, is accounted for statistically, but whose physical nature remains veiled" (EA 22-002).

Schrödinger eventually solved this problem by showing that, if we

employ configuration space, rather than physical space, for the representation of many-particle systems, we can construct there joint wave functions that are not factorizable into separate wave functions for the constituent systems, and that these nonfactorizable wave functions are necessary to account correctly for interference effects in quantum mechanical interactions. But precisely this feature of the Schrödinger formalism troubled Einstein deeply, as he made clear first in a letter to Schrödinger of 16 April 1926 (EA 22-012), and then, even more clearly, in a note "Added in Proof" to the never published manuscript of a talk to the Prussian Academy of Sciences on 5 May 1927. In his talk to the Academy, entitled "Does Schrödinger's Wave Mechanics Determine the Motion of a System Completely or Only in the Statistical Sense?" Einstein had attempted a kind of hidden-variables interpretation of the Schrödinger formalism.[10] The attempt failed, in Einstein's eyes (presumably why the talk was never published), because of nonseparability. Einstein wrote in the note "Added in Proof":

I have found that the schema does not satisfy a general requirement that must be imposed on a general law of motion for systems.

Consider, in particular, a system Σ that consists of two energetically independent subsystems, Σ_1 and Σ_2; this means that the potential energy as well as the kinetic energy is additively composed of two parts, the first of which contains quantities referring only to Σ_1, the second quantities referring only to Σ_2. It is then well known that

$$\Psi = \Psi_1 \cdot \Psi_2,$$

where Ψ_1 depends only on the coordinates of Σ_1, Ψ_2 only on the coordinates of Σ_2. In this case we must demand that the motions of the composite system be combinations of possible motions of the subsystems.

The indicated scheme does not satisfy this requirement. In particular, let μ be an index belonging to a coordinate of Σ_1, ν an index belonging to a coordinate of Σ_2. Then $\Psi_{\mu\nu}$ does not vanish. (EA 2-100)

From this time on, Einstein ceased being an active contributor to the development of the quantum theory he had been helping to shape since 1905. Instead, he turned his energies both to the development of a unified field theory, which he believed would satisfy the mentioned "general requirement that must be imposed on a general law of motion for systems"—in essence the separability principle—and to the progressive refinement of his argument for why the quantum theory's failure to satisfy this requirement is so objectionable.

The next phase of this story, namely, Einstein's encounters with Niels Bohr at the 1927 and 1930 Solvay conferences, has been told many times by many authors. However, most of them, including, most notably, Max Jammer (1974, 1985) and Bohr himself (1949), get the story wrong. The old view—encouraged by both Jammer and Bohr—was that Einstein first sought to show the quantum theory incorrect, as with the famous "photon-box" thought experiment, purportedly designed to exhibit violations of the Heisenberg uncertainty principle, and that he shifted to arguing for the theory's incompleteness only after Bohr had cleverly refuted all of his attempts to prove it incorrect. We now know that from the start Einstein was trying instead to formulate thought experiments that would bring to the fore the peculiar consequences of quantum nonseparability. Even the photon-box thought experiment was intended to show that, *if* one assumed the separability of the systems involved in two spacelike separated measurement events (the box and the emitted photon), *then* the quantum mechanical account of these measurements would be incomplete. (See Howard 1990, 98–100, for a discussion of Ehrenfest's letter to Bohr, 9 July 1931.)

Arthur Fine was the first to draw our attention to the way Einstein's concern with the separability problem informed his critique of the quantum theory, especially in his pioneering reanalysis of the Einstein-Podolsky-Rosen (EPR) argument for the incompleteness of quantum mechanics (Fine 1981; see also Howard 1985). Fine showed that, in correspondence with Schrödinger starting in June 1935, immediately after publication of the EPR paper (Einstein, Podolsky, and Rosen 1935), Einstein repudiated the published EPR paper, noting, "For reasons of language, this was written by Podolsky after many discussions. But still it has not come out as well as I really wanted; on the contrary, the main point was, so to speak, buried by the erudition" (Einstein to Schrödinger, 19 June 1935, EA 22-047). Einstein went on to explain what that "main point" was, elaborating a very different incompleteness argument similar in concept to the earlier photon-box thought experiment and based explicitly upon what Einstein there dubs the "separation principle."

Einstein begins with a simple example: two boxes and a single ball always located in one or the other of the boxes. We make an "observation" simply by lifting the lid on a box and looking inside. Consider now a state description: "The probability that the ball is in the first box is 1/2." Is it a complete description? Einstein says that one who subscribes to the "Born" interpretation must answer no, since, from that point of view, a complete description would be a categorical assertion that the ball *is* (or *is not*) in the first box. One who subscribes to the "Schrödinger" interpretation, by contrast, would say yes, claiming that before we make an observation, the ball is not really in either box, and that this "being in a definite box" is, in fact, the result of the observation, so that the state of the first box before we make an observation is described completely by the probability 1/2.

At least "Born" and "Schrödinger" will talk about the *real* state of the system. Not so Bohr, who is too much a positivist in Einstein's eyes: "The talmudic philosopher doesn't give a hoot for 'reality,' which he regards as a hobgoblin of the naive, and he declares that the two points of view differ only as to their mode of expression." What is Einstein's view? He writes:

> My way of thinking is now this: properly considered, one cannot get at the talmudist if one does not make use of a supplementary principle: the "separation principle." That is to say: "the second box, along with everything having to do with its contents, is independent of what happens with regard to the first box (separated partial systems)." If one adheres to the separation principle, then one thereby excludes the second ("Schrödinger") point of view, and only the Born point of view remains, according to which the above state description is an *incomplete* description of reality, or of the real states. (EA 22-047)

One cannot "get at" the "talmudist"—the antirealist Bohr—without the "separation principle."[11]

The actual argument for the incompleteness of quantum mechanics, developed in the 19 June 1935 letter to Schrödinger, proceeds as follows. We start with a definition of the "completeness" of a state description, Ψ: "Ψ is correlated one-to-one with the real state of the real system. . . . If this works, then I speak of a complete description of reality by the theory. But if such an interpretation is not feasible, I call the theoretical description 'incomplete'" (EA 22-047). Einstein then sketches the typical EPR-type thought experiment involving spatially separated, but previously-interacting systems *A* and *B*. According to the orthodox quantum-mechanical formalism, depending upon the *kind* of measurement we choose to perform on system *A* (choice of observable, not actual outcome), we ascribe different state functions, Ψ_B or $\underline{\Psi}_B$, to system *B*. It follows that quantum mechanics is incomplete:

Now what is essential is exclusively that Ψ_β and Ψ_β are in general different from one another. I assert that this difference is incompatible with the hypothesis that the Ψ description is correlated one-to-one with the physical reality (the real state). After the collision, the real state of (AB) consists precisely of the real state of A and the real state of B, which two states have nothing to do with one another. *The real state of B thus cannot depend upon the kind of measurement I carry out on A.* ("Separation hypothesis" from above.) But then for the same state of B there are two (in general arbitrarily many) equally justified Ψ_β, which contradicts the hypothesis of a one-to-one or complete description of the real states. (EA 22-047)

Among the many important outgrowths of the Einstein-Schrödinger correspondence from the summer of 1935 was the famous Schrödinger "cat paradox" (Schrödinger 1935a; see Fine 1986a). Perhaps the most important result, however, was Schrödinger's now classic formulation of the quantum mechanical interaction formalism, in which the nonseparability of the formalism—in the form of the nonfactorizability of the post-interaction joint state—is so prominently highlighted (1935b, 1936).

Over the next fourteen years, Einstein repeated and refined this argument in a number of publications (for details, see Howard 1985). Most important for our purposes is the way in which he clarified his understanding of the kind of independence intended in the "separation principle," gradually disentangling what is now more commonly designated the "separability principle" from the "locality principle." The separability principle is a fundamental ontological principle according to which any two systems separated by a non-null spatiotemporal interval, regardless of their history of interaction, possess their own separate real states that completely determine the joint state. In the language of the orthodox quantum mechanical formalism, this is simply the requirement (not generally satisfied in the quantum mechanical account of interactions) that $\Psi_{AB} = \Psi_A \otimes \Psi_B$. The "locality principle"—essentially a statement of the special relativistic prohibition on superluminal influences—says that, given any two space-like separated systems, A and B, the separate real state of one cannot be influenced by events (such as the choice of an observable to measure, for example, by rotating a Stern-Gerlach apparatus plus detectors) in the vicinity of the other.

Einstein explained the distinction in a restatement of the incompleteness argument for a 1948 special issue of the Swiss journal *Dialec-*

tica, edited by Pauli, that was devoted to the interpretation of quantum mechanics:

If one asks what is characteristic of the realm of physical ideas independently of the quantum-theory, then above all the following attracts our attention: the concepts of physics refer to a real external world, *i.e.*, ideas are posited of things that claim a "real existence" independent of the perceiving subject (bodies, fields, *etc.*), and these ideas are, on the other hand, brought into as secure a relationship as possible with sense impressions. Moreover, it is characteristic of these physical things that they are conceived of as being arranged in a space-time continuum. Further, it appears to be essential for this arrangement of the things introduced in physics that, at a specific time, these things claim an existence independent of one another, insofar as these things "lie in different parts of space." Without such an assumption of the mutually independent existence (the "being-thus") of spatially distant things, an assumption that originates in everyday thought, physical thought in the sense familiar to us would not be possible. Nor does one see how physical laws could be formulated and tested without such a clean separation. Field theory has carried out this principle to the extreme, in that it localizes within infinitely small (four-dimensional) space-elements the elementary things existing independently of one another that it takes as basic, as well as the elementary laws it postulates for them.

For the relative independence of spatially distant things (A and B), this idea is characteristic: an external influence on A has no *immediate* effect on B; this is known as the "principle of local action," which is applied consistently only in field theory. The complete suspension of this basic principle would make impossible the idea of the existence of (quasi-) closed systems and, thereby, the establishment of empirically testable laws in the sense familiar to us. (Einstein 1948, 321–22)

These are very rich paragraphs. Separability, "the mutually independent existence (the 'being thus') of spatially distant things," is distinguished from locality, "the principle of local action." Both are tied in special ways to field theory, which is said to carry out the separability principle "to the extreme" and to be the only place where the locality principle is applied "consistently."

Elsewhere I have speculated that what Einstein meant by saying that field theory, like general relativity, provides an extreme embodiment of the separability principle is that, by assigning a well-defined value of the fundamental field quantity—such as the metric tensor in general relativity—to every point of the spacetime manifold, and by taking the joint reality associated with any two such points to be completely determined by the values of the field quantities associated with those two points, a field theory implicitly regards every point of the manifold as representing a separable system, a separable bit of reality (Howard 1985, 1989). Here we need only attend to the specific manner in which Einstein discusses the first principle, separability, including even the vocabulary he employs.

It is characteristic of physics that its concepts "refer to a real external world," which is to say that "ideas are posited of things that claim a 'real existence' independent of the perceiving subjects (bodies, fields, *etc.*)." Beyond that, the elements of our ontology, "these physical things," are represented "as being arranged in a space-time continuum." And it is "essential" that "these things claim an existence independent of one another, insofar as these things 'lie in different parts of space.'" This "assumption of the mutually independent existence (the 'being-thus') of spatially distant things . . . originates in everyday thought." Without it, "physical thought in the sense familiar to us would not be possible." Keep these formulations in mind when, in the next section, we learn how Schopenhauer discussed space and time as the *principium individuationis*, the ground for the individuation of empirical objects.

To conclude this survey of what Einstein said about separability and individuation, consider one final remark from handwritten comments he added to the manuscript of Max Born's 1949 Waynflete Lectures, *Natural Philosophy of Cause and Chance* (Born 1949), answering Born's request to respond to his representation of their discussions of quantum mechanics. The subject is once again reality and the fundamental ontology of any possible, future, fundamental physical theory. Einstein writes:

> I just want to explain what I mean when I say that we should try to hold on to physical reality. We are, to be sure, all of us aware of the situation regarding what will turn out to be the basic foundational concepts in physics: the point-mass or the particle is surely not among them; the field, in the Faraday-Maxwell sense, might be, but not with certainty. But that which we conceive as existing ("actual") should somehow be localized in time and space. That is, the real in one part of space, *A*, should (in theory) somehow "exist" independently of that which is thought of as real in another part of space, *B*. If a physical system stretches over the parts of space *A and B*, then what is present in *B* should somehow have an existence independent of what is present in *A*. What is actually present in *B* should thus not depend upon the type of measurement carried out in the part of space, *A*; it should also be independent of whether or not, after all, a measurement is made in *A*.

> If one adheres to this program, then one can hardly view the quantum-theoretical description as a *complete* representation of the physically real. If one attempts, nevertheless, so to view it, then one must assume that the physically real in *B* undergoes a sudden change because of a measurement in *A*. My physical instincts bristle at that suggestion.
>
> However, if one renounces the assumption that what is present in different parts of space has an independent, real existence, then I do not at all see what physics is supposed to describe. For what is thought to be a "system" is, after all, just conventional, and I do not see how one is supposed to divide up the world objectively so that one can make statements about the parts. (Einstein to Born, March 1948, in Born 1969, 223–24)

The point-particle and the field are dispensable. The one thing that cannot be given up is the idea of separability: "The real in one part of space, *A*, should (in theory) somehow 'exist' independently of that which is thought of as real in another part of space, *B*." Deny this assumption, and "I do not at all see what physics is supposed to describe." Why? Because "what is thought to be a 'system' is, after all, just conventional, and I do not see how one is supposed to divide up the world objectively so that one can make statements about the parts." But it is precisely nonseparability that is the distinguishing, nonclassical feature of the quantum mechanical interaction formalism. And there's the rub. Einstein says that separability is indispensable, and quantum mechanics denies it.[12]

Schopenhauer on Space, Time, Causality, and Individuation

Schopenhauer represents the first generation of the nineteenth-century neo-Kantian reaction to the idealism of Hegel, Fichte, and, to some extent, Schelling (see Köhnke 1986; Wiley 1978). What he says about space, time, and causality can only be understood against this background, although he is no strict Kantian.

Consider first the doctrine as developed in Schopenhauer's major work, *Die Welt als Wille und Vorstellung* (1859). According to Schopenhauer, the will is the thing-in-itself, and, as such, is not knowable through either experience or reason. A kind of knowledge of the will is possible through aesthetic experience, however, in moments when the subject, slipping the bonds of its individuality and freeing itself momentarily from the urgings of the will, manages a fleeting insight, as much a feeling as anything else, into the non-empirical Ideas that are, as it were, the first stage of the objectification of the will in nature. What we know through intuition and experience — empirical objects — are those things in nature that are the ultimate objectifications of the will. The most basic necessary forms of this objectification are space and time, the *principium individuationis*, through which empirical objects are originally distinguished from one another and thus represented to us in experience: "It is only by means of time and space that something which is one and the same according to its nature and the concept appears as different, as a plurality of coexistent and successive things. Consequently, time and space are the *principium individuationis*" (1859, vol. 1, 113). Or, as we read in a later passage: "We know that *plurality* in general is necessarily conditioned by time and space, and only in these is conceivable, and in this respect we call them the *principium individuationis*" (ibid., 127). Or, again, "We have called time and space the *principium individuationis*, because only through them and in them is plurality of the homogenous possible. They are the essential forms of natural knowledge" (ibid., 331). What we know through reason are concepts, all of them formed by abstraction from intuited empirical objects thus individuated in space and time.

If space and time are the *principium individuationis*, how do they differ from one another, aside from the obvious fact that, as for Kant, space is the form of outer intuition and time the form of inner intuition — space being thus the ground for the possibility of geometry, time the ground for the possibility of arithmetic? Schopenhauer says that space is "absolutely nothing else but the possibility of the reciprocal determinations of its parts by one another, which is called *position*" and "succession is the whole essence and nature of time" (ibid., 8). What does this imply? "Space renders possible the persistence of substance" and "time renders possible the change of accidents" (1859, vol. 2, 50). What Schopenhauer seems to mean by this, to employ a modern idiom, is that space individuates *objects* or *systems* in nature, while time individuates the *states* of those objects. In the more scholastic vocabulary that Schopenhauer often employs, space is that against which we distinguish individual substances, while time is that against which we distinguish the changing accidents or properties born by those substances. Hence there can be no ontology of empirical objects and their changing characteristics without space and time as the *principium individuationis.*

Many things are therefore individuated in space and time: tables and

chairs, planets and stars, chemical atoms and birds in the sky. But most importantly, for Schopenhauer, the human body, the individuated, objective correlate of the knowing subject, exists in space and time:

> That which knows all things and is known by none is the *subject*. . . . Everyone finds himself as this subject. . . . But his body is already object, and therefore from this point of view we call it representation. For the body is object among objects and is subordinated to the laws of objects, although it is immediate object. Like all objects of perception, it lies within the forms of all knowledge, in time and space, through which there is plurality. (1859, vol. 1, 5)

Thus, the very *object-ivity* of human knowledge is bound up with the fact that the human body, the phenomenal side of the human subject, is an object individuated in space and time like all of the other objects in space and time that are the objects of human knowledge.

Added to space and time is the one category of the understanding that Schopenhauer takes over from Kant, namely, *causality.* More so than Kant, Schopenhauer sees causality standing in an intimate relation with space and time, uniting them together in the perception of *matter*, in a manner presupposing the individuating character of space and time. The result is the *reality* that is investigated by empirical science:

> For matter is absolutely nothing but causality. . . . Thus, its being is its acting; it is not possible to conceive for it any other being. Only as something acting does it fill space and time. . . . Thus cause and effect are the whole essence and nature of matter; its being is its acting. . . . The substance of everything material is therefore very appropriately called in German *Wirklichkeit* [i.e., activeness], a word much more expressive than *Realität*. . . . Time and space . . . each by itself, can be represented in intuition even without matter; but matter cannot be so represented without time and space. The form inseparable from it presupposes *space*, and its action, in which its entire existence consists, always concerns a change, and hence a determination of *time*. But time and space are not only, each by itself, presupposed by matter, but a combination of the two constitutes its essential nature, just because this, as we have shown, consists in action, causality. (Ibid., 8–9)

According to Schopenhauer, causality derives its meaning and necessity from the fact that change is not mere variation. Instead, the essence of change

> consists in the fact that, at the *same place in space*, there is now *one* condition or state and then *another*, and at *one* and the same point of time there is *here* this state and *there* that state. Only this mutual limitation of time and space by

each other gives meaning, and at the same time necessity, to a rule according to which change must take place. What is determined by the law of causality is therefore not the succession of states in mere time, but that succession in respect of a particular space, and not only the existence of states at a particular place, but at this place at a particular time. Thus change, i.e., variation occurring according to the causal law, always concerns a particular part of space and a particular part of time, *simultaneously* and in union. Consequently, causality unites space and time. (Ibid., 9–10)

Finally, causality's unification of space and time grounds our a priori knowledge of matter:

On this derivation of the basic determinations of matter from the forms of our knowledge, of which we are *a priori* conscious, rests our knowledge *a priori* of the sure and certain properties of matter. These are space-occupation, i.e., impenetrability, i.e., effectiveness, then extension, infinite divisibility, persistence, i.e., indestructibility, and finally, mobility. On the other hand, gravity, notwithstanding its universality, is to be attributed to knowledge *a posteriori*, although Kant in his *Metaphysical Rudiments of Natural Science* . . . asserts that it is knowable *a priori.* (Ibid., 11)

It was in his first book, *Ueber die vierfache Wurzel des Satzes vom zureichenden Grunde* (1813), that Schopenhauer originally developed the idea of space and time as principles of individuation. They are distinguished as the third of the four forms of the principle of sufficient reason—causality is the first—and are together designated the "ground of being," or the "principle of sufficient reason of being":

Space and time are so constituted that all their parts stand in mutual relation and, on the strength of this, every part is determined and conditioned by another. In space this relation is called *position*, in time *succession.* These relations are peculiar and differ entirely from all other possible relations of our representations. Therefore neither the understanding nor the faculty of reason by means of mere concepts is capable of grasping them, but they are made intelligible to us simply and solely by means of pure intuition a priori. . . . Now the law whereby the parts of space and time determine one another as regards these relations is what I call the *principle of sufficient reason of being, principium rationis sufficientis essendi.* (1813, 194)

As later, he characterizes matter as "the perceptibility of time and space" and "causality that has become objective" (ibid., 193), but in this earlier text we find some helpful additional detail about why space and time are needed as a ground for the individuation of objects in

experience, and about the manner in which such individuation, especially in space, is effected.

Why are space and time, as pure intuitions, necessary for individuation? The answer harkens back to Kant's famous "incongruent counterparts" argument, which we will consider more fully in the next section:

It is impossible to explain clearly from mere concepts what are above and below, right and left, front and back, before and after. Kant quite rightly confirms this by saying that the difference between the right and left gloves cannot possibly be made intelligible except by means of intuition. (Ibid., 194)

As Kant argued, the only way to distinguish "incongruent counterparts," like a right- and left-handed glove, is by their different situations with respect to absolute space. Assuming an otherwise empty space, and assuming two such objects that are qualitatively identical, their internal and external relational properties would not suffice to tell them apart. That there are two objects, rather than one, we might establish via their mutual relations with one another, but we could not determine, in this way, which is which. It is only against the background of absolute space, so argues Kant, that we can tell them apart; hence, space is necessary as a ground for the individuation of physical objects.

How, according to Schopenhauer, does space do the work of distinguishing physical objects? The answer comes in a discussion of how we know the truth of geometrical axioms not on the basis of rational demonstration, but of pure intuition. Space and time themselves are pure intuitions. Figures and numbers in space and time, respectively, are dubbed "normal intuitions" (*Normalanschauungen*), and most of Euclid's axioms are said to depend for their truth on one fact about these "normal intuitions":

What Plato says of his Ideas would hold good of these normal intuitions, even in geometry, as well as of concepts, namely that two cannot exist exactly alike because such would be only one. I say that this would hold good also of normal intuitions in geometry if it were not that, as exclusively *spatial* objects, they differ through mere juxtaposition and hence through *place.* (Ibid., 198–99)

Such spatial individuation is surely necessary to ground the possibility of congruent, but not identical figures, but otherwise it is not clear how this fact about "normal intuitions" can, by itself, assure the truth of Euclid's axioms. Schopenhauer merely says:

Now the mere view that such a difference of place does not abolish the rest of the identity seems to me to be capable of replacing those nine axioms, and of being more suitable to the true nature of science whose purpose is to know the particular from the general, than is the statement of nine different axioms that are all based on one view. (Ibid., 199)

However Schopenhauer imagined the grounding of the axioms to work, the significance of these remarks will loom larger when, again in the next section, we try to locate Schopenhauer's views on space, time, and individuation in a larger historical context, including, most importantly, the Leibniz-Clarke debate. For Schopenhauer asserts here that otherwise qualitatively identical objects (remember that material objects are just figures made perceptible by activity or causality) may be distinguished from one another solely by virtue of their different spatial locations, their separation in space. This is precisely what Leibniz denies.

It should be stressed that Schopenhauer is no philosopher of geometry, no philosopher of physics. He does not write about space and time as the *principium individuationis* out of an interest in phoronomy or the rational foundations of mechanics. On the contrary, Schopenhauer's larger aims concern aesthetics, ethics, and social philosophy. By the time of *Die Welt als Wille und Vorstellung* (1819), he is interested in space and time as the *principium individuationis* mainly because of what it can tell us about the way in which the embodied human subject lives in the phenomenal world as a human individual, subject to the law of causality, and ontologically separated, by necessity, from all other human individuals. By contrast, the will, the thing-in-itself, not being subject to the pluralizing forms of space and time, is a kind of unity. Much of what is most interesting in Schopenhauer's philosophy—in his ethics, his aesthetics, and his social philosophy—turns around the tension between the unity of the will and the plurality of objectifications of the will. It is here, at this point of creative tension, that Schopenhauer makes such masterful use of the image of the Veil of Maya, an idea borrowed from Vedic thought as a representation of the way in which space and time as the *principium individuationis* draw a veil across the deeper unity of the world as will, forcing us to apprehend that world only in the will's dispersed objectifications in space and time, except for those brief moments when, freeing ourselves from the will's urgings and transcending our individuality, we catch a glimpse behind the Veil of Maya. Let me illustrate by quoting at length from section 63 of *Die Welt als Wille und Vorstellung*, where the issue is "eternal justice":

The eyes of the uncultured individual are clouded, as the Indians say, by the veil of Maya. To him is revealed not the thing-in-itself, but only the phenomenon in time and space, in the *principium individuationis*, and in the remaining forms of the principle of sufficient reason. In this form of his limited knowledge he sees not the inner nature of things, which is one, but its phenomena as separated, detached, innumerable, very different, and indeed opposed. For pleasure appears to him as one thing, and pain as quite another; one man as tormentor and murderer, another as martyr and victim. . . . He sees one person living in pleasure, abundance, and delights, and at the same time another dying in agony of want and cold at the former's very door. He then asks where retribution is to be found. . . . He often tries to escape by wickedness, in other words, by causing another's suffering, from the evil, from the suffering of his own individuality, involved as he is in the *principium individuationis*, deluded by the veil of Maya. . . . This separation, however, lies only in the phenomenon and not in the thing-in-itself; and precisely on this rests eternal justice. . . . The person is mere phenomenon, and its difference from other individuals, and exemption from the sufferings they bear, rest merely on the form of the *principium individuationis*. According to the true nature of things, everyone has all the sufferings of the world as his own; indeed, he has to look upon all merely possible sufferings as actual for him, so long as he is the firm and constant will-to-live, in other words, affirms life with all his strength. For the knowledge that sees through the *principium individuationis*, a happy life in time, given by chance or won from it by shrewdness, amid the sufferings of innumerable others, is only a beggar's dream, in which he is a king, but from which he must awake, in order to realize that only a fleeting illusion has separated him from the suffering of his life. (1859, vol. 1, 352–53)

What passages like this should help to make clear is that the conception of space and time as the *principium individuationis* runs like a Leitmotiv throughout Schopenhauer's writings. Until one has read *Parerga und Paralipomena* or *Die Welt als Wille und Vorstellung* in their entirety, one cannot appreciate just how ubiquitous the idea is—how it tends to pop up every few pages, as a sort of reminder, even in places where one might not expect to find it. One cannot read even a small part of Schopenhauer's oeuvre without encountering it many times over.[13] The regular emphasis on space and time as the *principium individuationis* is thus one of the chief distinguishing features of Schopenhauer's philosophy. It sets him apart from virtually all other nineteenth-century philosophers, including even the later generation of Marburg neo-Kantians. Although they derived this idea from Kant, none of them highlights it in the same way as Schopenhauer.

Of course, all of this is still no proof that Einstein borrowed the idea of spatiotemporal separability from Schopenhauer, or even that he was influenced in some small way by what Schopenhauer wrote about space and time as the *principium individuationis*. After all, on the face of it, there are significant differences between the two conceptions, not the least of which is the fact that Schopenhauer, writing in the early nineteenth century, assumes the classical distinction between space and time, whereas Einstein sees them as aspects of a single, four-dimensional spacetime. But there is a context in which Schopenhauer was writing, a long-standing controversy about space, time, and individuation. An appreciation of the significance of that context will make it appear more likely that Einstein could have been influenced by Schopenhauer's position in this controversy.

From Suárez to Schopenhauer: The Historical Background

Schopenhauer tells us himself where to look for the earliest roots of this controversy:

> As we know, time and space belong to this principle [of sufficient reason], and consequently plurality as well, which exists and has become possible only through them. In this last respect I shall call time and space the *principium individuationis*, an expression borrowed from the old scholasticism, and I beg the reader to bear this in mind once and for all. For it is only by means of time and space that something which is one and the same according to its nature and the concept appears as different, as a plurality of coexistent and successive things. Consequently, time and space are the *principium individuationis*, the subject of so many subtleties and disputes among the scholastics which are found collected in Suárez (*Disp*. 5, sect. 3). (1859, vol. 1, 112–13)

The reference is to the fifth of Francisco Suárez's *Disputationes Metaphysicae* (1597), "De unitate individuali ejusque principio," a compendious summary of the various medieval debates on individuation (see Gracia 1982).

The medieval debate over individuation, summarized by Suárez, had its origins in the mid-thirteenth century, in Aquinas's *Commentary on Boethius' "De Trinitate"* (see Maurer 1987, xxiii–xxxv). The problem was to understand how the Father, the Son, and the Holy Spirit could be one God, not three. For Boethius, things can be different in three ways: in genus (like human and stone), in species (like human and horse), or in number (like Cato and Cicero). To show that none of these differences is present in God, Boethius must explain in what they consist. For our purposes, difference in number is the important case, and Boethius says that this is the result of the diversity of the accidental characteristics. But what about differences in accidents that do not affect the identity of individual substances, such as the difference between a person sitting and standing? Does this mean that accidents are not relevant to individuation? No. "For if we mentally remove all [other] accidents, still each occupies a different place, which we cannot possibly conceive to be the same" (Thomas Aquinas 1987, 57).

Aquinas takes up the Question in Question 4, Article 4 of his *Commentary*, "Does a Difference in Place Have Some Bearing on a Difference in Number?" Aquinas was writing in a very different intellectual climate, one in which Aristotle loomed far larger than he did for the neo-Platonist, Boethius. He therefore could not simply endorse Boethius's position, holding instead that the ultimate cause of individuation is "matter, itself, as existing under dimensions." He seems to mean that matter as that which is susceptible to division is the cause of individuation, of diversity in number. But it is equally the cause of difference in place, making difference in place a *sign* of difference in number—indeed, the surest sign: "Taken in this sense, difference of place plays the greatest role because it is the sign most closely related to diversity in number" (ibid., 109–10).

Suárez's fifth *Disputation*, which Schopenhauer cites, reviews the "subtleties and disputes" that arose in response to Aquinas's *Commentary*. We need not follow this history in detail here, except to note that by the time Suárez was writing, in the late sixteenth century, the issue had become quite complicated, owing to the intrinsic difficulty of even posing the question clearly in the vocabulary available to the Scholastics. For, happily, the nature of the debate was about to change, thanks largely to the revival of atomism in Renaissance natural philosophy and to related late-Renaissance developments in the conception of space and the void.

Among the first generation of classical Greek atomists, Leucippus and Democritus, the void was not simply the infinite, all-pervading arena or container within which the atoms moved; which is to say that it was not yet identified with the space of the geometers, as was to become the case by the time of Epicurus and, later, Lucretius. Instead, the void, the empty (κενὸν) is that which separates the atoms; it consists of the intervals (διαστήματα) between the atoms. The function of the void, then, is to be the principle making possible plurality (see Bailey 1928, 69–76). It retains this function when it later becomes the space of the geometers. For without a void or a space capable of thus separating, there would be no basis for distinguishing, one from another, the otherwise qualitatively identical atoms.

The way was made easier for the revival of classical Greek atomism in Renaissance natural philosophy by the growing tendency to regard *quantitative*, as opposed to *qualitative*, properties as ontologically primary. This tendency was especially in evidence in the neo-Platonic, Averroistic atmosphere of Padua, starting from the fifteenth-century debate between Biagio Pelacani and Gaetano of Thiene over whether the primary accident of a substance is quantitative or qualitative, to the late-sixteenth century continuation of this debate between Galileo and Cremonini (Dijksterhuis 1961, 235). Galileo gave the classic statement of the primacy of quantitative, mathematical properties when he described natural philosophy as being written "in the language of mathematics," explaining that "its characters are triangles, circles, and other geometric figures, without which it is humanly impossible to understand a single word of it; without these, one wanders about in a dark labyrinth" (*Il Saggiatore*; as quoted in ibid., 362). Of course, Descartes carried this tendency to its logical extreme by identifying matter with extension itself. In such a metaphysical context, where the only properties, or at least the primary qualities, are geometrical properties, the only possible basis for distinguishing otherwise identical (congruent) parts of the universe is difference in place, or spatial separation. There could be no diversity, no plurality, no change, no motion, no physics, were it not for the most basic metaphysical fact of individuation by spatial separation.

From this point of view, the otherwise large differences between the Cartesians and the Newtonians appear relatively insignificant. For whether one identifies matter with extension, or distinguishes space from matter, individuation via spatial separation remains a necessary part of one's world view, if one takes quantitative geometrical properties to be the metaphysically fundamental properties of material substance.

In Newton's case, a helpful text in this regard is his *De Gravitatione et aequipondio fluidorum.* (Hall and Hall 1962, 89–156), an unpublished manuscript that the Halls date to between 1664 and 1668, evidently at the time of Newton's first serious critical reaction to Descartes. Newton's principal aim in the heart of this manuscript is to refute the Cartesian identification of material substance with extension. Toward that end, he sets forth his own conception of the nature of extension and body.

From the start, the radical break with Scholasticism is clearly marked. "Perhaps now it may be expected that I should define extension as substance or accident or else nothing at all. But by no means, for it has its own manner of existence which fits neither substances nor accidents" (ibid., 131–32). What manner of existence is this? To begin with, there is the universal divisibility of space:

> 1. In all directions, space can be distinguished into parts whose common limits we usually call surfaces. . . . Furthermore spaces are everywhere contiguous to spaces, and extension is everywhere placed next to extension, and so there are everywhere common boundaries to contiguous parts. . . . And hence there are everywhere all kinds of figures, everywhere spheres, cubes, triangles, straight lines, everywhere circular, elliptical, parabolical and all other kinds of figures, and those of all shapes and sizes, even though they are not disclosed to sight. For the material delineation of any figure is not a new production of that figure with respect to space, but only a corporeal representation of it, so that what was formerly insensible in space now appears to the senses to exist. (Ibid., 132–33)

Thus the first, most fundamental property of space is that it can be divided anywhere, in any way, into parts. The second property of space is infinity. The third is that the parts of space are motionless. And to this point Newton comments as follows:

> Moreover the immobility of space will be best exemplified by duration. For just as the parts of duration derive their individuality from their order, so that (for example) if yesterday could change places with today and become the later of the two, it would lose its individuality and would no longer be yesterday, but today; so the parts of space derive their character from their positions, so that if any two could change their positions, they would change their character at the same time and each would be converted numerically into the other. The parts of duration and space are only understood to be the same as they really are because of their mutual order and position; nor do they have any hint of individuality apart from that order and position which consequently cannot be altered. (Ibid., 136)

The individuality of the parts of space consists solely in their order and position. The same will be true of bodies, because, as we shall see, they are nothing more than the aforementioned "material delineations" of parts of space, "*determined quantities of extension which omnipresent God endows with certain conditions.*" These include mobility—God successively determines immediately adjacent, congruent parts of space; impenetrability—two different determined quantities of extension cannot coincide; and the capacity to excite "perceptions of the senses" (ibid., 140). This is the meaning of the fourth fundamental property of space:

4. Space is a disposition of being *qua* being. No being exists or can exist which is not related to space in some way. God is everywhere, created minds are somewhere, and body is in the space that it occupies; and whatever is neither everywhere nor anywhere does not exist. And hence it follows that space is an effect arising from the first existence of being, because when any being is postulated, space is postulated. And the same may be asserted of duration: for certainly both are dispositions of being or attributes according to which we denominate quantitatively the presence and duration of any existing individual thing. (Ibid., 136)

Space is a disposition of being *qua* being in the sense that it is the ground of individuation, that "according to which we denominate quantitatively the presence . . . of any existing individual thing." The fifth property of space is that "the positions, distances and local motions of bodies are to be referred to the parts of space," a fact that "will be more manifest if you conceive that there are vacuities scattered between the particles," and the sixth and last property is that space is "eternal in duration and immutable in nature" (ibid., 137).

Whatever absolute space becomes for Newton in the *Principia* and later, whatever dynamical properties it must possess in order to make sense of the first and second laws—in order, that is, to ground the concept of acceleration and to make possible the identification of inertial motions—the more basic role of space as the ground of individuation remains.

This was not lost on Newton's contemporaries. We find John Locke, in 1694, writing in the second edition of the *Essay Concerning Human Understanding* (1694):

When therefore we demand whether anything be the *same* or no, it refers always to something that existed such a time in such a place, which it was certain, at that instant was the same with itself, and no other. From whence it

follows, that one thing cannot have two beginnings of existence, nor two things one beginning; it being impossible for two things of the same kind to be or exist in the same instant, in the very same place; or one and the same thing in different places. That, therefore, that had one beginning, is the same thing; and that which had a different beginning in time and place from that, is not the same, but diverse. . . . From what has been said, it is easy to discover what is so much inquired after, the *principium individuationis*; and that, it is plain, is existence itself; which determines a being of any sort to a particular time and place, incommunicable to two beings of the same kind. (1894, 439–41)

This passage is from book II, chapter 27, "Of Identity and Diversity," a chapter added in the second edition at the urging of William Molyneux (Molyneux to Locke, 2 March 1693, in De Beer 1979, vol. 4, 647–52; see also Locke to Molyneux, 23 August 1693, ibid., 719–23; and 8 March 1695, ibid., vol. 5, 284–88).

Whether or not Molyneux's motivation was a concern about Leibniz's reaction, the latter did react, leaving no uncertainty as to where he differed from Locke. Leibniz writes in the *Nouveaux essais sur l'entendement humain* (1704–1705), with specific reference to the quoted passage from chapter 27 of the *Essay*:

In addition to the difference of time or of place there must always be an internal *principle of individuation*: although there can be many things of the same kind, it is still the case that none of them are ever exactly alike. Thus, although time and place (i.e. the relations to what lies outside) do distinguish for us things which we could not easily tell apart by reference to themselves alone, things are nevertheless distinguishable in themselves. Thus, although diversity in things is accompanied by diversity of time or place, time and place do not constitute the core of identity and diversity, because they impress different states upon being. To which it can be added that it is by means of things that we must distinguish one time or place from another, rather than *vice versa*; for times and places are in themselves perfectly alike, and in any case they are not substances or complete realities. . . . The "principle of individuation" reduces, in the case of individuals, to the principle of distinction of which I have just been speaking. If two individuals were perfectly similar and equal and, in short, *indistinguishable* in themselves, there would be no principle of individuation. I would even venture to say that in such a case there would be no individual distinctness, no separate individuals. That is why the notion of atoms is chimerical and arises only from men's incomplete conceptions. For if there were atoms, i.e. perfectly hard and perfectly unalterable bodies which were incapable of internal change and could differ from one another only in size and in shape, it is obvious that since they could have the same size and shape they would then be indistinguishable in themselves and discernible only by means of external denominations with no internal foundation; which is contrary to the greatest principles of reason. . . . One can see from these considerations, which have until now been overlooked, how far people have strayed in philosophy from the most natural notions, and at what a distance from the great principles of true metaphysics they have come to be. (1981, 230–31)

But even though the new chapter 27 of the 1694 second edition of Locke's *Essay* was the immediate occasion of these remarks, the basic idea is of older vintage in Leibniz's thinking, as evidenced by the following remarks from a 1696 note "On the Principle of Indiscernibles":

A consideration which is of the greatest importance in all philosophy, and in theology itself, is this: that there are no purely extrinsic denominations, because of the interconnexion of things, and that it is not possible for two things to differ from one another in respect of place and time alone, but that it is always necessary that there shall be some other internal difference. So there cannot be two atoms which are at the same time similar in shape and equal in magnitude to each other; for example, two equal cubes. Such notions are mathematical, that is, they are abstract and not real. For all things which are different must be distinguished in some way, and in the case of real things position alone is not a sufficient means of distinction. This overthrows the whole of purely corpuscularian philosophy. (1696, 133)

Leibniz's critique of Locke on individuation looms in the background of the Leibniz-Clarke correspondence of 1715–1716. Whatever else might be the points of difference separating Leibniz and the Newtonians regarding the nature of space, this much is without doubt at the very heart of the controversy: The Newtonians want to defend the idea that space is the principle of individuation, that spatial separation is a sufficient condition for marking two bodies — even two qualitatively and geometrically identical atoms — as different individuals, with their own, separate identities, whereas Leibniz and his followers, like Christian Wolff, want to deny this.

Consider one of the most famous and oft-quoted passages from the Leibniz-Clarke correspondence, this being Leibniz's main argument against the Newtonian concept of space in his "Third Paper" of 25 February 1716:

I have many demonstrations, to confute the fancy of those who take space to be a substance, or at least an absolute being. But I shall only use, at the present, one demonstration, which the author here gives me occasion to insist upon. I say then, that if space was an absolute being, there would something happen

for which it would be impossible there should be a sufficient reason. Which is against my axiom. And I prove it thus. Space is something absolutely uniform; and, without the things placed in it, one point of space does not absolutely differ in any respect whatsoever from another point of space. Now from hence it follows, (supposing space to be something in itself, besides the order of bodies among themselves,) that 'tis impossible there should be a reason, why God, preserving the same situations of bodies among themselves, should have placed them in space after one certain particular manner, and not otherwise; why every thing was not placed the quite contrary way, for instance, by changing East into West. But if space is nothing else, but that order of relation; and is nothing at all without bodies, but the possibility of placing them; then those two states, the one such as it now is, the other supposed to be the quite contrary way, would not at all differ from one another. Their difference therefore is only to be found in our chimerical supposition of the reality of space in itself. But in truth the one would exactly be the same thing as the other, they being absolutely indiscernible; and consequently there is no room to enquire after a reason of the preference of the one to the other. (Alexander 1956, 26)

Yes, the issue is an absolute versus a relative conception of space. But what is it about Newton's absolute space that so troubles Leibniz? It is not — or certainly it is not *only* — that absolute space is endowed with dynamical properties sufficient to distinguish inertial motions. It is rather that, all the other characteristics of absolute space aside, Newton uses his absolute space as a ground of individuation. That is what really troubles Leibniz. Notice how he begins this passage: "I have many demonstrations, *to confute the fancy of those who take space to be a substance, or at least an absolute being*" (emphasis added). It is neither the substantival character of space — which, recall, Newton *denied* ("for it has its own manner of existence which fits neither substances nor accidents") — nor the dynamical properties of absolute space. What is objectionable is simply the claim that position or spatial separation is a sufficient condition for individuation.

Make Leibniz's argument simpler by considering a universe containing just two atoms, *A* and *B*. Now let God change "East into West." Why would it be impossible to distinguish the two states of the universe? Because, the spatial separation of *A* and *B* not being a sufficient condition for establishing their separate identities, we could not, even before the change of "East into West" tell *which* atom was *A* and *which* atom was *B*. That there were *two* atoms we may be able to determine by their mutual order; but *which* is *which* we cannot say. And therefore, there is no discernible difference between the two states of the universe before and after changing "East into West." It is worth pondering the similarity between Leibniz's position, vis-à-vis Newton's, and the position of Bose-Einstein statistics, vis-à-vis Boltzmann statistics. In both the Leibnizian ontology and in Bose-Einstein statistics, the exchange of the positions of the two particles, the changing of "East into West," makes no difference at all.

Let me venture at this point a strong historiographical claim. Too much of the historical literature on the Leibniz-Clarke correspondence misses this absolutely central point in Leibniz's critique of the Newtonian conception of space because of the error of anachronism. Recent interest in the Leibniz-Clarke correspondence arose mainly because the development of relativity theory awakened our curiosity about the controversy over absolute versus relative conceptions of space. We went back looking for anticipations of this twentieth-century controversy in the seventeenth and eighteenth centuries, not pausing to consider that what really mattered to Newton in the conception of space, as well as what really bothered Leibniz about Newton's position, was perhaps quite different from what our twentieth-century categories led us to expect. More recently, we have picked over the Leibniz-Clarke correspondence again, interested now in the more subtle question of substantival versus relational conceptions of space. But the anachronistic error has been repeated: Even Newton's explicit *denial* that space is something substantial is not enough to deter scholars from trying to turn him into a substantivalist.

Anachronism has similarly infected the literature on Leibniz's most penetrating critic, Immanuel Kant. Much has been written on the "incongruent counterparts" argument, but again, too much of this literature sees the argument as part of the absolute-relative or substantival-relational controversies. In fact, Kant was trying to defend Newton on *precisely* the point we have been tracking: that space is the ground of individuation.

The argument appeared first in Kant's 1768 essay, "Von dem ersten Grunde des Unterschiedes der Gegenden im Raume." It was used again in the 1770 *Inaugural Dissertation,* and is mentioned both in the *Prolegomena* (1783) and in the *Metaphysische Anfangsgründe der Naturwissenschaft* (1786), although, curiously, it is not mentioned in either the first (1781) or second (1787) editions of the *Kritik der reinen Vernunft.* There are genuine puzzles about the place of this argument in

the development of Kant's thought. It puzzled some commentators that, when the argument first appears in 1768, it is employed as an argument for the reality of absolute space, whereas already at the time of the *Inaugural Dissertation* and thereafter it appears as an argument for the transcendental ideality of space (Buroker 1981, 3–4). But this much is beyond dispute. The argument first appears at the crucial moment of Kant's final turn away from Leibniz and Wolff, a time when, according to Cassirer, Kant was intensely engaged with a careful reading of the Leibniz-Clarke correspondence (Cassirer 1918, 111).

What is the argument? Kant asks us to consider two objects, such as our right and left hands or a right- and left-handed screw, objects that are "perfectly like and similar and yet . . . in themselves so different that the limits of the one cannot at the same time be the limits of the other." Such objects are called "incongruent counterparts" (1768, 41). What does the existence of such objects demonstrate?

> It is already clear from the everyday example of the two hands that the figure of a body can be completely similar to that of another, and that the size of the extension can be, in both, exactly the same; and that yet, however, an internal difference remains: namely, that the surface that includes the one could not possibly include the other. As the surface limiting the bodily space of the one cannot serve as a limit for the other, twist and turn it how one will, this difference must, therefore, be such as rests on an inner principle. This inner principle cannot, however, be connected with the different way in which the parts of the body are connected with each other. For, as one sees from the given example, everything can be perfectly identical in this respect. (Ibid., 42)

What, then, is this "inner principle"? Kant explains:

> If one accepts the concept of modern, in particular, German philosophers, that space only consists of the external relations of parts of matter, which exist alongside one another, then all real space would be, in the example used, simply that *which this hand takes up*. However, since there is no difference in the relations of the parts to each other, whether right hand or left, the hand would be completely indeterminate with respect to such a quality, that is, it would fit on either side of the human body. But that is impossible.
>
> From this it is clear that the determinations of space are not consequences of the situations of the parts of matter relative to each other; rather are the latter consequences of the former. It is also clear that in the constitution of bodies differences, and real differences at that, can be found; and these differences are connected purely with *absolute and original space*, for it is only through it that the relation of physical things is possible. (Ibid., 43)

Or, as Kant announced the conclusion at the beginning of the essay, "*absolute space has its own reality independently of the existence of all matter and that it is itself the ultimate foundation of the possibility of its composition*" (ibid., 37).

What is it about absolute space that Kant means here to defend against Leibniz? Again, it has nothing whatsoever to do with the dynamical properties of Newtonian absolute space. It is rather that more basic property of space that it is, first, independent of the existence of matter, and is, second, "*the ultimate foundation of the possibility of its composition*," that is to say, that space is the ground of individuation. It was this that Newton and Locke held to be the most fundamental property of space; it was this that Leibniz attacked; and it is this that Kant defends.

Realizing that individuation is the issue actually helps us to resolve some of the puzzles about the role of the "incongruent counterparts" argument in the development of Kant's thought. For I think it wrong to see a major difference between the 1768 employment of the argument and its later employments. Already in 1768, where Kant says that he is arguing that "absolute space has its own reality independently of the existence of all matter," he nevertheless concludes:

> It is also clear that since absolute space is not an object of external sensation, but rather a fundamental concept, which makes all these sensations possible in the first place, we can only perceive through the relation to other bodies that which, in the form of a body, purely concerns its relation to pure space. (1768, 43)

This is not so very different from the way the "incongruent counterparts" argument is used two years later in the *Inaugural Dissertation* to prove that "the concept of space is therefore a pure intuition" (Kant 1770, sec. 15, 69). Indeed, what is common to each of the occasions when Kant invokes the argument is the idea that we require a ground for the individuation of objects in experience. In the *Prolegomena*, Kant puts the point this way:

> Space is the form of the external intuition of this sensibility, and the internal determination of every space is possible only by the determination of its external relation to the whole of space, of which it is a part (in other words, by its relation to the outer sense). That is to say, the part is possible only through the whole, which is never the case with things in themselves, as objects of the mere understanding, but which may well be the case with mere appearances. Hence the difference between similar and equal things which are not congruent (for instance, two symmetric helices) cannot be made intelligible by any concept, but only by the relation to the right and left hands which immediately refers to intuition. (1783, sec. 13, 34)

That space is a form of intuition follows from what is required for individuation, for the conditions for the possibility of the parts of space. He says later in the *Prolegomena*: "The mere universal form of intuition, called space, must therefore be the substratum of all intuitions determinable to particular objects; and in it, of course, the condition of the possibility and of the variety of these intuitions lies" (ibid., sec. 38, 68–69; see also 1786, 23–24).

We saw above that, when Schopenhauer first introduced the concept of space and time, the *principium individuationis*, as the "ground of being," the third form of the Principle of Sufficient Reason, in *Ueber die vierfache Wurzel*, it was precisely Kant's "incongruent counterparts" argument that he cited to begin the explanation of how spatial objects differ "through mere juxtaposition and hence through *place*" (1813, 194, 199; see also 40). When Schopenhauer writes, thus, about space and time as the *principium individuationis*, he writes in a well-defined historical context, in which the problem of space as the ground of individuation was the central issue in two centuries of controversy over the nature of space. Schopenhauer follows Kant in affirming the Newtonian, Lockean view in the face of the Leibnizian critique. It is in this context also that we must situate Einstein's reading of Schopenhauer.

Not that it is essential to Einstein's understanding of Schopenhauer, but could Einstein have understood that Schopenhauer was writing in this context? Newton's *De Gravitatione* would not have been available to Einstein. It is a good guess that he was familiar with the Leibniz-Clarke correspondence, although the issue of individuation is not mentioned in the discussion of the correspondence in Ferdinand Rosenberger's *Isaac Newton und seine physikalischen Principien* (1895), a copy of which was in Einstein's personal library. Einstein read Kant's first *Kritik* at least once, as a youth (see Howard 1994), but the "incongruent counterparts" argument, as noted above, is not explicitly mentioned there. It is, however, discussed at length, with generous quotations from the 1768 essay in which it first appeared, in the published version of August Stadler's lectures on Kant for which Einstein enrolled as a student at the ETH in the summer semester of 1897 (Stadler 1912, 104–05, 189–92).[14] Einstein also read the *Prolegomena* at least once, in 1918 (see Einstein to Max Born, summer 1918, in Born 1969, 25). And, finally, as noted above, there is evidence that Einstein read Locke (Grüning 1989, 247). Still, this does not mean that Einstein knew the history of the controversies over space as the ground of individuation; his reading of Schopenhauer could very well stand on its own.

Schopenhauer in the Eyes of Einstein's Contemporaries: Philosopher of Pessimism or Philosopher of Physics?

We have documented Einstein's extensive reading of Schopenhauer. We have examined what Schopenhauer had to say about space and time as the *principium individuationis.* We have explored the historical context in which Schopenhauer developed this doctrine. But we lack explicit documentation that Einstein's distinctive conception of spatiotemporal separability was influenced, in any way, by his reading of Schopenhauer. It is possible that Einstein did not read Schopenhauer as a thinker with interesting and important things to say about fundamental questions in physics. Lacking any specific documentation on this point, how might we proceed? What we can do is to ask how others read Schopenhauer at this time and, most important, how other physicists and philosophers of science contemporary with Einstein read Schopenhauer.

Mention was made above of many people who were, without question, deeply influenced by their reading of Schopenhauer, such as Wagner, Nietzsche, and Thomas Mann. But what about thinkers closer to Einstein in their interests and outlook? Robert Musil, author of *Der Mann ohne Eigenschaften*, is an interesting case, standing as he does between the philosophical and literary communities.[15] In his youth, he read Schopenhauer with keen interest. As David Luft puts it, "Musil's generation (from Hermann Broch to Thomas Mann) received the Kantian doctrine and romantic aesthetics via Schopenhauer" (1980, 39). Or consider the novelist and critic Max Brod, and his Prague friend, the philosopher Hugo Bergmann. They were at the core of the Fanta-Kreis, the Jewish, intellectual salon with which Einstein associated during his year in Prague, 1911–1912. Brod and Bergmann are among the many Jewish intellectuals who had an enthusiasm for Schopenhauer, according to Henry Brann (1975, 60–61).[16] Closer still to Einstein is Ludwig Wittgenstein, whose debt to and respect for Schopenhauer has been widely discussed (see, for example, Janik and Toulmin 1973; Magee 1983, esp. 286–315; Monk 1990).

What about Schopenhauer's reception among philosophers of science and physicists? Was he taken seriously as a thinker with interesting and important things to say about fundamental questions in physics? Lest we forget, Ludwig Boltzmann took Schopenhauer seriously enough to do him the honor of a harsh, witty, at times almost sarcastic, critique, "Über eine These Schopenhauers," before the Vienna Philosophische Gesellschaft in January 1905. Boltzmann's main target was Schopenhauer's ethical pessimism, but along the way he devoted special attention to Schopenhauer's views on the a priori status of space and time (Boltzmann 1905, 387–88). Ernst Mach had a different attitude toward Schopenhauer. Among many references to Schopenhauer, we find Mach writing the following in *Erkenntnis und Irrtum*, as a footnote to the remark that "the foundation of all knowledge is . . . *intuition*": "As it appears to me, after Kant, Schopenhauer accorded the highest value to the significance of intuition" (1905, 310).

Moritz Schlick went out of his way on many occasions to criticize Schopenhauer's conception of metaphysics, especially the possibility of a metaphysical knowledge different in kind from scientific knowledge (see, for example, Schlick 1932, 167–68). He nevertheless saw the relevance of Schopenhauer in other areas. Thus, in a discussion of causality and reality in his *Allgemeine Erkenntnislehre*, which Einstein read with care (see Howard 1984), Schlick writes:

> If previously we said that we call real whatever is the cause of experiences, we can now give up the relation to experience and still maintain the position that everything real is a *cause*. Anything that does not make itself noticeable in some way, never manifests itself, is in fact not *there*, is not real; whether we experience the manifestation of a thing, however, is accidental. Thus we capture the essential as opposed to the accidental if we accept the formulation: the real is that which *has an effect* (*wirklich ist, was wirkt*).
>
> Even our language seems to exert pressure in behalf of this interpretation and to demonstrate that it has caught the sense of the popular view. In German, the word "real" ("wirklich") is derived from the verb "to have an effect" ("wirken"). In Aristotle the concept *energeia* coincides with that of reality. And Leibniz, too, declared: "*quod non agit, non existit.*" The best known advocate of this conception is no doubt Schopenhauer. Of matter, he said: "its being is its acting on something; it is impossible even to think of its having any other being." In another passage, he wrote that matter is "causality itself, objectively conceived." The reality of things, he explained, is their materiality: reality is thus the "efficacy of things generally." Today we find the same definition in many thinkers. (Schlick 1918, 181–82)

What about the reception specifically of Schopenhauer's doctrine of space and time as the *principium individuationis*? Was he taken seriously by physicists and philosophers of science on this topic as well? Schlick was no doubt echoing Schopenhauer when he wrote, also in the *Allgemeine Erkenntnislehre*: "In material reality, space and time are the great uniters and dividers. In the end, all the determinations by which we mark off an object of the external world and distinguish it as an individual thing from other individual things consist of specifications of time and place" (1918, 53). But what of those who were most involved in the debates about space, time, and individuation, the debates in which Einstein deployed his principle of spatiotemporal separability? I would count among the members of this core group, aside from Einstein, Erwin Schrödinger, Wolfgang Pauli and Niels Bohr (see Howard 1990). Evidently Bohr did not read Schopenhauer (according to a personal communication from David Favrholdt), but Schrödinger and Pauli did.

Karl von Meyenn, the editor of Pauli's scientific correspondence, tells us that Pauli "admired [Schopenhauer] greatly" (Pauli 1985, 586). In a 1954 essay on scientific and epistemological aspects of the unconscious, written on the occasion of Carl Jung's eightieth birthday, Pauli wrote sympathetically about extrasensory perception, noting approvingly that "even such a thoroughly critical philosopher as Schopenhauer not only regarded parapsychological effects going far beyond what is secured by scientific evidence as possible, but even considered them as a support for his philosophy," and adding: "Schopenhauer speaks metaphysically of the 'Will' that breaks through space and time, the 'principium individuationis,' as he calls them, and opposes the 'nexus metaphysicus' to the customary 'nexus physicus'" (Pauli 1954, 124, referring to the chapter "Animalischer Magnetismus und Magie" in Schopenhauer 1836). This is the same Pauli who, in discussing the EPR argument with Werner Heisenberg, in a letter of 15 June 1935 (four days *before* the letter to Schrödinger in which Einstein first enunciated his "separation principle"), cut right to the essential point about separability:

> Quite independently of *Einstein*, it appears to me that, in providing a systematic foundation for quantum mechanics, one should *start* more from the composition and separation of systems than has until now (with Dirac, e.g.) been the case. — This is indeed — as Einstein has *correctly* felt — a very fundamental point in quantum mechanics, which has, moreover, a direct connection with your reflections about the *cut* and the possibility of its being shifted to an arbitrary place. (Pauli 1985, 404)

We glimpse here a difference in the way different thinkers read Schopenhauer. If I might venture a shaky generalization, I would say that those thinkers broadly sympathetic to quantum mechanics, with its nonseparable way of accounting for interacting systems, tended to be drawn toward the holism that lies on the side of the Will, the thing-in-itself, in Schopenhauer; whereas a thinker critical of quantum nonseparability, like Einstein, tended to emphasize the more Kantian aspect of Schopenhauer's thought — in particular, the insistence that objective knowledge of empirical objects is possible only as those objects are individuated in space and time. Thus, Schrödinger, a friend of quantum nonseparability, is drawn, like Pauli, to the mystical side of Schopenhauer, the Schopenhauer who was the prophet of Vedanta.

Schrödinger's biographer, Walter Moore, details the lifelong influence of Schopenhauer on Schrödinger, starting with the fact that "Erwin read everything written by Schopenhauer," and adding that "his direct influence on Schrödinger was considerable, but equally important was the introduction he provided to Indian philosophy" (Moore 1989, 111, 112). Schrödinger himself later recalled that in 1918 he was "with great enthusiasm becoming familiar with Schopenhauer and, through him, with the doctrine of unity taught by the Upanishads" (ibid., 109). The traces of this influence are everywhere in Schrödinger, such as in the very Schopenhauerian epilogue, "On Determinism and Free Will," that concludes his famous *What Is Life?* (1944), or the metaphysical fourth chapter, "The Arithmetical Paradox: The Oneness of Mind," of *Mind and Matter* (1958), or the Schopenhauerian label that Schrödinger put on one folder of papers in his files: "Sammlung der Gedanken über das physikalische Principium individuationis" (AHQP, Erwin Schrödinger Papers, 44, 9).

Most interesting, however, is the essay, "Seek for the Road," which Schrödinger wrote during the summer of 1925 when he was first coming to grips with the failure of classical assumptions about the individuality of spatially separated particles brought to light by Einstein's work on Bose-Einstein statistics in the winter and spring of 1925, and immediately before the conceptual breakthrough that led to wave mechanics in December 1925 (Moore 1989, 191–209), the wave mechanics that Schrödinger saw as "a method of dealing with the problem of many particles" (1950, 206), the problem being the new statistics required by the failure of individuality.

"Seek for the Road" is thoroughly Schopenhauerian in spirit. Its main point is to argue for the unity of all consciousness, in spite of "the spatiotemporal plurality of individuals" (1925, 31). "For philosophy," Schrödinger writes, "the real difficulty lies in the spatial and temporal multiplicity of individuals." The difficulty is not to be solved by logical means, he claims, "but it is quite easy to express the solution in words, thus: the plurality that we perceive is only *an appearance*; *it is not real*" (ibid., 18). Applied to the problem of consciousness, this "solution" implies that "this life of yours which you are living is not merely a piece of the entire existence, but is in a certain sense the *whole*" (ibid., 21). Schrödinger summarizes this view by quoting the very same mystic formula of the Brahmins, called the *Mahavakya*, "*Tat tvam asi*" ["This living thing art thou"], that Schopenhauer regularly quoted for the purpose of expressing the idea of the unity of the will (see, for example, Schopenhauer 1859, vol. 1, 220, 355, 374). What follows from the unity of all consciousness? Schrödinger says: "It is the vision of this truth (of which the individual is seldom conscious in his actions) which underlies all morally valuable activity" (1925, 22).

To the end of his life, a belief in the unity of consciousness was central to Schrödinger's philosophy. In the chapters appended to "Seek for the Road" at the time of its preparation for publication in 1960, just months before his death in January 1961, Schrödinger wrote: "We still have the lovely thought of unity, of belonging unqualifiedly together, of which . . . Schopenhauer said that it was his comfort in life and would be his comfort in death"; and "Schopenhauer's books are still beautiful" (1964, 104, 110).[17]

There is surely no straight logical line leading from a belief in the unity of consciousness to the invention of entangled quantum states. But Schrödinger himself was no friend of a misplaced logic ("In a considerable number of cases logical thinking brings us up to a certain point and then leaves us in the lurch" [1925, 19]), and it seems not unlikely that the metaphysical vision of unity preached in "Seek for the Road" helped prepare the ground for Schrödinger's construction in wave mechanics of a theoretical framework within which entangled many-particle physical states could be realized.

So Pauli and Schrödinger, both of whom focused on the failure of classical notions of spatial separability in quantum mechanics,

were avid readers of Schopenhauer, much influenced by his holism, his glimpse of unity on the side of the Will, the thing-in-itself. Still, we have not yet found any of Einstein's philosophical and scientific contemporaries making *explicit* the connection between Schopenhauer's views on space and time as the *principium individuationis* and the failure of separability in quantum mechanics. Did anyone make this point?

Hermann Weyl came very close. To begin with, he clearly understood the role of space (if not also time) as the *principium individuationis*. Thus, he opens his seminal 1922 Barcelona and Madrid lectures, *Mathematische Analyse des Raumproblems*, as follows:

> With respect to reality, we follow Kant in distinguishing the *qualitative content* from its *Form*, the spatio-temporal extension that first makes possible a distinguishing of the qualitative. Without changing the nature of its content, in that it remains exactly such as it was, a body can find itself, instead of being *here*, at any other place in space. In this way it is possible, in the extensive medium of the external world (in which, in addition to space, we count time), for things that are, in their essences, in their natures, the same as one another, to be individually distinguished. (1923, 1)

And five years later, in his *Philosophie der Mathematik und Naturwissenschaft* (1927), he characterized "the essence of space" in this way:

> The penetration of the This (here-now) and the Thus is the general form of consciousness. A thing exists only in the indissoluble unity of intuition and sensation, through the superimposition of continuous extension and continuous quality. . . . Since the mere Here is nothing by itself that might differ from any other Here, space is the *principium individuationis*. It makes the existence of numerically different things possible which are equal in every respect. That is why Kant contradistinguishes it as the *form of intuition* from "the matter of phenomena, i.e. that which corresponds to sensation." Here lies the root of the concepts of similarity and congruence. (1927, 130–31)

Schopenhauer may not here be mentioned by name (he is discussed elsewhere by name; see ibid., 34, 210), but Weyl's use of the telltale Schopenhauerian expression, "*principium individuationis*," leaves little doubt as to the source. In the appendices added to the 1949 translation of this work, we find extensive discussions of quantum nonseparability (1949, 261–62), as well as of the failure of classical conceptions of particle individuation resulting from Bose-Einstein and Fermi-Dirac statistics (ibid., 245–47). Here again, the vocabulary he uses is interesting: "The upshot of it all is that the electrons satisfy Leibniz's *prin-*

cipium identitatis indiscernibilium. . . . In a profound and precise sense physics corroborates the Mutakallimûn; neither to the photon nor to the (positive and negative) electron can one ascribe individuality" (ibid., 247). The reference is to certain practitioners of *kalam* (speculative theology) in early Islam whose way of denying becoming involved a denial of the enduring identities of individual things. But even Weyl does not make the explicit connection we seek.

The one thinker I have so far found who does make the connection in the most unmistakable terms is Ernst Cassirer. The place is Chapter II.ii, "Zum Problem des 'materiellen Punktes,'" of *Determinismus und Indeterminismus in der modernen Physik.* Cassirer writes:

Quantum mechanics has emphasized again and again that within it a strictly mathematical schema exists, but that this schema is not to be imagined as a simple interconnection of things in space and time. If this is so, however, some very definite conclusions follow concerning the "individuality" of the elements with which quantum mechanics constructs nature. Schopenhauer declares that space and time are to be regarded as the real *principium individuationis.* In other places also in the philosophical history of the problem of individuation, we encounter this determination frequently. "It is easy to discover," Locke observed, "what is so much inquired after, the *principium individuationis*; and that, it is plain, is existence itself, which determines a being of any sort to a particular time and place incommunicable to two beings of the same kind. Let us suppose an atom . . . existing in a determined time and place; it is evident that, considered in any instant of its existence, it is, in that instant, the same with itself." Conversely it follows, however, that, when an object is no longer determinable by means of a "here" and "now," when it is not denotable as a τόδε τι, its "individuality" can no longer be maintained in the conventional sense, valid for things in space and time. . . . If then we continue to talk about the individuality of single particles, this can only be done indirectly; not insofar as they themselves, as individuals, are given, but so far as they are describable as "points of intersection" of certain relations. If we scrutinize the development of modern quantum mechanics, we will find this assumption fully confirmed. In de Broglie's wave theory of matter and in Schrödinger's wave mechanics the concepts of proton and electron are maintained, but they are defined no longer as "material points" in the sense of classical mechanics, but instead, as centers of energy. We may thus continue to talk of the electron as a definite object but it no longer possesses that individuation that could be designated by a simple "here" and "now." Waves are not tied to a single spatiotemporal point; they enjoy a kind of omnipresence. Each extends through the entire space—which, however, is no longer to be considered as an empirical space but as a configurational space. (1936, 180–81)

Cassirer goes on to note that the situation regarding individuation is the same in Heisenberg's matrix mechanics and in Born's statistical interpretation, adding, in a long footnote:

The impossibility of delimiting different electrons from one another, and of ascribing to each of them an independent individuality, has been brought into clear light through the evolution of the modern quantum theory, and particularly through the considerations connected with the Pauli exclusion principle. Considered solely from the standpoint of its methodological significance in the construction of the quantum theory, Pauli's exclusion principle is strangely analogous to the general principle introduced into philosophy by Leibniz under the name of *principium identitatis indiscernibilium.* This principle states that there cannot be two objects which completely correspond to each other in every determining characteristic, and thus are indistinguishable except by mere number. There are no things that differ from one another *solo numero*; rather every true difference must be definable as a qualitative difference, a distinction of the attributes and conditions that constitute the object. Cf. Leibniz, *Briefwechsel mit Clarke*, Letter 4, sec. 4; Letter 5, 6, etc. The Pauli principle is, as it were, the *principium identitatis indiscernibilium* of quantum theory. (Ibid., 184–85)

And, finally, Cassirer relates all of this to the problems of nonseparability and non-Boltzmann statistics:

A system consisting of two electrons determines, from the point of view of quantum mechanics, the state of these electrons, but the reverse does not follow. A knowledge of the states of the two parts does not determine the state of the joint system, and a derivation of the latter from the former is out of the question. The question, how, within a given whole, the separation into parts may be accomplished and how a certain aggregate may be differentiated and "individualized," accordingly always constitutes a difficult problem for quantum theory. The ordinary method of counting, which presupposes that it is known from the beginning what is to constitute one thing and what two or more things, is here insufficient. Individual things are not separate from each other in as simple a manner as in the sensuous-spatial view; complicated theoretical considerations are thus always required in order to determine precisely what is to be treated as an individual, what is to be counted as "one." According to the premises chosen, entirely different forms of quantum statistics may arise. . . . Here also we see clearly that the determination of the individual, or that which truly figures as "one" being, is not the *terminus a quo*, but always only the *terminus ad quem* for quantum theory. (Ibid., 187–88)

The large conclusion that Cassirer draws from this analysis is that we need not regard the quantum theory as threatening to impugn the universal validity of the causal principle, as long as we get clear about what counts as an individual in the ontology of the quantum theory. In other words, we can save causality, if we are willing to abandon the classical, spatiotemporal mode of individuating physical systems.

Einstein read Cassirer's *Determinismus und Indeterminismus in der modernen Physik* in March 1937. He recorded his reaction in a letter to Cassirer of 16 March:

I have read your book carefully and with sincere admiration. I do not know whether one should admire more the sagacity, the skill in presentation, or the depth of your knowledge of the subject. Moreover, it dawned on me for the first time in reading this book what a towering intellect Leibniz was. That he did not find the hypothesis of forces acting at a distance satisfactory is not so surprising; even Newton, himself, did not believe that this postulate was to be conceived as final, irreducible. His rejection of an absolute space is even more admirable. But that he realized that the atomic theory was to be rejected as a foundation for physics, because it is, in principle, irreconcilable with a representation by means of continuous functions (the law of impact as an elementary law), that required at that time true genius. (EA 8-394)

(The discussion of Leibniz is in the chapter immediately preceding that from which the above quotations were taken.) Einstein then goes on to rehearse for Cassirer the version of the argument for the incompleteness of quantum mechanics that he had first presented in correspondence with Schrödinger in June 1935, where he introduced his "separation principle." This time he concluded that one either recognizes that the quantum theoretical description of separated subsystems in the EPR-type experiment is incomplete, or "one would make up one's mind to believe that there is some kind of 'telepathic' interaction between the separated masspoints 1 and 2, something to which no theorist known to me could subscribe" (EA 8-394).

Did physicists and philosophers of science contemporary to Einstein take Schopenhauer seriously as someone with interesting and important things to say about fundamental questions in physics? Yes. Did they understand the relevance of what Schopenhauer said about space and time as the *principium individuationis* to the problems of nonseparability in quantum theory? Yes. Could Einstein have seen this connection? How could he not have seen it if he read Schopenhauer as thoroughly as the evidence suggests, and if, besides, he read Cassirer's *Determinismus und Indeterminismus in der modernen Physik* as carefully as he told Cassirer he had? The evidence is, of course, still only circumstantial. But it is not wildly implausible to assume that Einstein understood the connection. If he did, it would help us to understand why he accorded Schopenhauer's portrait a place of honor alongside those of Faraday and Maxwell.

Conclusion

What, then, have we learned? We have learned that, whether or not Einstein learned about space and time as the *principium individuationis* from Schopenhauer, his invocation of the separability principle in the course of explicating his concerns about the quantum theory occurs in a context. The context is a centuries-old tradition, which found clear expression in the writings of thinkers like Newton, Locke, and Kant, a tradition that regards space as the ground against which physical systems are individuated, a tradition according to which "in material reality, space and time are the great uniters and dividers" (Schlick 1918, 53). And we have seen that it was Schopenhauer, more than anyone else, who made this idea a commonplace in the minds of early twentieth-century physicists and philosophers of science.

Understanding this context makes it plausible that Einstein, like many of his contemporaries, learned the lesson about space and time as the *principium individuationis* from Schopenhauer. At the very least, understanding the context should help us better to understand why Einstein clung so tenaciously to his separability principle. And it should also help us to understand why Einstein regarded field theory—which provides the most extreme embodiment of the separability principle—as the proper framework for fundamental physics.

Einstein clung to separability and the field concept because, in this tradition, space and time as *principium individuationis* function as a priori conditions for the very possibility of objective scientific knowledge of empirical objects. Notice that I said "space and time as *principium individuationis*." It is not the absolute space and time of Newton that have this a priori status. It is not space as described by Euclidean geometry. It is not the space and time of the turn-of-the-century neo-Kantianism that was so offended by general relativity. Instead, it is space and time as *principium individuationis*. Remember what Einstein said in his comments to Born in March 1948:

If one renounces the assumption that what is present in different parts of space has an independence, real existence, then I do not at all see what physics is supposed to describe. For what is thought to be a "system" is, after all, just conventional, and I do not see how one is supposed to divide up the world objectively so that one can make statements about the parts. (Born 1969, 224)

Einstein was certainly no friend of the Kantian a priori. He believed, with Kant, that we necessarily bring something of our own to our construction of scientific theories of the world—at least that our theories are not logical deductions from experience—but he denied that what we contribute is somehow fixed by the very nature of the human mind or knowledge itself (see Howard 1994). I would not want to overread a passage such as this. Still, the sentiment expressed is a strong one. Give up separability "and I do not see how one is supposed to divide the world objectively so that one can make statements about the parts."

From the point of view that regards separability as the fundamental metaphysical fact about space, there is a continuity in the evolution of our understanding of space since the seventeenth century (and earlier) that is largely overlooked in the standard histories. Thus, if it is separability, space as *principium individuationis*, that is the basic property of space, then it makes little difference whether one regards space as something absolute or relative. The space or spacetime of Einstein individuates physical systems just as well as the space of Newton. Of course the relative space of Leibniz does not. In asserting a relative conception of space in opposition to Newton's absolute space, Leibniz meant to deny something much more fundamental about Newtonian space than Einstein ever meant to deny. And that something is, again, separability. To put it in different words, Einstein's relativity is not Leibniz's relativity.

It also makes no difference whether one takes the material particle or the field as ontologically primary. The idea of separability, space as *principium individuationis*, first grew up in the context of atomism. But Einstein understood that it was field theory that gives the most extreme expression of the idea of separability, by its treating every point of space (or spacetime) as, in effect, a separable physical system endowed with its own, separable, real physical state. Even allowing the material contents of space to be swallowed up, as it were, by the geometry of space (spacetime) makes no difference in the capacity of space to act as the ground of individuation.

And, finally, it makes no essential difference whether one distinguishes space and time as Newton did, or unites them into one, four-dimensional spacetime manifold, as Einstein did. The only change this

brings about in the way we individuate things in nature is that, whereas classically space is the ground for the individuation of *systems*, *bodies*, *things*, or *substances*, and time is the ground for the individuation of the *states* of those systems, bodies, things, or substances, in Einsteinian spacetime the distinction between *system* and *state* lapses.

All that is necessary for a unified spacetime to fulfil its inherited role as *principium individuationis*, is that, in it, the basic idea of *separation* retains its objective significance. In the case of a four-dimensional continuum, the minimum necessary condition for the possibility of objective individuation is the objectivity, which is to say the invariance under arbitrary transformations, of the infinitesimal metric interval:

$$ds^2 = g_{ik}dx_i dx_k.$$

But this is precisely the fundamental invariant of the general theory of relativity. So one could say that general relativity is, simply, the most general way of doing physics that retains the basic idea of space and time, or spacetime, as the *principium individuationis*.

With this insight, history closes upon itself. For the principle underlying the definition of the infinitesimal metric interval is the Pythagorean theorem. It is the most abstract representation of the idea of *distance* and was the great secret of the Pythagoreans. But it was those same Pythagoreans to whom, in the end, we owe our idea of space. As F. M. Cornford reminds us, in "The Invention of Space":

Our first glimpse of the Void in philosophic literature we owe to a passage in Aristotle's *Physics*, recording a feature of the primitive Pythagorean cosmology: "The Pythagoreans too asserted that Void exists and that it enters the Heaven itself, which, as it were, breathes in from the boundless a sort of breath which is at the same time the Void. This keeps things apart, as if it constituted a sort of separation or distinction between things that are next to each other. . . ." (*Phys.*, IV, 6, 213b, 23). (Cornford 1928, 8)

There is more to be said about Einstein's reading of Schopenhauer. For example, the characteristic way in which Schopenhauer combines space and time with causality, as equally essential a priori conditions for the possibility of objective scientific knowledge, suggests Einstein's equally firm commitments to both separability and causality. Of course, one learns a similar lesson from Kant himself, but it is not at all impossible that, like Robert Musil, and so many other young thinkers of that day, Einstein learned his Kant filtered through the lens of Schopenhauer. More and more I see the ghost of Schopenhauer in the whole debate over how the quantum theory might force us to choose between spacetime representation and causality, or at least to regard them as complementary modes of description. But this deserves a fuller study in its own right.

Finally, we should, perhaps, draw a lesson about how an Einstein must be regarded as a whole thinker. We should resist the positivist prejudice (so much in evidence in Frank's characterization of Einstein's reading of Schopenhauer) that regards some thinkers from the past, such as Hume, Mach, and (grudgingly) Kant, as legitimate candidates for being influences on an Einstein's scientific development, and consigns thinkers such as Schopenhauer to the category of the merely literary. Schrödinger, for example, would have been offended at our denying the importance of Schopenhauer to his intellectual development. Who are we to say, by the lights of our own puny prejudices, how a thinker of Einstein's stature is to be allowed to weave together, from a hundred different sources, an original picture of the natural world? We should be content to be visitors in the gallery, not bluenose censors of the life of the mind.

NOTES

The intellectual debts that I hope in some small measure to repay with this essay are old ones. Two of my graduate school teachers, Milič Čapek and Judson Webb, will see important parts of themselves in what I write here. They may not want to claim credit for it, and I do not want to implicate them in any errors, oversights, or misinterpretations the essay may contain. But I do want, sincerely and generously, to express my thanks to them for the seeds they planted long ago. More specific thanks are owing to a variety of other colleagues, including Mara Beller, David Cartwright, Catherine Chevalley, Bob Cohen, Dan Breazeale, Arthur Fine, Dan Frank, Don Giles, John Inglis, Michel Janssen, John Norton, Ze'ev Rosnkranz, Tom Ryckman, Henry Schankula, Robert Schulmann, Abner Shimony, John Stachel, and Jim Wilkinson. I received many helpful suggestions from audiences at the University of Pittsburgh, Northwestern University, the University of Notre Dame, Boston University, the Universität Göttingen, and the 1994 meeting of the North American Division of the Schopenhauer Society, where versions of this paper were read.

My sincere thanks go to the University of Kentucky for granting me a University Research Professorship for the 1992–1993 academic year, during which time this essay was written. Finally, I thank the Hebrew University of Jerusalem, which holds the copyright, for permission to quote from Einstein's unpublished correspondence.

1. This report is seconded by Konrad Wachsmann, who also explicitly disputes the assertion by Helen Dukas and Banesh Hoffmann (Hoffmann-Dukas 1972, 46) that the portraits were of Newton, Faraday, and Maxwell; see the interview with Wachsmann in Grüning 1989, 144. On the other hand, in an undated photograph of Einstein in his study reproduced in Grüning 1989, 136, one sees only a portrait of Newton, on the wall (the same photo is also reproduced in Sugimoto 1989, 102).

2. For Einstein's early reading of these individuals and others named in this paragraph, see the respective references in Einstein 1987, 1989.

3. For a representative sampling of testimony to this influence, see the many essays and "testimonials" conveniently collected in Haffmans 1977, including Nietzsche's "Schopenhauer als Erzieher," originally published in 1874 as part of the *Unzeitgemäße Betrachtungen*, and Mann's "Schopenhauer," originally written in 1938 as an introduction to *Living Thoughts of Schopenhauer* (New York: Longmans, Green, 1939).

4. Einstein wrote in the preface: "The author of this book is one who knows me rather intimately in my endeavor, thoughts, beliefs—in bedroom slippers. I have read it to satisfy, in the main, my own curiosity. . . . I found the facts of the book duly accurate, and its characterization, throughout, as good as might be expected of one who is perforce himself, and who can no more be another than I can."

5. The question actually came from Einstein's second son, Eduard, then housed in a Swiss mental institution, where he was befriended by Seelig; see Seelig 1960, 191–92.

6. Einstein owned the twelve-volume 1894–1896 edition by Cotta, which was based on the Julius Frauenstädt edition of 1873–1874 (see Hübscher 1981, 37) and carried an introduction by Rudolf Steiner. My thanks to Ze'ev Rosenkranz, director of the Albert Einstein Archives, for locating and identifying this edition.

7. For a healthy antidote to Frank's reading of Einstein, see Holton 1968. For a critical response to Holton, see Howard 1993. Frank is also wrong about Einstein's attitude toward Kant, a thinker whom Einstein took seriously, even while disagreeing with him (see Howard 1994). What is convincingly "Einsteinian" in Frank's account is the use of the musical image to disparage a second-rate thinker. But I know of only one case where Einstein himself deployed this image for such a purpose, and Hegel, not Schopenhauer, was the target. On 16 May 1951, Einstein wrote to G. Broggi: "In my eyes, a philosopher of Hegel's type is a man who juggles words that correspond to no clear concepts, a kind of word-music" (EA 25-428). I thank Arthur Fine for drawing my attention to this remark.

8. In a very different intellectual tradition, Schopenhauer's influence on Einstein's world view was long taken seriously, but for all the wrong reasons. In 1951, the Soviet philosopher M. M. Karpov wrote the following in a contribution to a long-running discussion of Einstein's philosophy in *Voprosy filosofii*: "Einstein's views and opinions were formed under the influence of such idealist philosophers as Hume, Mach, and Schopenhauer. That could not help influencing his philosophical views. Einstein answers the basic question of philosophy idealistically" (1951, 130, quoted in Gribanov 1987, 38).

9. Schopenhauer says that this need for metaphysics arises from wonder experi-

enced by the will made objective in man: "And its wonder is the more serious, as here for the first time it stands consciously face to face with *death*, and besides the finiteness of all existence, the vanity and fruitlessness of all effort force themselves on it more or less" (1859, vol. 2, 160). It is perhaps no coincidence that Einstein's first really serious physical collapse, in the form of an attack of pericarditis, had occurred only shortly before, in the spring of 1928, forcing him to endure a long period of bed rest (see Clark 1971, 348–50).

10. For a reconstruction of the hidden variables model attempted by Einstein, see Cushing 1994, chap. 8, sec. 3, and app. 3.

11. It is clear from a letter of 9 August 1939 that the "talmudist" is intended to be Bohr. After once again distinguishing the "Born" and "Schrödinger" interpretations of the Ψ-function, Einstein writes: "There are also, however, the mystics, who altogether prohibit, as unscientific, any question about something existing independently of experience (Bohr). Then the two conceptions flow together in a soft haze, in which I do not feel any better, however, than in one of the aforementioned conceptions, which take a position on the reality concept" (EA 22-060).

12. There is a way of reading the implications of Bell's theorem and the Bell experiments that implies that Einstein was wrong, that, in order to avoid violations of the locality principle, which, recall, encapsulates special relativistic constraints on superluminal signals, one must give up separability. And there is the further argument that, if separability is so intimately connected with the concept of a field theory like general relativity, then giving up separability threatens also the very foundations of general relativity (see Howard 1989, 1993).

13. See the lengthy entries under "Individuation," "Individuum," and "Individualität," in Wagner 1909, 184–86.

14. For Einstein's enrollment in Stadler's lectures, see Einstein 1987, 46, 364. As noted above, however, there is some question about Einstein's actual attendance at lectures.

15. In 1908, Musil wrote a doctoral dissertation on Ernst Mach, *Beitrag zur Beurteilung der Lehren Machs*, under Karl Stumpf in Berlin; Luft 1980, 81–88.

16. For Brod's enthusiasm for Schopenhauer, see the references in Brod 1947, 1966, 1979.

17. The unity of *consciousness* is, to be sure, a more specifically Vedantic theme; Schopenhauer insists on the unity of the *will*. Schrödinger's emphasis on the unity of consciousness thus may reflect his reading of Paul Deussen's *Das System der Vedanta* (1906) and other works on Indian philosophy and religion (see Moore 1989, 113), as much as it reflects his reading of Schopenhauer, Schrödinger's interpretation of Schopenhauer being influenced, perhaps, by his interest in the Vedanta. My thanks to David Cartwright on this point.

REFERENCES

Alexander, H. G., ed. 1956. *The Leibniz-Clarke Correspondence. With Extracts from Newton's "Principia" and "Opticks."* Manchester: Manchester University Press; New York: Barnes & Noble.

Bailey, C. 1928. *The Greek Atomists and Epicurus. A Study.* Oxford: Clarendon Press.

Bernstein, A. 1853–1857. *Aus dem Reiche der Naturwissenschaft. Für Jedermann aus dem Volke.* 12 vols. Berlin: Besser's Verlagsbuchhandlung. Reissued as *Naturwissenschaftliche Volksbücher.* 20 vols. Berlin: F. Duncker, 1867–1869.

Bohr, N., 1949. "Discussion with Einstein on Epistemological Problems in Atomic Physics." In Schilpp 1949, 201–41.

Boltzmann, L. 1905. "Über eine These Schopenhauers." In *Populäre Schriften.* Leipzig: Johann Ambrosius Barth, 385–402.

Born, M. 1949. *Natural Philosophy of Cause and Chance.* Oxford: Oxford University Press.

Born, M., ed. 1969. *Albert Einstein — Hedwig und Max Born. Briefwechsel 1916–1955.* Munich: Nymphenburger.

Bose, S. N. 1924. "Plancks Gesetz und Lichtquantenhypothese." *Zeitschrift für Physik* 26: 178–81.

Brann, H. W. 1975. *Schopenhauer und das Judentum.* Bonn: Bouvier Verlag Herbert Grundmann.

Brod, M. 1947. *Diesseits und Jenseits.* 2 vols. Zurich: Mondial-Verlag.

———. 1966. *Der Prager Kreis.* Stuttgart: Kohlhammer.

———. 1979. *Streitbares Leben. Autobiographie 1884–1968.* Frankfurt am Main: Insel Verlag.

Büchner, L. 1855. *Kraft und Stoff. Empirisch-naturphilosophische Studie.* Frankfurt am Main: Meidinger Sohn & Cie.

Buroker, J. V. 1981. *Space and Incongruence: The Origin of Kant's Idealism.* Dordrecht: D. Reidel.

Cassirer, E. 1918. *Kants Leben und Lehre.* Vol. 11 (Ergäzungsband) of *Immanuel Kants Werke.* Ernst Cassirer, ed. Berlin: Bruno Cassirer. Page numbers from *Kant's Life and Thought*, trans. James Haden. New Haven and London: Yale University Press, 1981.

———. 1936. *Determinismus und Indeterminismus in der modernen Physik. Historische und systematische Studien zum Kausalproblem.* Vol. 42, pt. 3, *Göteborgs Högskolas Ärsskrift.* Separatum: Göteborg: Elanders Boktryckeri Aktiebolag, 1937. Page numbers and translations from *Determinism and Indeterminism in Modern Physics: Historical and Systematic Studies of the Problems of Causality*, trans. O. Theodor Benfey. New Haven and London: Yale University Press, 1956.

Clark, R. W. 1971. *Einstein: The Life and Times.* New York and Cleveland: World Publishing.

Cornford, F. M. 1928. "The Invention of Space." In *Essays in Honor of Gilbert Murray.* London: Allen & Unwin. Page numbers from the partial reprinting in *The Concepts of Space and Time: Their Structure and Their Development.* Milič Čapek, ed. Dordrecht: D. Reidel, 1976.

Cushing, J. T. 1994. *Quantum Mechanics: Historical Contingency and the Copenhagen Hegemony.* Chicago: University of Chicago Press.

De Beer, E. S., ed. 1979. *The Correspondence of John Locke.* 8 vols. Oxford: Clarendon Press.

Deussen, P. 1906. *Das System des Vedanta. Nach den Brahma-Sutra's des Badara-*

yana und dem Kommentare des Cankara über dieselben als ein Kompendium der Dogmatik des Brahmanismus vom Standpunkte des Cankara aus dargestellt. 2nd ed. Leipzig: F. A. Brockhaus.

Dijksterhuis, E. J. 1961. *The Mechanization of the World Picture.* Trans. C. Dikshoorn. London: Oxford University Press.

Einstein, A. 1905. "Über einen die Erzeugung und Verwandlung des Lichtes betreffenden heuristischen Gesichtspunkt." *Annalen der Physik* 17: 132–48.

——. 1909a. "Zum gegenwärtigen Stand des Strahlungsproblems." *Physikalische Zeitschrift* 10: 185–93.

——. 1909b. "Über die Entwickelung unserer Anschauungen über das Wesen und die Konstitution der Strahlung." *Deutsche Physikalische Gesellschaft. Verhandlungen* 11: 482–500. Reprint. *Physikalische Zeitschrift* 10 (1909): 817–25.

——. 1918. "Motive des Forschens." In *Zu Max Plancks sechzigstem Geburtstag. Ansprachen, gehalten am 26. April 1918 in der Deutschen Physikalischen Gesellschaft.* Karlsruhe: C. F. Müller, 29–32. Page numbers from "Principles of Research," in Einstein 1954, 224–27.

——. 1924. "Quantentheorie des einatomigen idealen Gases." *Preussische Akademie der Wissenschaften. Physikalisch-mathematische Klasse. Sitzungsberichte*: 261–67.

——. 1925a. "Quantentheorie des einatomigen idealen Gases. Zweite Abhandlung." *Preussische Akademie der Wissenschaften. Physikalisch-mathematische. Klasse. Sitzungsberichte*: 3–14.

——. 1925b. "Zur Quantentheorie des idealen Gases." *Preussische Akademie der Wissenschaften. Physikalisch-mathematische Klasse. Sitzungsberichte*: 18–25.

——. 1930. "Religion and Science." *New York Times Sunday Magazine*, 9 November 1930, 1–4. A German version, "Religion und Wissenschaft," was published in the *Berliner Tageblatt*, 11 November 1930, suppl., p. 1, and in Einstein 1934, 36–45. Page numbers from Einstein 1954, pp. 36–40.

——. 1931. "The World as I See It." *Forum and Century* 84: 193–94. Reprinted in *Living Philosophies.* New York: Simon & Schuster, 1931, 3–7. Page numbers from Einstein 1954, 8–11.

——. 1934. *Mein Weltbild.* Amsterdam: Querido.

——. 1946. "Autobiographisches—Autobiographical Notes." In Schilpp 1949, 1–94. Page numbers and translations from the corrected version, *Autobiographical Notes: A Centennial Edition.* Trans. and ed. Paul Arthur Schilpp. La Sable, Ill.: Open Court, 1979.

——. 1948. "Quanten-Mechanik und Wirklichkeit." *Dialectica* 2: 320–24.

——. 1954. *Ideas and Opinions.* New York: Bonanza Books.

——. 1955. "Erinnerungen-Souvenirs." *Schweizerische Hochschulzeitung* 28 (*Sonderheft*): 145–53. Page numbers from the reprinting as "Autobiographische Skizze," in Seelig 1956, 9–17.

——. 1987. *The Collected Papers of Albert Einstein.* Vol. 1, *The Early Years, 1879–1902.* John Stachel et al., eds. Princeton: Princeton University Press.

——. 1989. *The Collected Papers of Albert Einstein.* Vol. 2, *The Swiss Years: Writings, 1900–1909.* John Stachel et al., eds. Princeton: Princeton University Press.

Einstein, A., B. Podolsky, and N. Rosen. 1935. "Can Quantum-Mechanical Description of Physical Reality Be Considered Complete?" *Physical Review* 47: 777–80.

Fine, A. 1981. "Einstein's Critique of Quantum Theory: The Roots and Significance of EPR." In *After Einstein: Proceedings of the Einstein Centennial Celebration at Memphis State University, 14–16 March 1979.* Peter Barker and Cecil G. Shugart, eds. Memphis, Tenn.: Memphis State University Press, 147–58. Reprinted in Fine 1986b, 26–39.

——. 1986a. "Schrödinger's Cat and Einstein's: The Genesis of a Paradox." In Fine 1986b, 64–85.

——. 1986b. *The Shaky Game: Einstein, Realism, and the Quantum Theory.* Chicago: University of Chicago Press.

Föppl, A. 1894. *Einführung in die Maxwell'sche Theorie der Elektrizität.* Leipzig: B. G. Teubner.

Frank, P. 1947. *Einstein: His Life and Times.* Trans. George Rosen. Shuichi Kusaka, ed. New York: Alfred A. Knopf.

Gracia, J. E., trans. and ed. 1982. *Suárez on Individuation. Metaphysical Disputation V: Individual Unity and Its Principle.* Milwaukee: Marquette University Press. The text is based on Carolo Berton's edition of Disputation V in Suárez's *Opera omnia*, vol. 25. Paris: Vivès, 1861, 145b–201a.

Gribanov, D. P. 1987. *Albert Einstein's Philosophical Views and the Theory of Relativity.* H. Campbell Creighton, trans. Moscow: Progress Publishers.

Grüning, M. 1989. *Ein Haus für Albert Einstein. Erinnerungen • Briefe • Dokumente.* Berlin: Verlag der Nation.

Haffmans, G., ed. 1977. *Über Arthur Schopenhauer.* Zurich: Diogenes.

Hall, A. R., and M. B. Hall, eds. and trans. 1962. *Unpublished Scientific Papers of Isaac Newton: A Selection from the Portsmouth Collection in the University Library, Cambridge.* Cambridge: Cambridge University Press.

Herneck, F. 1969. "Einstein und die Willensfreiheit." *Physik in der Schule*, no. 4: 145–49. Page numbers from the revised and expanded version, "Einstein und Schopenhauer." In *Einstein und Sein Weltbild. Aufsätze und Vorträge.* Berlin: Buchverlag der Morgen, 1976, 199–210.

Hoffmann, B., and H. Dukas. 1972. *Albert Einstein: Creator and Rebel.* New York: Viking Press.

Holton, G. 1968. "Mach, Einstein, and the Search for Reality." *Daedalus* 97: 636–73. Reprinted in *Thematic Origins of Scientific Thought: Kepler to Einstein.* Cambridge: Harvard University Press, 1973, 219–59.

Howard, D. 1984. "Realism and Conventionalism in Einstein's Philosophy of Science: The Einstein-Schlick Correspondence." *Philosophia Naturalis* 21: 618–29.

——. 1985. "Einstein on Locality and Separability." *Studies in History and Philosophy of Science* 16: 171–201.

——. 1989. "Holism, Separability, and the Metaphysical Implications of the Bell Experiments." In *Philosophical Consequences of Quantum Theory: Reflec-*

tions on Bell's Theorem. James T. Cushing and Ernan McMullin, eds. Notre Dame, Ind.: University of Notre Dame Press, 224–53.
——. 1990. "'Nicht sein kann was nicht sein darf,' or the Prehistory of EPR, 1909–1935: Einstein's Early Worries about the Quantum Mechanics of Composite Systems." In *Sixty-two Years of Uncertainty: Historical, Philosophical, and Physical Inquiries into the Foundations of Quantum Mechanics.* Arthur I. Miller, ed. New York and London: Plenum, 61–111.
——. 1993. "Was Einstein Really a Realist?" *Perspectives on Science: Historical, Philosophical, Social* 1: 204–51. Revised version of "Einstein era realamente un realista?" In *Realismo/Antirealismo: Aspetti del dibattito epistemologico contemporaneo.* Alessandro Pagnini, ed. Florence: La Nuova Italia, 1995, 93–141.
——. 1994. "Einstein, Kant, and the Origins of Logical Empiricism." In *Language, Logic, and the Structure of Scientific Theories: The Carnap-Reichenbach Centennial.* Wesley Salmon and Gereon Wolters, eds. Pittsburgh: University of Pittsburgh Press; Konstanz: Universitätsverlag, 1994, 45–105.
Hübscher, A. 1981. *Schopenhauer-Bibliographie.* Stuttgart-Bad Cannstatt: Frommann-Holzboog.
Jammer, M. 1974. *The Philosophy of Quantum Mechanics: The Interpretation of Quantum Mechanics in Historical Perspective.* New York: John Wiley.
——. 1985. "The EPR Problem in Its Historical Context." In *Symposium on the Foundations of Modern Physics.* P. Lahti and P. Mittlestaedt, eds. Singapore: World Publishing, 129.
Janik, A., and S. Toulmin. 1973. *Wittgenstein's Vienna.* New York: Simon and Schuster.
Kant, I. 1768. "Von dem Ersten Grunde des Unterschiedes der Gegenden im Raume." *Königsberger Frag- und Anzeigungsnachrichten.* In Kant 1900–1942, vol. 2, pp. 375–84. Translations and page numbers from "Concerning the Ultimate Foundations of the Differentiation of Regions in Space." In Kant 1968, pp. 36–43.
——. 1770. *De mundi sensibilis atque intelligibilis forma et principiis.* Königsberg: Königl. Hof- und akadem. Druckerei. In Kant 1900–1942, vol. 2, 385–419. Translations and page numbers from "On the Form and Principles of the Sensible and Intelligible World." In Kant 1968, 45–92.
——. 1781. *Kritik der reinen Vernunft.* Riga: Johann Friedrich Hartknoch. In Kant 1900–1942, vol. 4, 1–252.
——. 1783. *Prolegomena zu einer jeden künftigen Metaphysik, die als Wissenschaft wird auftreten können.* In Kant 1900–1942, vol. 4, 253–384. Page numbers from *Prolegomena to any Future Metaphysics.* Lewis White Beck, trans. Indianapolis: Bobbs-Merrill, 1950.
——. 1786. *Metaphysische Anfangsgründe der Naturwissenschaft.* In Kant 1900–1942, vol. 4, pp. 465–566.
——. 1787. *Kritik der reinen Vernunft,* 2nd ed. Riga: Johann Friedrich Hartknoch. In Kant 1900–1942, vol. 3, pp. 1–552.
——. 1900–1942. *Kants gesammelte Schriften.* 22 vols. Berlin: Preussische Akademie der Wissenschaften.

——. 1968. *Kant: Selected Pre-Critical Writings and Correspondence with Beck.* G. B. Kerferd and D. E. Walford, trans. Manchester: Manchester University Press; New York: Barnes & Noble.
Karpov, M. M. 1951. "On the Philosophical Views of A. Einstein." *Voprosy filosofii 1.*
Köhnke, K. C. 1986. *Entstehung und Aufstieg des Neukantianismus. Die deutsche Universitätsphilosophie zwischen Idealismus und Positivismus.* Frankfurt am Main: Suhrkamp.
Kollros, L. 1955. "Erinnerungen-Souvenirs." *Schweizerische Hochschulzeitung* 28 (*Sonderheft*): 169–173. Page numbers from translation as "Erinnerungen eines Kommilitonen," in Seelig 1956, 17–31.
Laban, F. 1880. *Die Schopenhauer-Literatur. Versuch einer chronologischen Uebersicht derselben.* Reprint, New York: Burt Franklin, 1970.
Leibniz. G. W. 1696. "On the Principle of Indiscernibles." In *Leibniz: Philosophical Writings.* G. H. R. Parkinson, ed. London: J. M. Dent, 133–35.
——. 1981. *New Essays on Human Understanding.* Peter Remnant and Jonathan Bennett, trans. and eds. Cambridge: Cambridge University Press.
Locke, J. 1894. *An Essay Concerning Human Understanding.* 2 vols. Alexander Campbell Fraser, ed. Oxford: Oxford University Press. Reprint, New York: Dover, 1959.
Luft, D. S. 1980. *Robert Musil and the Crisis of European Culture, 1880–1942.* Berkeley: University of California Press.
Mach, E. 1905. *Erkenntnis und Irrtum. Skizzen zur Psychologie der Forschung.* Leipzig: Johann Ambrosius Barth.
Magee, B. 1983. *The Philosophy of Schopenhauer.* Oxford: Clarendon Press.
Maurer, A. 1987. "Translator's Introduction." In Thomas Aquinas 1987, vii–xxxviii.
Monk, R. 1990. *Ludwig Wittgenstein: The Duty of Genius.* New York: Penguin.
Moore, W. 1989. *Schrödinger: Life and Thought.* Cambridge: Cambridge University Press.
Moszkowski, A. 1921. *Einstein. Einblicke in seine Gedankenwelt. Gemeinverständliche Betrachtungen über die Relativitäts-Theorie und ein neues Welt-System entwickelt aus Gesprächen mit Einstein.* Hamburg: Hoffmann und Campe; Berlin: F. Fontane. Translated as *Einstein the Searcher: His Work Explained from Dialogues with Einstein.* Henry L. Brose, trans. New York: E. P. Dutton, 1921. Page numbers from *Conversations with Einstein.* New York: Horizon Press, 1970.
Pauli, W. 1954. "Naturwissenschaftliche und erkenntnistheoretische Aspekte der Ideen vom Unbewußten." *Dialectica* 8. Reprinted in *Aufsätze und Vorträge über Physik und Erkenntnistheorie.* Braunschweig: Friedrich Vieweg & Sohn, 113–28.
——. 1985. *Wissenschaftlicher Briefwechsel mit Bohr, Einstein, Heisenberg u.a.* Vol. 2, *1930–1939.* Karl von Meyenn, ed. Berlin: Springer-Verlag.
Reiser, A. [Rudolf Kayser, pseud.] 1930. *Albert Einstein: A Biographical Portrait.* New York: Albert and Charles Boni.
Rosenberger, F. 1895. *Isaac Newton und seine physikalischen Principien. Ein*

Hauptstück aus der Entwickelungsgeschichte der modernen Physik. Leipzig: Johann Ambrosius Barth.
Safranski, R. 1990. *Schopenhauer and the Wild Years of Philosophy*. Trans. Ewald Osers. Cambridge: Harvard University Press.
Schilpp, P. A., ed. 1949. *Albert Einstein: Philosopher-Scientist*. Evanston, Ill.: Library of Living Philosophers.
Schlick, M. 1918. *Allgemeine Erkenntnislehre*. Berlin: Julius Springer. Translations and page numbers from *General Theory of Knowledge*. Albert E. Blumberg, trans. La Sable, Ill.: Open Court, 1985.
——. 1932. "Gibt es ein materiales Apriori?" In *Wissenschaftlicher Jahresbericht der Philosophischen Gesellschaft an der Universität zu Wien—Ortsgruppe Wien der Kant-Gesellschaft für das Vereinsjahr 1931/32*. Vienna, 55–65. Page numbers from "Is There a Factual *A Priori*?" In *Philosophical Papers*, vol. 2 [1925–1936]. Henk L. Mulder and Barbara F. B. van de Velde-Schlick, eds. Trans. Peter Heath, Wilfrid Sellars, Herbert Feigl, and May Brodbeck. Dordrecht: D. Reidel, 1979, 161–70.
Schopenhauer, A. 1813. *Ueber die vierfache Wurzel des Satzes vom zureichenden Grunde. Eine philosophische Abhandlung*. Rudolstadt: in Commision der Hof-Buch-und-Kunsthandlung. Translations and page numbers from *On the Fourfold Root of the Principle of Sufficient Reason*. Trans. E. F. J. Payne. La Sable, Ill.: Open Court, 1974.
——. 1819. *Die Welt als Wille und Vorstellung*. Leipzig: F. A. Brockhaus.
——. 1836. *Ueber den Willen in der Nature. Eine Erörterung der Bestätigungen, welche die Philosophie des Verfassers, seit ihrem Auftreten, durch die empirischen Wissenschaften erhalten hat*. Frankfurt am Main: Siegmund Schmerber.
——. 1841a. "Ueber die Freiheit des menschlichen Willens." In Schopenhauer 1841b, 361–458.
——. 1841b. *Die beiden Grundprobleme der Ethik*. Frankfurt am Main: Joh. Christ. Hermann'sche Buchhandlung, F. E. Suchsland, 1–100. Page numbers from the reprinting in *Arthur Schopenhauers Werke in fünf Bänden*. Ludger Lütkehaus, ed. Vol. 3, *Kleinere Schriften*. Zurich: Haffmanns Verlag, 1988.
——. 1844. *Die Welt als Wille und Vorstellung*. 2 vols. 2nd ed. Leipzig: F. A. Brockhaus.
——. 1851. *Parerga und Paralipomena. Kleine philosophische Schriften*. 2 vols. Berlin: A. W. Hayn. Page numbers and quotations from the edition supported by the Schopenhauer-Gesellschaft and the Schopenhauer-Archiv, *Parerga und Paralipomena. Kleine philosophische Schriften*. 2 vols. Zurich: Diogenes, 1977, which is based on the 3d ed. of Arthur Hübscher's critical-historical edition of the original text, Wiesbaden: Brockhaus, 1972.
——. 1859. *Die Welt als Wille und Vorstellung*, 3d impr. and enl. ed. Leipzig: F. A. Brockhaus. Translations and page numbers from *The World as Will and Representation*. 2 vols. E. F. J. Payne, trans. Indian Hills, Colo.: Falcon's Wing, 1958; reprint, New York: Dover, 1969. The Payne translation is based on Arthur Hübscher's German edition, Leipzig: F. A. Brockhaus, 1937.
——. 1874. *Parerga und Paralipomena. Kleine philosophische Schriften*. 2 vols.

3rd ed. Julius Frauenstädt, ed. Leipzig: Brockhaus. [4th ed., 1878; 5th ed., 1888; 6th ed., 1888; 7th ed., 1891.]
——. 1886. *Aphorismen zur Lebensweisheit. Separatausgabe aus "Parerga und Paralipomena."* 2 vols. Wilhelm Gwinner, ed. Leipzig: Brockhaus.
——. 1985. *Der handschriftliche Nachlass in fünf Bänden*. Vol. 1, *Frühe Manuskripte (1804–1818)*. Arthur Hübscher, ed. Munich: Deutscher Taschenbuch Verlag. Translations and page numbers from the English translation: *Manuscript Remains in Four Volumes*. Vol. 1, *Early Manuscripts (1804–1818)*. E. F. J. Payne, trans. Oxford, New York, and Hamburg: Berg, 1988.
Schrödinger, E. 1925. "Seek for the Road." In Schrödinger 1964, 1–58.
——. 1935a. "Die gegenwärtige Situation in der Quantenmechanik." *Die Naturwissenschaften* 23: 807–12, 824–28, 844–49.
——. 1935b. "Discussion of Probability Relations between Separated Systems." *Cambridge Philosophical Society. Proceedings* 31: 555–63.
——. 1936. "Probability Relations between Separated Systems." *Cambridge Philosophical Society. Proceedings* 32: 446–52.
——. 1944. *What Is Life? The Physical Aspect of the Living Cell*. Cambridge: Cambridge University Press.
——. 1950. "What Is an Elementary Particle?" *Endeavor* 7. Page numbers from the reprinting in *Science, Theory, and Man*. New York: Dover, 1957, 193–223.
——. 1958. *Mind and Matter*. Cambridge: Cambridge University Press.
——. 1964. *My View of the World*. Cecily Hastings, trans. Cambridge: Cambridge University Press, 1964; reprint, Woodbridge, Conn.: Ox Bow Press, 1983. English trans. of *Meine Weltansicht*. Hamburg: P. Zsolnay, 1961.
Seelig, C. 1960. *Albert Einstein. Leben und Werk eines Genies unserer Zeit*. Zurich: Europa Verlag.
Seelig, C., ed. 1956. *Helle Zeit—Dunkle Zeit. In Memoriam Albert Einstein*. Zurich: Europa Verlag.
Stadler, A. 1912. *Kant. Akademische Vorlesungen*. Leipzig: Johann Ambrosius Barth.
Sugimoto, K. 1989. *Albert Einstein: A Photographic Biography*. Barbara Harshav, trans. New York: Schocken.
Thomas Aquinas. 1987. *Faith, Reason, and Theology: Questions I–IV of his Commentary on the "De Trinitate" of Boethius*. Trans. Armand Maurer. Toronto: Pontifical Institute of Medieval Studies, 1987.
Violle, J. 1892–1893. *Lehrbuch der Physik*. E. Gumlich et al., eds. Berlin: Julius Springer.
Voss, A. 1901. "Die Prinzipien der rationellen Mechanik." In *Encyklopädie der mathematischen Wissenschaften, mit Einschluss ihrer Anwendungen*. Vol. 4, *Mechanik*, pt. 1. Felix Klein and Conrad Müller, eds. Leipzig: B. G. Teubner, 1901–1908, 3–121.
Wagner, G. F. 1909. *Encyklopädisches Register zu Schopenhauer's Werken, nebst einem Anhange, der den Abdruck der Dissertation von 1813, Druckfehlerverzeichnisse u.a.m. enthält*. Karlsruhe im Breisgau: G. Braunsche Hofbuchdruckerei und Verlag. Reprinted as *Schopenhauer-Register*. Arthur Hübscher, ed. Stuttgart-Bad Cannstatt: Günther Holzboog, 1960.

Weyl, H. 1923. *Mathematische Analyse des Raumproblems. Vorlesungen gehalten in Barcelona und Madrid.* Berlin: Julius Springer.

——. 1927. *Philosophie der Mathematik und Naturwissenschaft.* Munich and Berlin: R. Oldenbourg. Page numbers and translations from Weyl 1949.

——. 1949. *Philosophy of Mathematics and Natural Science.* Olaf Helmer, trans. Princeton: Princeton University Press; reprint, New York: Athenaeum, 1963.

Wiley, T. E. 1978. *Back to Kant: The Revival of Kantianism in German Social and Historical Thought, 1860–1914.* Detroit: Wayne State University Press.