Problems covered in recitation on Monday, Oct 6, 2003

October 8, 2003

On Monday, we talked about Lagrange multipliers. We also did the following problems.

Problem 1 Let $w(x, y) = x^3y^2$. Find a parametric equation for the line tangent to the curve $w = 1$ at the point $(1, 1)$. Use the gradiant.

Solution We compute

$$\nabla w = \langle 3x^2y^2, 2x^3y \rangle$$

so we have $\nabla w(1, 1) = \langle 3, 2 \rangle$. Now, the curve $w = 1$ is a level curve for w, and we know that ∇w is perpendicular to level curves. So if we rotate ∇w by $90°$, we get a vector tangent to the curve. This rotated vector is

$$\nabla w(1, 1)^{\perp} = \langle -2, 3 \rangle.$$

A parametric equation for a line passing through $(1, 1)$ in the direction of $\langle -2, 3 \rangle$ is given by

$$\begin{aligned} x(t) &= -2t + 1 \\ y(t) &= 3t + 1. \end{aligned}$$

Problem 2 Find the point of the plane $x + 2y + 3z = 1$ closest to the origin.

Solution We use Lagrange multipliers. Our constraint equation is $g(x, y, z) = 1$ where

$$g(x, y, z) = x + 2y + 3z.$$

We want to minimize distance to the origin, which is really the same as minimizing the square of the distance (get rid of that nasty square root). So the function we want to minimize is

$$f(x, y, z) = x^2 + y^2 + z^2.$$

Now we do Lagrange multipliers. We look for solutions to

$$\nabla f = \lambda \nabla g.$$

We compute

$$\begin{aligned} \nabla f &= \langle 2x, 2y, 2z \rangle \\ \lambda \nabla g &= \lambda \langle 1, 2, 3 \rangle \end{aligned}$$

Comparing the components of these vectors, we see that we have a system of equations

$$\begin{aligned} 2x &= \lambda \\ 2y &= 2\lambda \\ 2z &= 3\lambda \end{aligned}$$

Of course, we also have the constraint equation $x + 2y + 3z = 1$. Solving for x, y, and z in terms of λ, we get $x = \lambda/2$, $y = \lambda$, and $z = 3\lambda/2$. Plugging into the constraint equation, we see that

$$\lambda/2 + 2\lambda + 9\lambda/2 = 7\lambda = 1$$

so $\lambda = 1/7$. Therefore,

$$\begin{aligned} x &= 1/14 \\ y &= 1/7 \\ z &= 3/14. \end{aligned}$$