Biologically-inspired Linkage Design

Computing Form From Function

By Jonathan D. Hauenstein and J. Michael McCarthy

For several decades, engineers have been designing robots resembling the general structure of a human, called humanoid robots. In response to the nuclear disaster at Fukushima, Japan in 2011, the DARPA Robotics Challenge was created to accelerate the development of robots that could perform tasks in areas too dangerous for humans. A total of 23 teams participated in the finals of this challenge in June 2015, which was won, along with a $2 million prize, by the humanoid DRC-HUBO (derived from "HUmanoid roBOt") from Team KAIST (Daejeon, Republic of Korea). Coincidentally, the National Institute for Mathematical Sciences (NIMS), also in Daejeon, hosted the SIAM Conference on Applied Algebraic Geometry in August 2015. Although this Daejeon commonality is fortuitous, there is a long and storied history of the relationship between algebraic geometry and mechanical engineering, particularly in the area of kinematics, which studies the motion of systems such as a robotic arm.

The arm of the DRC-HUBO robot consists of four linkages coupled together: a hand linkage, a wrist linkage, an elbow linkage, and a shoulder linkage. In contrast to Rube Goldberg machines, biologically-inspired linkages should be minimalist based on the function required and thus follow an "Occam's razor" mentality. That is, the form should be derived to yield the functionality required in a simple way, as unnecessarily complicated designs increase the chances of failure.

In addition to mimicking human motion, linkages may also be inspired by other organisms. One such example is the elegance of a bird flapping its wings. The development of this linkage [2], which received the A.T. Yang Memorial Award in Theoretical Kinematics at the ASME 2015 International Design Engineering Technical Conferences, provides an excellent case study of the symbiotic relationship between mechanism design and numerical algebraic geometry, the area of algebraic geometry that designs and implements numerical algorithms for solving and manipulating solution sets to systems of polynomial equations. *Numerically Solving Polynomial Systems with Bertini*, published by SIAM [1], provides an overview of numerical algebraic geometry with a focus on using the software Bertini.

This bird-flapping linkage, as shown in Figure 1 and in the video (http://y2u.be/7aXmze9Ynis), was designed to take a constant rotational input from a motor. From this functionality, the general form of the linkage began to take shape, in this case, four coordinated six-bar linkages. Figure 2 (left) shows a schematic drawing of a special kind of six-bar linkage used in the bird-flapping linkage, called a Stephenson II six-bar linkage.

Working with a family of four coordinated six-bar linkages, the next step for the engineer is to synthesize the mechanism, that is, find a member of the family that accomplishes specific tasks. Starting with data from the flight of black-billed magpies and pigeons [3] (see Figure 2 (right)), a set of tasks was developed to impose constraints on the family. In this case, the constraints are represented as polynomial equations that must be satisfied, thereby providing the "link" between mechanical engineering and algebraic geometry. This is commonly referred to as algebraic kinematics [4].

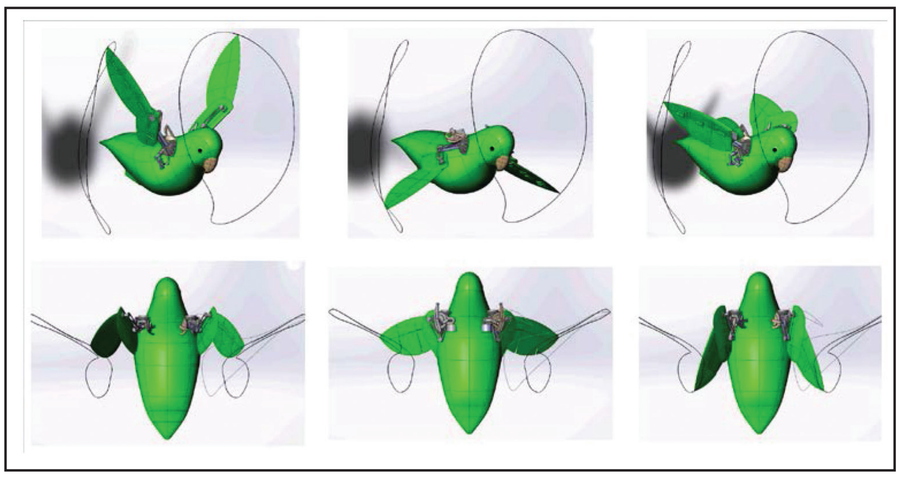

Figure 1. *Four coordinated six-bar linkages designed to take a constant rotational input and transform it to angular outputs that match the wing movement of a bird.*

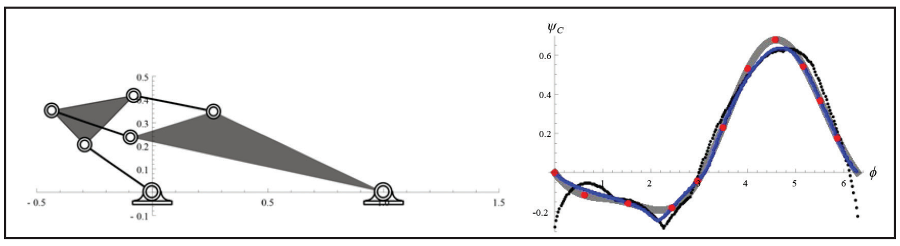

Figure 2. *At left, a Stephenson II six-bar linkage designed to transform uniform rotation to a specific joint angle function. At right, comparing the angle of the elbow joint of a bird-wing [3] (black dots), fitted Fourier function (gray), and the function obtained from the synthesized six-bar linkage (blue) passing through 8 of the 11 task points (red dots). Both figures reprinted from [2] with permission.*

One common synthesis approach is to specify task points where the corresponding curve generated by the linkage, called a coupler curve, must pass through. To yield finitely many six-bar linkages, 15 task points are needed, yielding a system of 154 quadratic equations in 154 variables. Since computing all solutions of this polynomial system was beyond the current reach of available computing resources, a simplification was made to produce a polynomial system consisting of 70 quadratic equations in 70 variables specifying up to 11 task points. Although such a polynomial system could have $2^{70} \approx 10^{21}$ isolated solutions, algebraic geometry in the form of the multihomogeneous Bézout theorem shows that there can be at most roughly 3×10^8 isolated solutions. Solving such a system is within reach and all of its isolated solutions were obtained using Bertini running for over 300 hours on a cluster consisting of 256 processing cores. After solving this once, numerical algebraic geometry via a parameter homotopy permits one to solve other synthesis problems on this family of mechanisms in 2 hours on a typical desktop workstation or under 30 seconds on the cluster described above.

After computing the full solution set, in this case consisting of over 1.5 million linkages, the engineer culls through to locate a linkage that suitably accomplishes the prescribed function. That is, selecting a linkage that is free of branch and circuit defects and evaluating other transmission characteristics. In the case of the bird-flapping linkage, this produced 7,363 candidate designs, one of which is shown in Figure 1, which passes through 8 of the 11 selected task points as seen in Figure 2 (right). This is a rich haul for the designer to select a proper linkage, particularly since current optimization methods yield a single design.

This synthesis calculation of the form of a six-bar linkage to achieve a given function was simply not possible before, and demonstrates the opportunities for innovation that can arise from the collaboration of numerical algebraic geometry and mechanical design.

References

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Jonathan D. Hauenstein is an associate professor in the Department of Applied and Computational Mathematics and Statistics at the University of Notre Dame. J. Michael McCarthy is a professor in the Department of Mechanical and Aerospace Engineering at the University of California, Irvine.