Math 526 – Algebraic Geometry
Homework # 2
Due: Tuesday, September 10, 2013 8:30 am

Problem 1. *Suppose that $V \subset \mathbb{A}^n(k)$ is a variety.*

a. *Show that* $\underbrace{V \times \cdots \times V}_{j \text{ times}} = \{(v_1, \ldots, v_j) \mid v_i \in V\} \subset \underbrace{\mathbb{A}^n(k) \times \cdots \times \mathbb{A}^n(k)}_{j \text{ times}}$ *is a variety.*

b. *Show that* $\Delta_V = \{(v, \ldots, v) \mid v \in V\} \subset \underbrace{\mathbb{A}^n(k) \times \cdots \times \mathbb{A}^n(k)}_{j \text{ times}}$ *is a variety.*

Problem 2. *Compute $I(\ell_1 \cup \ell_2 \cup \ell_3) \subset \mathbb{R}[x, y, z]$ where*

$$\ell_1 = \{(t, 0, 0) \mid t \in \mathbb{R}\},\ \ell_2 = \{(0, t, 0) \mid t \in \mathbb{R}\},\ \text{and } \ell_3 = \{(0, 0, t) \mid t \in \mathbb{R}\}.$$

Problem 3. *Show there is a unique monomial order on $\mathbb{C}[x]$.*

Problem 4. *For the polynomial ring $k[x_1, \ldots, x_n]$ and $w \in \mathbb{R}^n$, consider the ordering $>_w$ defined by $x^\alpha >_w x^\beta$ if and only if $w(\alpha) > w(\beta)$ where*

$$w(\gamma) = w_1\gamma_1 + \cdots + w_n\gamma_n.$$

a. *If $n = 2$, $w_1 = 3$, and $w_2 = 7$, is $>_w$ a monomial order?*
b. *If $n = 2$, $w_1 = 1$, and $w_2 = \pi$, is $>_w$ a monomial order?*
c. *Develop and prove necessary and sufficient conditions on w such that $>_w$ is a monomial order.*