Math 526 – Algebraic Geometry
Homework # 3
Due: Thursday, September 19, 2013 8:30 am

Problem 1. *On $k[x, y, z]$, let $>$ be the graded lexicographic ordering defined by $x > y > z$. Apply the division algorithm to $g = x^2y^2 + x^3 - x^2y - x^2z - yz + x$ given the following collection of polynomials:*

a. $\{x^2y - z, xy - 1\}$,
b. $\{x - z, xy - 1\}$,
c. $\{x - z, yz - 1\}$.

Even though each collection generates the same ideal (which contains g), explain why your remainders are not all zero.

Problem 2. *Let $F = \{x^3 - 2xy, x^2y - 2y^2 + x, x^2, xy, 2y^2 - x\} \subset k[x, y]$ and $I = \langle F \rangle$. Consider the standard graded lexicographic ordering ($x > y$).*

a. *Use Buchburger's criterion to show that F is a Gröbner basis for I.*
b. *Compute a minimal generating set of $LT(I)$.*
c. *Compute the set of standard monomials of I.*
d. *Compute the normal form of $g = 3xy^2 + 2xy - 4y^2 + 3x - y + 2$.*

Problem 3. *For $I = \langle x^2 - y, x^3 - z \rangle \subset k[x, y, z]$, use Buchburger's algorithm to compute a Gröbner basis*

a. *with respect to the standard lexicographic ordering ($x > y > z$),*
b. *with respect to the lexicographic ordering defined by $y > z > x$,*
c. *with respect to the standard graded lexicographic ordering,*
d. *with respect to the standard graded reverse lexicographic ordering.*