Hw 11

Note Title 5/4/2009

20.1 (1) [6 points]

Suppose $a < x' < x'' < b$

Since F' is continuous, it is int'ble on $[x', x'']$

(i) If $F' \geq 0$

$$\underbrace{\int_{x'}^{x''} F'(x)\,dx}_{F(x'') - F(x')} \geq 0 \qquad \text{(comparison thm)}$$

1st fund thm of calc

$\Rightarrow \quad F(x'') \geq F(x')$

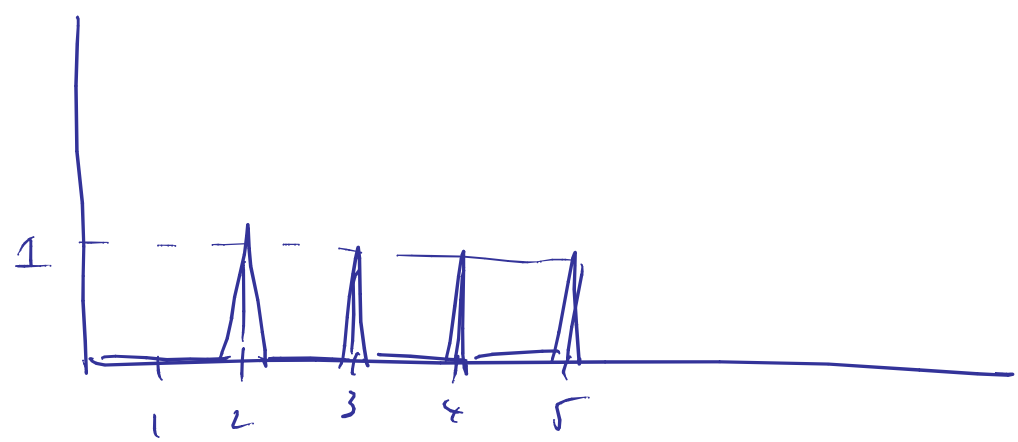
(ii) Since F' is continuous on $[x', x'']$, the minimum thm $\Rightarrow$ F' achieves its minimum $m = \min_{[x', x'']} F'(x)$

Note Title 5/12/2009

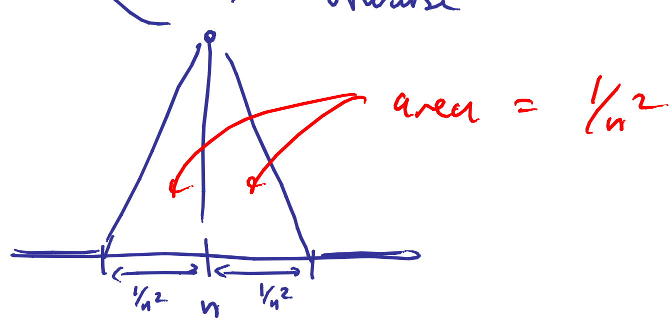
21.2 (4) [5 points]

4.) It is not true!

Consider the function:



$$f(x) = \begin{cases} n^2\left(x - \left(n - \frac{1}{n^2}\right)\right), & x \in [n - 1/n^2, n) \\ & n \geq 2 \\ 1 - n^2(x - n), & x \in [n, n + 1/n^2] \\ & n \geq 2 \\ 0, & \text{otherwise} \end{cases}$$



Then $\displaystyle\int_0^\infty f(x)\,dx = \sum_{n=2}^{\infty} \frac{1}{n^2}$

converges

Yet $\displaystyle\lim_{n\to\infty} f(x)$ does not exist.

22.1 (1a) [5 points]

(a) If $x = 0$

$$\frac{1}{1+nx} = 0 \xrightarrow[n\to\infty]{} 0$$

If $x > 0$

$$0 \le \frac{x}{1+nx} = \frac{1}{\frac{1}{x}+n} < \frac{1}{n} \to 0$$

$$\Longrightarrow \frac{1}{\frac{1}{x}+n} \to 0 \quad \text{(algebraic limit thm, } \infty\text{-version)}$$

Squeeze thm

The sequence converges uniformly.

Pick $\varepsilon > 0$. Suppose $n > 1/\varepsilon$

Then $$\left|\frac{x}{1+nx}\right| = \begin{cases} 0 \ , & x = 0 \quad (\mathrm{I}) \\ \dfrac{1}{\frac{1}{x}+n} \ , & x > 0 . \quad (\mathrm{II}) \end{cases}$$

In case I, $\dfrac{x}{1+nx} \approx_\varepsilon 0$

In case II,

$$\left|\frac{x}{1+nx}\right| = \frac{1}{\frac{1}{x}+n} < \frac{1}{n} < \varepsilon$$

So $\dfrac{x}{1+nx} \approx_\varepsilon 0$

$\square$

22.1 (3) [5 points]

Pick $\varepsilon > 0$

Let $S_n(x)$ be the n^{th} partial sum.

Clearly $\quad S_n(x) \longrightarrow g(x)$

$$g(x) = \begin{cases} 0, & x < 0 \\ 1, & x \geq 1 \end{cases}$$

<u>Suppose</u>: $\quad n > R$.

Then $\quad S_n(x) = g(x) \quad$ for $\quad x \in [-R, R]$

$$\Rightarrow \quad S_n(x) \approx_\varepsilon g(x) \quad \text{for} \quad x \in [-R, R]$$

$$\Rightarrow \quad S_n(x) \rightrightarrows g(x) \quad \text{on} \quad [-R, R]$$

$\Rightarrow \quad \sum_i u_k(x)$ converges uniformly on $[-R, R]$

<u>However</u>: Suppose $\sum_i u_k(x)$ converges uniformly to $g(x)$ on $(-\infty, \infty)$

Then, for $\varepsilon = 1/2$, there $\exists$ an n such that $\quad S_n(x) \approx_{1/2} g(x)$ for all x.

Take $x > n+1$

then $S_n(x) = 0$

$g(x) = 1$

and $S_n(x) \not\to_{1/2} g(x)$ $\rightarrow\!\!\!\!\times$

So $S_n(x) \not\rightrightarrows g(x)$, on $(-\infty, \infty)$

$\Rightarrow$ $\sum_1^\infty u_k$ does not converge uniformly on $(-\infty, \infty)$

22.2 (2a) [4 pts]

$$\left| \frac{n}{x+n} - 1 \right| = \left| \frac{x}{x+n} \right|$$

$$= \begin{cases} 0, & x = 0 \\ \frac{1}{1+\frac{n}{x}}, & x > 0 \end{cases}$$

In either case,

$$\left|\frac{n}{x+n} - 1\right| < \frac{x}{n} \underset{\substack{\uparrow \\ x \le R}}{\le} \frac{R}{n} = B_n$$

but $B_n \to 0$.

By the elementary criterion for uniform convergence,

$$\frac{n}{x+n} \rightrightarrows 1$$

22.2 (2d) [4 points]

$$\left|\frac{\sin(nx)}{x^2+n^2}\right| \le \frac{1}{x^2+n^2} \le \frac{1}{n^2} = M_n$$

$\sum M_n$ converges.

$\underset{\text{Weierstrass M-test}}{\Longrightarrow}$ $\sum \frac{\sin nx}{x^2+n^2}$ converges uniformly.